Matthias Ludwig, Simone Jablonski,
Amélia Caldeira and Ana Moura
(Editors)

Research on Outdoor STEM Education in the digiTal Age

Proceedings of the ROSETA Online Conference
in June 2020

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RESEARCH ON OUTDOOR STEM EDUCATION IN THE DIGITAL AGE – BACKGROUND AND INTRODUCTION

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"*Can we have class outside?*" Teaching and learning outside the classroom – what sounds like a student's dream, becomes more relevant in every days' classes. Leaving the classroom provides new perspectives on subjects and connects the school contents with real world problems (Karpa, Lübbecke, & Adam, 2015). Even though learning is not limited to the school building, teaching outdoors requires additional considerations and organization, which naturally provokes the question of its added value. Different efforts have been made in order to provide guidelines, templates, ideas and research results – summarized under the heading of "outdoor education". In the big number of new science centers which are created for learning outside the school, we see that there is a need for a new learning format.

Especially the combination of outdoor education with digital tools seems contemporary and brings benefits for students and teachers. Technology allows new dimensions of mobility which outdoor education asks for. Under the name "mobile learning", digital tools are used to enable learning in different contexts and independent from a location. Simultaneously, technology can assist students in their outdoor learning activities, either through guidance or feedback.

With both topics being relevant for different subjects, it has been our intention to especially disseminate outdoor education with digital tools primarily for mathematics education through math trails. A math trail is a walk where one can discuss and solve mathematical tasks (Shoaf, Pollack, & Schneider, 2004). Originally, math trails were created for tourists and families. Nevertheless, their idea suits the claim for authentic mathematics education. In contrast to mathematics tasks in school books, mathematics can be discovered in authentic, relevant contexts with real objects. By using math trails in the educational context, new challenges arise. Especially the preparation and organization of math trails can be challenging and time-consuming. By means of digital tools, these challenges can be met. With this intention, the co-funded Erasmus+ project "Mobile Math Trails in Europe (MoMaTrE)" started in 2017 with a project duration of three years. The consortium contains seven partners (see also Figure 1 and 2):

- Goethe University Frankfurt (Germany)
- Autentek GmbH Berlin (Germany)
- University Claude Bernard Lyon 1 (France)
- School of Engineering, Polytechnic of Porto (Portugal)
- Institute of Systems Engineering and Computers, Research and Development, INESC-ID (Portugal)
- University Constantin the Philosopher Nitra (Slovakia)
- Federation of Mathematics Teachers Societies (Spain)

Ludwig, M., Jablonski, S., Caldeira, A. & Moura, A. (2020). Research on Outdoor STEM Education in the Digital Age – Background and Introduction. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 5-9). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.01

Figure 1: The Location of all the MoMaTrE Partners in Europe.

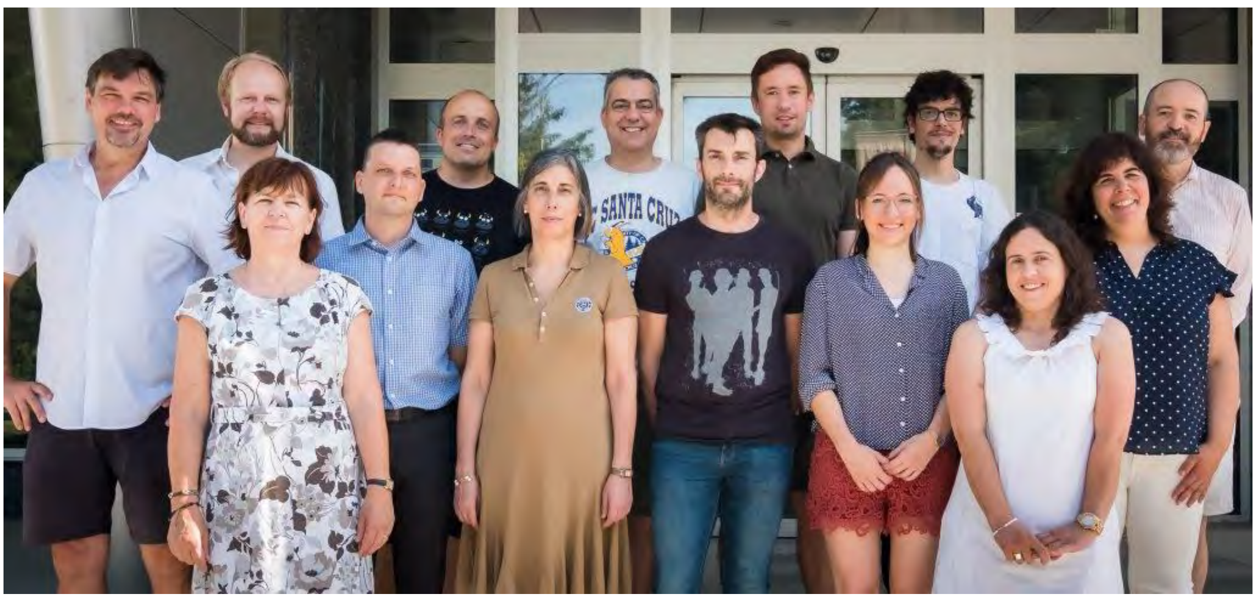

Figure 2: The MoMaTrE Consortium during the third Project Meeting in Nitra, Slovakia (from left to right: Matthias Ludwig, Goethe University Frankfurt, Johannes Scheerer, autentek, Sona Ceretkova, University Constantin the Philosopher Nitra, Radoslav Omelka, University Constantin the Philosopher Nitra, Imrich Jakab, University Constantin the Philosopher Nitra, Claudia Lázaro, Federation of Mathematics Teachers Societies, Pedro Santos, Institute of Systems Engineering and Computers, Research and Development, Bienvenido Espinar, Federation of Mathematics Teachers Societies, Iwan Gurjanow, Goethe University Frankfurt, Simone Jablonski, Goethe University Frankfurt, Miguel Ferreira, autentek, Ana Moura, School of Engineering, Polytechnic of Porto, Amélia Caldeira, School of Engineering, Polytechnic of Porto, Christian Mercat, University Claude Bernard Lyon 1).

The main intention of MoMaTrE is to support and educate teachers in conducting outdoor mathematics activities. Therefore, the project is structured in Intellectual Outputs on different levels:

- Technical Outputs

  Mobile Application: The mobile application "MathCityMap" for navigation and running math trails on the one hand, and for creating math trails on the other hand is developed and improved.

  Web Portal: An interactive web portal which provides authoring tools to create math trails is developed. The web portal should further allow interaction between users in order to create a community in which the users share their work.

- Material

  Generic Tasks: The consortium develops a catalogue of generic tasks, which is a collection of tasks ideas that can be found frequently outside. These tasks can easily be adapted to different locations and support the interactive web portal with numerous ideas for mathematical problems.

- Education

  Long-Term Curriculum: A long-term curriculum for a university course is developed and evaluated. It educates students of mathematics education in their outdoor teaching and is accredited with 3 ECTS for Erasmus students amongst the partner universities.

During the project, the ideas and advantages of outdoor mathematics are disseminated through research, workshops, articles and events. One of these events is the "Research on Outdoor STEM Education in the digiTal Age (ROSETA)" conference. With the ROSETA conference, the previously discussed relevance of outdoor learning and digital learning should not be limited to mathematics and math trails, but extended to further subjects, namely Science, Technology, Engineering and Mathematics (STEM).

STEM education has always been essential to the wealth of the mankind. Nowadays, there are many initiatives on integrating outdoor learning and digital technologies in STEM classes.

With the ROSETA conference, we intend to make these initiatives visible by

- receiving and sharing inputs from the STEM community,
- connecting different approaches from the community,
- sharing outdoor learning experiences,
- disseminating the MoMaTrE intellectual outputs and the MathCityMap system to the scientific and educational community.

Originally being scheduled in Porto from 16th to 19th June 2020, the conference could not take place physically due to the Corona pandemic. So the conference had to become true to its name and use the benefits of the "digital age" that allowed a virtual exchange of experiences and research results.

It is our pleasure to present and combine these results in the proceedings of the ROSETA conference. The proceedings contain 27 articles by 50 authors from eleven different nations. Being structure in three invited papers, 19 papers and five poster presentations, the authors present their research, experiences and ideas on outdoor learning and digital tools.

In "Contextualizing STEM Learning: Frameworks & Strategies", Helen Crompton gives an overview on an effective use of technology in STEM education by presenting different frameworks and strategies.

Through the influence and the context of the MoMaTrE project, a major focus of the conference are digital tools that can be used for outdoor mathematics. Especially the math trail idea is presented by the use of the digital tools

- MathCityMap, e.g. in "MathCityMap – Popularizing Mathematics around the Globe with Math Trails and Smartphone" by Iwan Gurjanow, Joerg Zender and Matthias Ludwig,
- Actionbound, in "The Norwegian Study Math & The City on Mobile Learning with Math Trails" by Nils Buchholtz,
- LabStar™ in "Mathematical Modelling in STEM Education: A Math Trail using LabStar™" by Defne Yabas, Hayriye Sinem Boyacı and M. Sencer Corlu.

Further, the potential and use of Augmented Reality is evaluated, either in connection to math trails and/or with GeoGebra 3D as in "Disocvering Everyday Mathematical Situations Outside the Classroom with MathCityMap and GeoGebra 3D" by Zsolt Lavicza, Ben Haas and Yves Kreis.

Another focus of the proceedings are considerations for the task design and experiences on outdoor learning tasks. On the one hand, this includes theoretical and exemplary perspectives on task design as in "Some Remarks on 'Good' Tasks in Mathematical Outdoor Activities" by Philipp Ullmann. On the other hand, evaluated outdoor tasks and learning environments are presented for

- Chemistry, e.g. "Developing and Assessing E-Learning Settings by Digital Technologies" by Christiane S. Reiners, Laurence Schmitz and Stefan Mueller,
- Technology and Engineering, e.g. in "Teaching Privacy Outdoors – First Approaches in the Field in Connection with STEM Education" by Sandra Schulz,
- Mathematics, e.g. in "Learning Math Outdoors: Graph Theory using Maps" by Aaron Gaio, Laura Branchetti and Roberto Capone.

The proceedings contain ideas for the curriculum design and the education of university students and (pre-service) teachers as in "Math Trails through Digital Technology: An Experience with Pre-Service Teachers" by Ana Barbosa and Isabel Vale.

With this international collaboration, it is our intention to contribute to the current and prospective research in the Outdoor STEM Education in the Digital Age.

Acknowledgment

The strategic partnership "Mobile Math Trails in Europe" and the "Research on Outdoor STEM Education in the digiTal Age Conference" as one of its Multiplier Events have been co-funded by the European Union. It is part of the Erasmus+ Programme, Key Action 2 – Strategic Partnerships under the number: 2017-1-DE01-KA203-003577. The consortium thanks especially the German national agency (DAAD) for the financial and advising support during the last three years.

References

Karpa, D., Lübbecke, G., & Adam, B. (2015). Außerschulische Lernorte. Theoretische Grundlagen und praktische Beispiele [Extracurricular Learning. Theoretical background and practical examples]. *Schulpädagogik heute*, *6*(11), 1-13.

Shoaf, M. M., Pollak, H., & Schneider, J. (2004). *Math Trails.* The Consortium for Mathematics and Its Applications (COMAP).

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INVITED PAPERS

CONTEXTUALIZING STEM LEARNING: FRAMEWORKS & STRATEGIES

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Abstract. *STEM studies provide crucial knowledge and understandings for students to perform tasks as part of daily life. However, there are issues with regards to students showing a lack of interest, persistence, and fail to see the relevance of STEM concepts. Empirical evident show that contextualized outdoor learning approaches can connect students to STEM learning in a meaningful way increasing interest, motivation, and relevance to all students. This paper highlights the factors involved in effectively using technologies for outdoor STEM learning. This includes an examination of the technology integration frameworks of the Technological, Pedagogical, Content Knowledge (TPACK) framework, and the Mlearning integration ecological framework. Followed by an examination and examples of outdoor contextualized technological pedagogies, including mobile learning, situated learning, authentic learning, outdoor experiential learning, and context-aware ubiquitous learning.*

INTRODUCTION

Science, Technology, Engineering, & Mathematics (STEM) are a collective group of subjects that each involve a network of complex systems, theories, and axiomatic concepts. While having an understanding of STEM concepts are critical in providing students with an awareness of modern society, fundamental concepts from each area, and application fluency to perform tasks relevant to daily life (NAE & NRC, 2014), students often show a lack of persistence, and interest in STEM studies (Koul, Lerdpornkulrat, & Chantara, 2011; Vedder-Weiss & Fortus, 2011). STEM enrollments are low, dropouts are high, and the general STEM pipeline of students has been highlighted as an issue that needs to be addressed (Cannady, Greenwald, & Harris, 2014; van den Hurk, Meelissen, & van Langen 2019).

These lack of interest and the discrepancies in STEM are often connected to curricula (PCAST, 2010), stereotype, implicit bias (McGee, 2013; McGee, & Pearman, 2015), social dynamics, students feeling disconnected from STEM (Perry, & Morris, 2014; DeNisco, 2015), and students not understanding STEM relevance (Habiba & Odis, 2019). Empirical evident show that context based outdoor learning approaches can connect students to STEM learning in a meaningful way increasing interest (Swirski, Baram-Tsabari & Yarden, 2018), motivation (Pilot & Bulte, 2006), and relevance to all students (Eliks & Hofstein, 2015).

CONTEXTUALIZED OUTDOOR LEARNING

STEM concepts are often presented to students in abstract forms, rather than contextualized to provide meaning to students. For example, angles are often presented as lines on paper, rather than contextualized as angles on window frames, door ways and many other architectural forms recognizable to students, and this can cause misconceptions, and errors in understanding the true nature of the content knowledge (Crompton, 2015a; b; 2017a). Scholars have advocated for students to learn by connecting to the physical phenomena to provide meaning to these abstract concepts.

Crompton, H. (2020). Contextualizing STEM Learning: Frameworks & Strategies. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp.13-22). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.02

Contextualizing outdoor learning is not a new idea. For many years, scholars, such as Dewey (1916), Mumford (1946), and Orr (1992) have advocated for locally situated, culturally and environmentally informed pedagogies. Dewey in particular is known from his early work that he wrote about the need for curriculum to have "real-life" relevancy (Dewey, 1902, 1938). Outdoor contextualized learning is a pedagogical method of bridging the gap to difficult STEM concepts, and the world we live in, to make learning more meaningful (Kortland, 2007). It is not just the "place", but place has a locus of shared social values and norms (Williams, 2014), place is not must equivalent to location (Semken, Ward, Moosavi & Chinn, 2017). Pedagogies connecting students to real-world contexts, especially those connected to students' own knowledge and experiences can promote conceptual understanding and motivation to learn (Sugimoto, Turner & Stoehr, 2017).

Understanding best practices for teaching STEM in contextualizing learning is important, the next step is supporting educators in facilitating this pedagogical approach. Scholars have uncovered a variety of difficulties educators can encounter in trying to incorporate relevant real-world contexts into learning (e.g. Sugimoto et al., 2017). Within academia and practicing teachers, a plethora of evidence show that technology is a tool that extends and enhances in the process of contextualizing learning. These findings will be explored in the following section.

TECHNOLOGY TO SUPPORT CONTEXTUALIZED LEARNING

Empirical findings reveal that the incorporation of technology into learning activities provides new ways of teaching and learning, which are often more hands-on, active learning approaches, improving student focus and understanding, (Alijwarneh, Radhakrishna & Cheruvu, 2017). Technological tools are especially beneficial for students learning outdoors in authentic environments (Blackburn, 2017, Hwang & Chen, 2017; Liu, Chen, & Hwang 2018).

Technologies for Contextualizing Learning

With the advent of portable technologies, students no longer need to be tethered by the use of technologies plugged into electrical sockets, such as desktop computers. Mobile phones, then the expanded societal use of tablets in 2010, provided learners with Internet connected technologies that are easily portable and can be used across spatial and temporal domains (Crompton, 2015c). Systematic reviews of the literature show a trend towards the use of mobile devices to facilitate contextualize outdoor learning in science (Crompton, Burke, Gregory & Gräbe, 2016), mathematics (Crompton & Burke, 2015; 2014), STEM subjects (Crompton & Burke, 2018; Crompton, Burke, & Gregory, 2017) and reveal extended opportunities for learning outdoors (e.g. Crompton, Burke & Lin, 2019; Crompton, & Burke 2018).

Technology Integration Frameworks

When incorporating mobile devices into learning, a variety of educational components need to be considered, such as the device, how the device is used, curriculum, policies, technical support, infrastructure. To highlight these factors Crompton, (2017b) developed the Mlearning integration ecological framework, (see Figure 1).

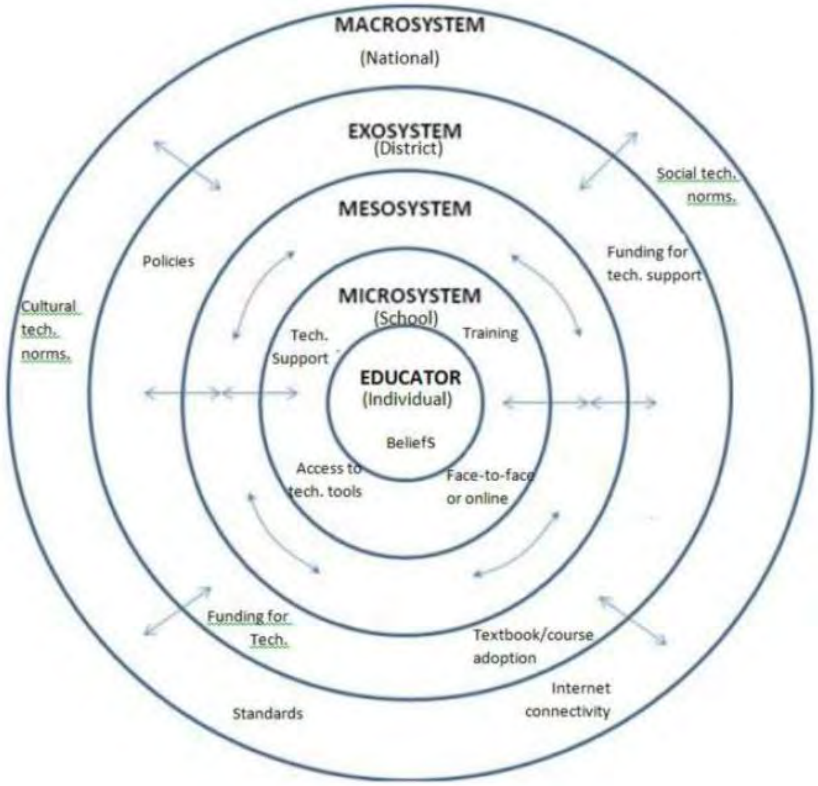

Figure 1: Mlearning integration ecological framework (Crompton, 2017).

This framework is based on Bronfenbrenner's (1979) Ecological Framework, that has the child at the center of the framework and describes ecological influences on the child. Crompton's framework places the educator at the center of the framework and the concentric circles represent the different systems influencing how the educator integrates technology. The ecological influences include the social ecology of interactions with people and ecology relating to environmental factors of the physical environment.

The educator in the center is influenced by their own beliefs on mobile devices, such as their effectiveness for learning and methods of use. The microsystem represents the school and includes factors such as training, access to technology, and modality of teaching. The exosystem is the district, including policies and funding, and the macrosystem is the national level that involves influences, such as social, and cultural technology norms, and Internet connectivity. The mesosystem in the middle of the frameworks with arrows pointing different directions, highlight that a factor in one area may also be present in other areas. For example, policies are included in the exosystem (district level) and policies may also be at the school and national level.

As educators are considering outdoor education, it is important to first start with the educator in the center. The beliefs of the educator in the efficacy of the outdoor learning approach and the use of technology can ensure the activity is a success or a failure as their actions will often hinge on these beliefs. At the microsystem – school, devices are needed, technology support to ensure the devices are working effectively out of school, and training on what pedagogies would be effective in mobile assisted outdoor educational activities. The various support factors need to be in place across the school, district, and national level. Teacher education on how to use mobile devices for outdoor learning is important.

Other technology integration frameworks were developed to support educators in integrating technology into teaching and learning, such as the technological, pedagogical and content knowledge (TPACK) framework (Mishra and Koehler, 2006). Based on Shulman's (1986) model, the TPACK framework highlights the three knowledge groups that educators have as separate entities (technology, pedagogy, and content knowledge), then how the three should be considered working together to be effective (see Figure 2).

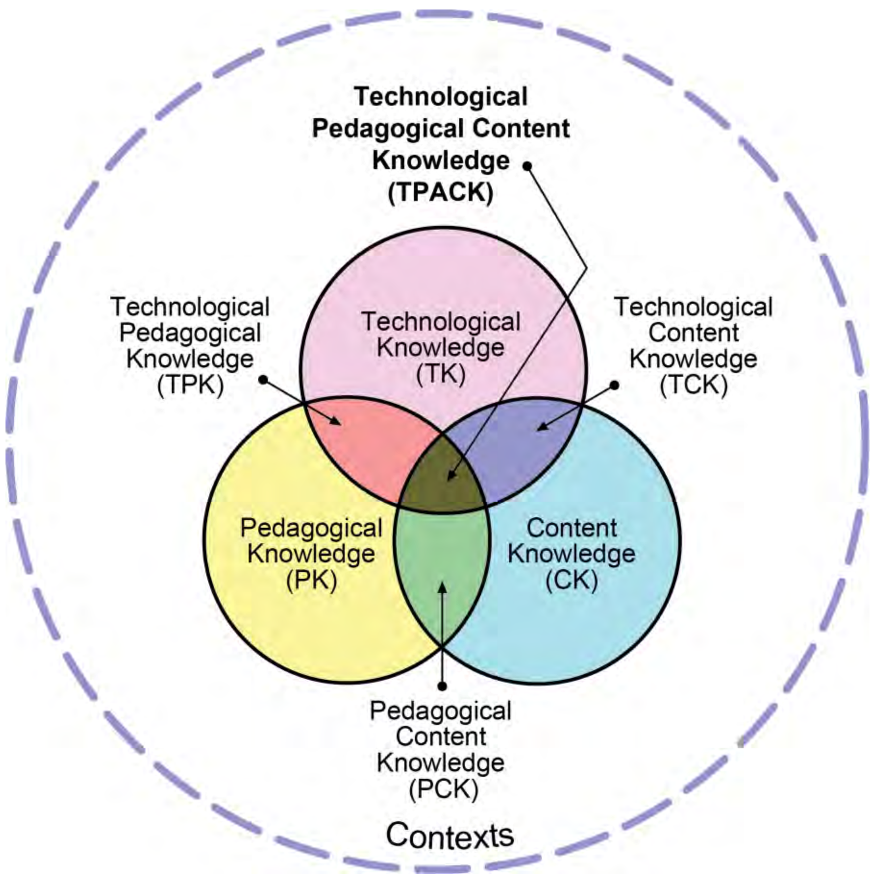

Figure 2: TPACK framework (Mishra and Koehler, 2006).

This framework has the educator thinking about the content knowledge they are to teach. In this case what STEM concepts the student is to learn. Then the educator thinks about the best pedagogy to teach that content knowledge and the technologies that can support in that activity. Pedagogy is the Greek word meaning "to lead the child". It is how you organize learning: is it with the students working individually or in pairs; are the students learning outdoors, or in the classroom; are they using technologies, manipulatives, or other tools. The context circle around the Venn diagram reminds the educator to think about other aspects such as the ages and interests of students.

The content knowledge in this paper is focused on STEM with topics such as angles, place value, ecosystems etc. As discussed earlier, these concepts are often presented in abstract form on paper in textbooks with little to no concrete connection to the students real-world that allows them to gain a firm understanding of the topic. Therefore, this paper advocates for the pedagogical choices that the educator would make are those making that concrete connection with outdoor learning. This outdoor education would use those mobile devices to extend and enhance learning. These three decisions on the content, pedagogy, and technology working effectively together are the center point of Figure 2 as TPACK. While this framework for technology integration may be helpful for the educator in thinking about bringing together the three aspects, further support is needed in what type of pedagogies work well outdoors with the use of technology (Sugimoto et al., 2017).

Outdoor Contextualized Technological Pedagogies

Scholars have connected learning theories to the use of contextualize learning as students solve problems within information-rich settings, such as discovery learning, problem based learning, inquiry learning, experiential learning and constructivist learning (Cheng, Hwang & Chen, 2019) and using mobile devices (Sharples, Taylor & Vavoula, 2016), with personalized, learner centered, situated, collaborative, ubiquitous, lifelong learning.

Pedagogies and technology can work effectively together for outdoor contextualized learning (Blackburn, 2017, Hwang & Chen, 2017). There are a variety of contextualized outdoor pedagogical approaches that use technology and these terms have changed over time as they have following trends in pedagogy and the emergence of new digital technologies (Crompton, 2013a; 2015). Following the authors review of recent literature, the outdoor contextualized technological pedagogies include *mobile learning, situated learning, authentic learning, outdoor experiential learning, and context-aware ubiquitous learning.* These are explained with examples below. These pedagogies can have many similarities and some names may be used interchangeably.

Mobile learning by definition is “learning across multiple contexts, through social and content interactions, using personal electronic devices” (Crompton 2013b, p. 4). This highlights the very nature of learners moving across and within contexts. The mobility, ubiquity, and connectivity of mobile devices help to foster meaningful learning experience across contexts unrestricted by environmental restraints. For example, students were tasked with designing an engineering solution for how to water an outdoor classroom garden (Apul, & Philpott, 2011) could use mobile devices to photograph, collect data, and measure distance and angles.

Situated learning technology has been shown to improve learning outcomes and increase student motivation in situated learning environments (Hwang & Chen, 2017). This is learners studying while within an environment relevant to the content knowledge. For example, Pfeiffer, Gemballa, Jarokzka, Scheiter, and Gerjets (2009), had students learn about fish biodiversity via mobile devices in a situated learning scenario. Students received video support from mobile device during a snorkeling activity.

Authentic learning is having the learner connected to authentic environments and involved in practical situations (Chen, Hwang & Tsai, 2014). For example, Fessakis, Karta, and Kozas (2018), that had students learning mathematics in an authentic context. Primary students took part in a math trail. The students were guided through the trail using a digital map and guided to a set of preselected sites of a park where they explored and solved math problems using data from the environmental context.

Outdoor experiential learning is learners obtaining scientific knowledge from the phenomena of conceptualization and transfer of experience (García-Sánchez & Luján-García, 2016). The experiential learning process has four parts, concrete experience, abstract conceptualization, reflective observation and active experimentation (Kolb, 2014). Schnepp and Rogers developed an app that delivered reflection prompts and content before, during, and after an experiential learning activity. This idea could be used across a variety of STEM topics with educators using various tools to deliver information to students when relevant. For example, Chan & Tam (2018) developed an application that enabled students to use an electronic map which tracks the location of the student and highlights locations that the student needs to go to complete tasks. As the student reaches the location of the task they are presented with a connection and investigation scaffold of an authentic task the student has to complete. These are activities that involve, experimentation, data collection, investigation, and reflection using the mobile phone.

Context-aware ubiquitous learning refers to mobile technologies being used while connecting with real world phenomenon (Hwang, Wu & Chen, 2007). For example, Crompton (2014) had students study angles by connecting with their surrounding environment. In the school grounds and playground, students used a mobile application to take photographs of angles and then used a dynamic protractor to measure the angle and discuss the angles with peers while seeing the real-world version and the 2D version on the device.

CONCLUSION

STEM studies are crucial for students to perform tasks as part of daily life (NAE & NRC, 2014). However, there are issues with regards to students showing a lack of interest and persistence (Koul, Lerdpornkulrat & Chantara, 2011), and fail to see the relevance of STEM concepts to their lives (DeNisco 2015). Empirical evident show that contextualized outdoor learning approaches can connect students to STEM learning in a meaningful way increasing interest (Swirski, Baram-Tsabari & Yarden, 2018), motivation (Pilot & Bulte, 2006), and relevance to all students (Eliks & Hofstein, 2015). This paper highlights the factors involved in effectively using technologies for outdoor STEM learning. Technology integration frameworks, such as the TPACK (Mishra & Koehler, 2006) framework, and the Mlearning integration ecological framework Crompton, (2017b) provide educators with a lens to consider how technology can be used to extend and enhance learning outside the classroom.

The mlearning integration ecological framework highlights the many aspects involved in using mobile devices outdoors and the TPACK frameworks presents an overarching consideration of learning as three parts, technology, pedagogy, and content. The pedagogy is then covered as the final part of the three as there are various teaching approaches to contextualize the often abstractly presented concepts in STEM. The pedagogies of mobile learning, situated learning, authentic learning, outdoor experiential learning, and context-aware ubiquitous learning are offered with examples for educators, school leaders, policy makers, and funders to use as a springboard to advocate for outdoor contextualized STEM learning.

References

Aljawarneh, S. A., Radhakrishna, V., & Cheruvu, A. (2017). VRKSHA: a novel tree structure for time profiled temporal association mining. *Neural Computing and Applications,* 1–29.

Apul, D. S., Philpott, S. W., (2011). Use of outdoor living spaces and Fink's taxonomy of significant learning in sustainability engineering education. *Journal of Professional Issues in Engineering Education and Practice 137*(2), 69-77.

Blackburn, G. (2017). A university's strategic adoption process of an PBL-aligned eLearning environment: An exploratory case study. *Educational Technology Research and Development, 65*, 147–176.

Bronfenbrenner, U. (1979). *The Ecology of Human Development: Experiments by Nature and Design*. Harvard University Press, Cambridge, MA.

Cannady, M. A., Greenwald, E., & Harris, K. N. (2014). Problematizing the STEM pipeline metaphor: Is the STEM pipeline metaphor serving our students and the STEM workforce? *Science Education, 98*(3), 443–460. doi:10.1002/sce.21108

Chan, T., & Tam, V. (2018). Gamifying and mobilizing social enquiry-based learning in authentic outdoor environments. *Journal of Educational Technology & Society 21*(4). 277-292.

Chen, C. H., Hwang, G. J., & Tsai, C. H. (2014). A progressive prompting approach to conducting contextual ubiquitous learning activities for natural science courses. *Interacting with Computers, 26*(4), 348–359.

Cheng, S-C., Hwang, G-J., Chen, C-H. (2019). From reflective observation to active learning: A mobile experiential learning approach for environmental science education. *British Journal of Educational Technology 50*(5), 2251-2270.

Crompton, H. (2013a). Mobile learning: New approach, new theory. In Z. L. Berge & L. Y. Muilenburg (Eds.), *Handbook of mobile learning* (pp. 47-57). Florence, KY: Routledge.

Crompton, H. (2013b). A historical overview of mobile learning: Toward learner-centered education. In Z. L. Berge & L. Y. Muilenburg (Eds.), *Handbook of mobile learning* (pp. 3-14). Florence, KY: Routledge.

Crompton, H. (2015a). Using context-aware ubiquitous learning to support students understanding in geometry. *Journal of Interactive Media in Education 1*(13), 1-11,

Crompton, H. (2015b). Understanding angle and angle measure: A design-based research study using context-aware ubiquitous learning. *International Journal for Technology in Mathematics Education 22*(1).

Crompton, H. (2015c). A theory of mobile learning. In B. H. Khan, & M. Ali, *International Handbook of E-Learning Volume 2* (pp. 309-319). Volume 1. Florence, KY: Routledge.

Crompton, H. (2017a). Using mobile learning to support students' Understanding in geometry: A design-based research study. Journal of Education, *Technology, & Society 20*(3), 207- 219.

Crompton, H. (2017b). Moving toward a mobile learning landscape: Presenting a mlearning integration framework. *Interactive Technology and Smart Education 18*(2) 97-109. DOI is 10.1108/ ITSE-02-2017-0018

Crompton, H., & Burke, D. (2014). Review of trends in mobile learning studies in mathematics: A meta-analysis. *Communications in Computer and Information Science*, 479, 304–314.

Crompton, H., & Burke, D. (2015). Research trends in the use of mobile learning in mathematics [Special Issue]. *International Journal of Mobile and Blended Learning 7*(4), 1-15.

Crompton, H., & Burke, D. (2018). The use of mobile learning in higher education: A systematic review, *Computers & Education 123*, 53-64.

Crompton, H., Burke, D., & Gregory, K. (2017). The use of mobile learning in pk-12 education: A Systematic Review. *Computers & Education 110* 51-63.

Crompton, H., Burke, D., & Lin, Y-C. (2019). Mobile learning and student cognition: A systematic review of PK-12 research using Bloom's Taxonomy. *British Journal of Educational Technology 50*(2), 684-701.

Crompton, H., Burke, D., Gregory, K., & Gräbe, C. (2016). The use of mobile learning in science education: A systematic review. *Journal of Science Education and Technology, 25*, 149-160.

DeNisco, A. (2015). School suspensions widen achievement gap. District Administration Leadership Institute. Issue: November 2015 Report.

Dewey, J. (1902). *The child and the curriculum*. Chicago, IL: The University of Chicago.

Dewey, J. (1916). *Democracy and education: An introduction to the philosophy of education*. New York: Macmillan.

Dewey, J. (1938). *Experience and education.* New York: NY. Touchstone.

Eliks, I., & Hofstein, A. (2015). *Relevant chemistry education: From theory to practice*. Rotterdam: SensePublishers

Fessakis, G., Karta, P. & Kozas, K. (2018). Designing math trails for enhanced by mobile learning realistic mathematics education in primary education*. International Journal of Engineering Pedagogy* (2), 499-63.

García-Sánchez, S., & Luján-García, C. (2016). Ubiquitous knowledge and experiences to foster EFL learning affordances. *Computer Assisted Language Learning, 29*(7), 1169–1180.

Habiba, I., & Odis, J. (2019). School discipline, race-gender and STEM readiness: A hierarchical analysis of the impace of school discipline on math achievement in high school. *The Urban Review 52*, 75-99.

Hwang, G. J., & Chen, C. H. (2017). Influences of an inquiry-based ubiquitous gaming design on students' learning achievements, motivation, behavioral patterns, and tendency towards critical thinking and problem solving. *British Journal of Educational Technology, 48*(4), 950–971.

Hwang, G. J., Wu, T. T. and Chen, Y. J. (2007) Ubiquitous computing technologies in education, *Journal of Distance Education Technologies, 5*(4), 1-4.

Kolb, D. A. (2014). *Experiential learning: Experience as the source of learning and development* (2nd ed.). Upper Saddle River: Pearson Education.

Kortland, J. (2007). Context-based science curricula: Exploring the didactical frictions between context and science content. In The seventh ESERA conference, Malmö, Sweden. Retrieved from http://www.phys.uu.nl/~kortland/uk_publications.htm

Koul, R., Lerdpornkulrat, T., & Chantara, S. (2011). Relationship between career aspirations and measures of motivation toward biology and physics, and the influence of gender. *Journal of Science Education and Technology, 20*(6), 761–770.

Liu, G. Z., Chen, J. Y., & Hwang, G. J. (2018). Mobile-based collaborative learning in the fitness center: A case study on the development of English listening comprehension with a context-aware application. *British Journal of Educational Technology, 49*(2), 305–320.

McGee, E. (2013). High-achieving black students, biculturalism, and out-of-school STEM learning experiences: Exploring some unintended consequences. *Journal of Urban Mathematics Education, 6*(2), 20–41.

McGee, E. O., & Pearman, F. (2015). Understanding black male mathematics high achievers from the inside out: Internal risk and protective factors in high school. *Urban Review, 47*(3), 513–540.

Mishra, P. & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge, *Teachers College Record, 108*(6), 1017-1054.

Mumford, L. (1946). *Values for survival: Essays, addresses, and letters on politics and education.* New York: Harcourt, Brace.

National Academy of Engineering (NAE) and National Research Council (NRC). (2014). STEM integration in K-12 education: status, prospects, and an agenda for research. In M. Honey, G. Pearson, & H. Schweingruber (Eds.), *Committee on K-12 engineering education.* Washington, DC: National Academies Press.

Orr, D.W. (1992). *Ecological literacy: Education and the transition to a postmodern world.* Albany, NY: State University of New York Press.

Perry, B., & Morris, E. (2014). Suspending progress: Collateral consequences of exclusionary punishment in public schools. *American Sociological Review, 79*, 1067–1087.

Pfeiffer, V. D., Gemballa, S., Jarodzka, H., Scheiter, K., & Gerjets, P. (2009). Situated learning in the mobile age: mobile devices on a field trip to the sea. *Research in Learning Technology, 17*(3).

Pilot, A., & Bulte, A. M. W. (2006). Why do you 'need to know'? context-based education. *International Journal of Science Education, 28*(9), 953–956.

President's Council of Advisors on Science and Technology (PCAST). (2010). Prepare and inspire: K-12 education in STEM for America's future. Retrieved November 3, 2018 from http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-stemed-report.pdf.

Schnepp, J. C., & Rogers, C. B. (2017). Evaluating the acceptability and usability of EASEL: A mobile application that support guided reflection for experiential learning activities. *Journal of Information Technology Education 16*, 195-214.

Semken, S., Ward, E. G., Moosavi, S., & Chinn, P. W. U. (2017). Place-based education in geoscience: Theory, research, practice, and assessment. *Journal of Geoscience Education 65* 542-562.

Sharples, M., Taylor, J., & Vavoula, G. (2016). A theory of learning for the mobile age. In C. Haythornthwaite, R. Andrews, J. Fransman, & E. M. Meyers (Eds.). *The SAGE handbook of e-learning research* (pp. 63–81). (2nd ed.). SAGE.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Sugimoto, A. T., Turner, E. E. & Stoehr, K. J. (2017). A case study of dilemmas encountered when connection middle school mathematics instruction to relevant real world examples. *Middle Grades research Journal 11*(2), 61-82.

Swirski, H., Baram-Tsabari, A., & Yarden, A. (2018). Does interest have an expiration date? An analysis of students' questions as resources for context-based learning. *International Journal of Science Education, 40*(10), 1136-1153.

van den Hurk, A., Meelissen, M., & van Langen, A. (2019). Interventions in education to prevent STEM pipeline leakage. *International Journal of Science Education, 41*(2), 150-164.

Vedder-Weiss, D., & Fortus, D. (2011). Adolescents' declining motivation to learn science: Inevitable or not? *Journal of Research in Science Teaching, 48*(2), 199–216.

Williams, D. R. (2014). Making sense of 'place': Reflections on pluralism and positionality in place research. *Landscape and Urban Planning, 131*, 74–82.

DISCOVERING EVERYDAY MATHEMATICAL SITUATIONS OUTSIDE THE CLASSROOM WITH MATHCITYMAP AND GEOGEBRA 3D

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Abstract. *In elementary school, teaching and learning activities aim to develop, among others, students' skills to acquire deeper understanding of their living environments. There are numerous opportunities for students to recognize forms, shapes, and mathematical connections in everyday situations. These everyday situations can be simulated in classrooms; however, educational technologies offer new approaches to extend classroom activities, teachers can simulate and design shapes through Augmented Reality and 3D printing within or beyond the classroom. To stimulate students' everyday mathematical connections utilizing these technologies could assist in developing activities outside the classroom in urban or in natural environments. Through this approach students could utilize or enhance their mathematical and technical skills within their usual living environments. Utilising educational software such as MathCityMap, GeoGebra 3D Calculator, and other 3D modelling software we developed examples of tasks that could offer easy transitions from in- to out-side of classrooms. In this paper, we will describe learning and teaching aims of these tasks and outline further research and development directions to broaden opportunities to develop students' mathematical, design and modelling skills.*

Keywords: elementary school, geometry, educational technology, outside, mathematical thinking, trail

INTRODUCTION

In elementary school, numerous curricular topics are strongly related to students' everyday environments and lives. Hence, teachers recurrently attempt to integrate activities into their teaching encouraging students to explore mathematics in their surroundings and their experiment with their mathematical skills to better understand the world around themselves. In addition, new technologies offer new opportunities for such attempts and enable teachers to design challenging and relevant tasks for students. Several teachers, in Luxembourg, become committed to these opportunities and began using technologies to simulate forms, shapes, and mathematical processes in their classrooms. In these classes, students learned to apply content- and process-related skills in mathematical thinking (Selter & Zannetin, 2018) through the given tasks. Augmented reality was one of the novel technologies utilized to foster students' deeper understanding of forms and shapes (Berdik 2017; González, 2015). In parallel, physical modelling with 3D printing could make students to apply relevant mathematical ideas and interact with real-world problems (Carreira & Baioa, 2015). Nevertheless, research clearly shows that teachers play a crucial role in the teaching and learning processes and utilising these technologies could guide students' evolution from personal to mathematical meanings (Lieban & Lavicza, 2019a; 2019b). Students' living environments and experiences obviously are not limited into their schools and classrooms. They live in urban and natural environments where they daily encounter mathematical ideas and patterns, interact with them consciously or unconsciously. For example, commonly used personal objects and architecture surrounding them entail diverse geometric shapes and various mathematical connections,

Lavicza, Z., Haas, B. & Kreis, Y. (2020). Discovering Everyday Mathematical Situations Outside the Classroom with MathCityMap and GeoGebra 3D. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp.23-30) Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.03

orienting themselves to and from schools require spatial navigation skills, and building a hut with their friends in the forest necessitates hidden mathematical ideas to be utilised. Skills and knowledge, they learned in classrooms could be applied beyond school, but most often these connections are not recognised and mathematics considered out of reach from real life. Thus, if education aims to utilise out-of-classroom environments for teaching and learning, these situations would need to be stimulated or scaffolded making them apparent for students. This raises the question of how we could transfer classroom activities outdoors, into students' living environments and offer meaningful and inspiring mathematical tasks to enable new forms of mathematical learning. One of the approaches for such transformations of mathematics to outdoors is the idea of mathematical trails with the utilisation of technologies through a novel educational tool MathCityMap (Ludwig & Jesberg, 2015). In MathCityMap (MCM) teachers can define trails and create tasks suited to their students in their own environments. Furthermore, MCM also allow creators to design tasks with additional technologies, in our case, augmented reality (AR) and 3D printing, which was already trialled in classroom conditions and then could be utilised in MCM trails. In this paper, we will present examples of tasks based on augmented reality applications within the GeoGebra 3D Calculator, attached to MCM trails as well as possible applications of 3D printing associated with these tasks. We will describe initial tasks and discuss their potentials for further research and developments of theoretical approaches. Furthermore, we aim to develop opportunities to integrate MCM and GeoGebra 3D at a deeper level and internally connect this software for future benefits in outdoor task designs.

CONTEXT, DATA AND TASK DESIGNS

Creating tasks with augmented reality in the dynamic geometry software GeoGebra 3D Calculator has been part of recent studies we undertook (Lieban & Lavicza, 2019a; Haas, Kreis & Lavicza, in review). Based on our experiences, students are experiencing difficulties in the process of visualizing three-dimensional geometric shapes and offering support only in 2D showed similar problems in other studies (Ng, 2017; González, 2015). Neverheless, for our study we decided to experiment with designing tasks that are fully utilising augmented reality applications. Using AR features of the GeoGebra 3D calculator on phones and tablets, students could have opportunities to discover properties of their surroundings real-time and explore as well as modulate these shapes for their own needs. Through these activities we expected that students could foster their visual-spatial memories and visualisation abilities, which are well described in neuroscience research by Szucs et al., (2013). Based on the curricular requirements in Luxembourg (MENJE, 2011) and following the four basic principles of Dienes' theory of mathematics (Hirstein, 2008; Lieban & Lavicza, 2019b), as Dienes' approach fitted well for our tasks in previous 3D-related studies, we reproduced geometric shapes from students learning environments with AR in GeoGebra 3D. Initially students worked with pre-designed worksheets that they could modulate the shapes fitting real-world objects (Figure 1), but when they became more proficient with the tools, they were able to develop their own modelling applications (Figure 2).

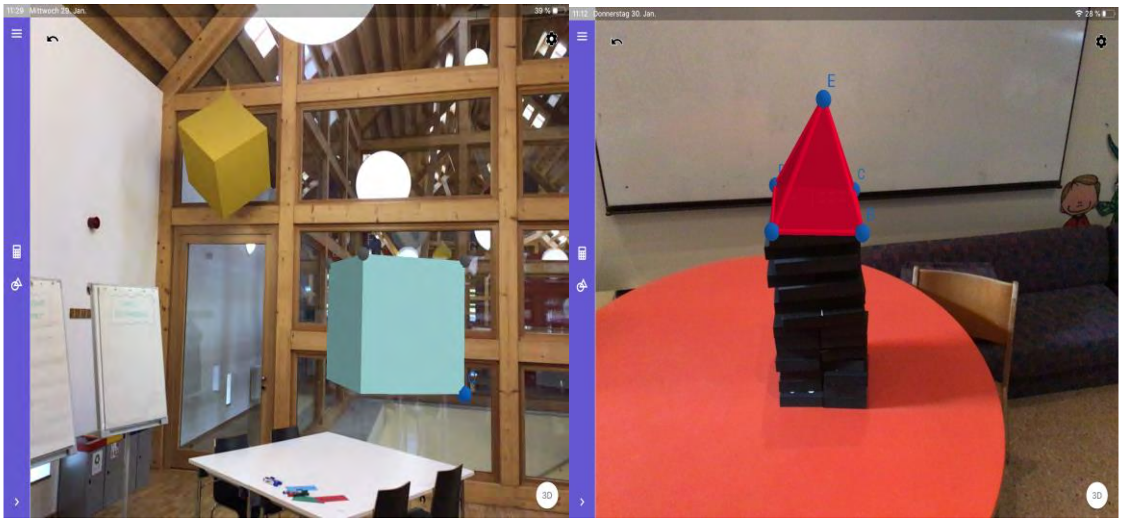

Figure 1: Reproduction of shapes with GeoGebra 3D Calculator.

To further engage the pupils in a deeper modulation process, we worked with them to recreate shapes with the augmented reality functions, objects, composed of different shapes. Figure 2 shows an example in which students created modelling of a paper plan with exact measurements in the software.

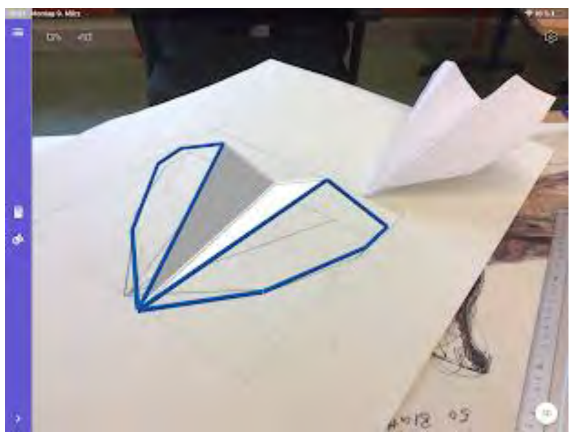

Figure 2: Recreation complex objects with GeoGebra 3D Calculator.

During the execution of different tasks, students were utilising their already learned knowledge in mathematics, software and design as well as their skills in exploration, modelling, and cooperation.

Within the experiments we collected various data from students and teachers. We carried out semi-structured interviews with students in two classes, four Grade-4 teachers, three Kindergarten teachers and a School Principal. In addition, we observed students' activities in school and partly on MCM trails as well as we could collect students' solutions. However, due to the sudden introduction of school closure and confinements due to the COVID-19 emergency, we could not continue working with students in school. Nonetheless, we worked on other AR and 3D Printing online activities that will be recorded in future papers (e.g. Haas, Kreis & Lavicza, in review). In this paper, we mainly report on the activities we carried out with some preliminary findings and will continue these experiments after schools reopen.

In the modelling of the previous two examples (Figure 1,2), for some of the groups we observed their uses of the software and recorded their arguments. Further analysis of these records will outline students' design arguments, their uses of mathematical language, descriptions of measurements, and explanations of lengths, areas and volumes. Overall, students' arguments of modelling were containing valid expressions of their mathematical knowledge and development of concepts, but we noticed that further scaffolding of tasks and then later discussion of modelling together with teachers are essential for future designs of such tasks.

The next step of our task design was to integrate our experiences from the initial tasks within the school and move learning to the outside. Thus, we attempted to connect the AR features of GeoGebra 3D Calculator and MCM. As GeoGebra 3D Calculator is available on iPads, we decided to initially utilise this setup and allow students to explore geometric modelling around their schools and at home. Both applications were installed on iPads and could be connected to enable students to explore their environments.

In this experiment, we chose three tasks that we wanted to add into trails in MCM:

- Discovering shapes in the environment (task on cubes)
- Reproducing more complex objects constructed from different geometric shapes (task on woodhouse)
- Designing complex objects created from different shapes (task on the barn gate)

We decided to create each task near a smaller district in our town, where most students from the school live. Thus, we designed the three tasks with GeoGebra 3D Calculator and MCM to engage students with mathematics outside of their classrooms and still benefit from interactions with each other and the AR tool. The tasks we integrated into a trail (Figure 3) and students' designs and solutions had to be posted on our dedicated online portal.

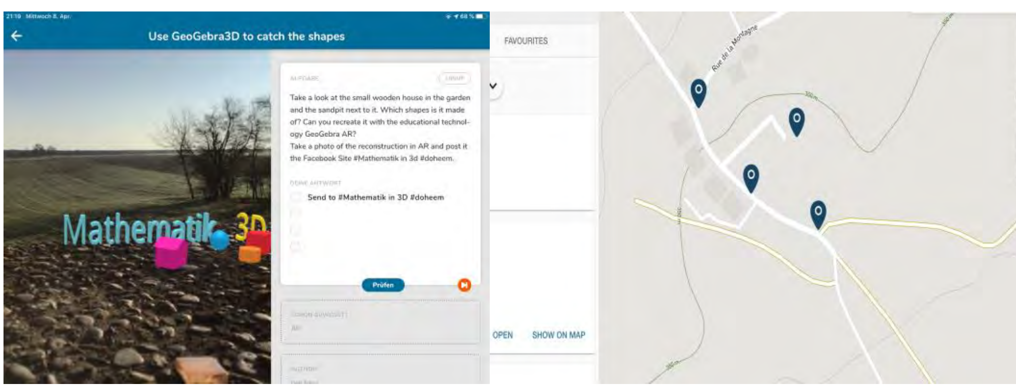

Figure 3: Task presentation in MCM.

In the first task, we wanted students to discover cubes in their environments. This task was similar to some of the exercises on which students had already worked in class. The main aim of this task was to prepare students for forthcoming tasks, further develop their software skills, and follow the different development stages described by Dienes (Hirstein, 2008).

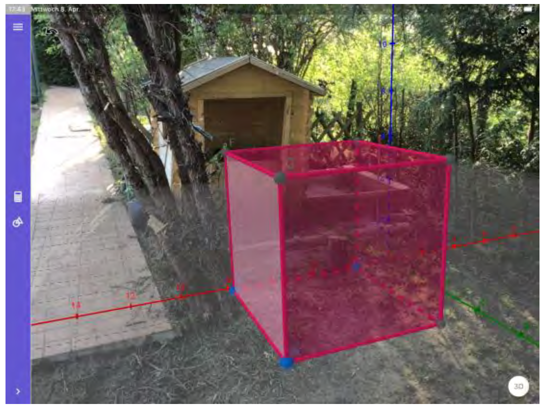

Figure 4: Discovering of shapes in the environment (base of the house).

In the second task, students needed to reconstruct a complex object made from different shapes; for instance, a small wooden house and a sandpit (Figure 5). Firstly, students had to identify the different shapes from which the object was made of. Then, secondly, they had to construct them one-by-one and then join them together on their iPad. To be able to develop a correct construction, they needed to recognise the different properties of shapes, create line segments, and modulate their constructions. Students employed process-related skills, such as modelling and argumentation to be able to solve this task. They also had the opportunity to discuss their tasks with each other and then compare solutions in school together with their teachers.

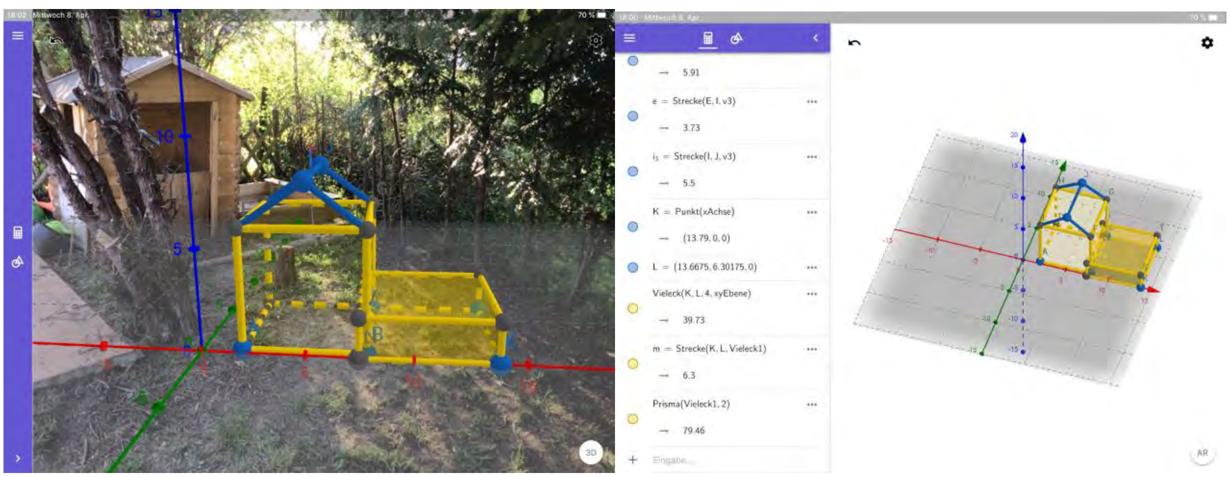

Figure 5: Reconstruction of a complex object made of different shapes in MCM.

In the third task, students needed to create a gate for a barn in 3D GeoGebra Calculator (Figure 6). Students had to identify shapes required for the construction in augmented reality. They had to adapt the size of the shapes, combine them with different shapes, and modulate their construction, so that the missing gate would fit to its place. Once they finished, they posted their results and received feedback both online and in class.

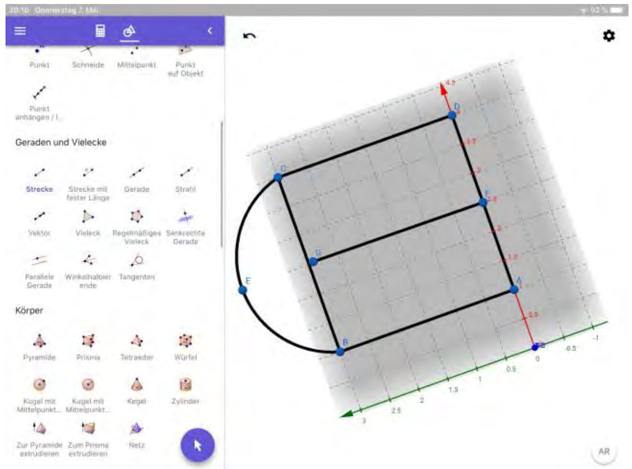

Figure 6: Designing a barn gate and the creation of a complex object.

The tasks above required students to recreate objects with digital modelling through utilising their mathematical and software knowledge. However, we also aimed to inspire students to use these skills to create objects meaningful and useful for their lives and environments. One of the submitted examples came from a trail close to a garden and the head of the watering can was broken next to the newly planted potatoes. Students wrote in their report that they wanted to help, took measurements with AR and designed the watering head with the AR software, using trial and error method, and then 3D printed several prototypes until they fixed the problem (Figure 7). According to students, this was one of the most inspiring part of their mathematics trail. These kinds of problems could be integrated into MCM trails, but possibly open-ended tasks encouraging students to be aware of their surroundings could be explored.

Figure 7: Fixing a watering can with 3D printing.

The gathered interview and reporting data from students and teachers suggest that 3D printing adds one more layer to the understanding of the correctness of students' solutions.

With these prints students acquire not only visual, but also concrete feedback after touching and observing their printed solutions. Through AR tools they can perform digital modelling and can evaluate their correctness, but assessing results from 3D prints, which also require additional mathematical and software skills, they could immediately recognise if their measurements calculations and modelling approach was correct or not. If some of the elements were incorrect, they would need to repeat modelling until they obtain their desired solution. We believe that this process cycle could be offer opportunities to offer a glance for scientific research methods and engineering processes.

Finally, an interesting solution was submitted by a student who created a castle, he had seen during their travelling, called it “the fortress and the blue magic tower” (Figure 8) and also used this model for his story in a language class on storytelling.

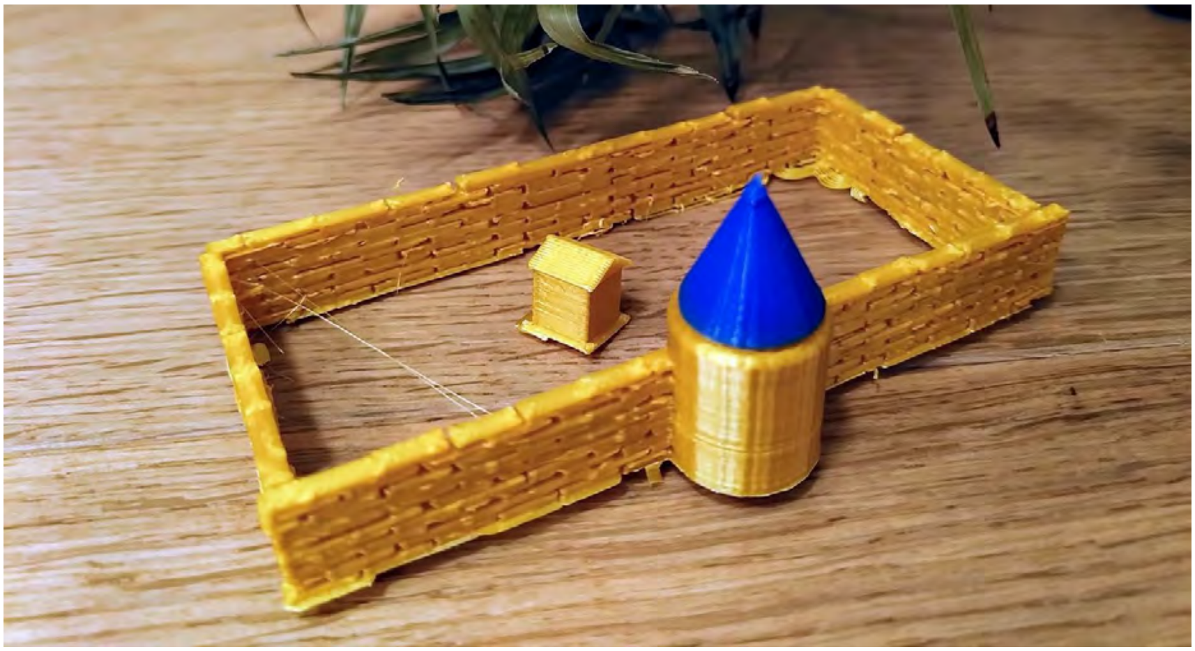

Figure 8: the fortress and the blue magic tower.

Throughout these tasks students acquired variety of knowledge in mathematics, technology and related subjects as well as they become more aware of the mathematical nature of their surroundings, demystifying some previously held beliefs about the subject.

DISCUSSIONS

Combining the AR features in the GeoGebra 3D Calculator with MCM offered us new opportunities to experiment with novel mathematical approaches beyond classrooms. Teachers can create trails to let the students explore their living environments and make them discover these places with mathematical perspectives. The examples presented in this paper could assist teachers to create new kinds of activities and trails to make mathematics more integrated into students’ lives and contribute to reducing the perceived disconnection of mathematics from the real world. Our preliminary data analyses, together with our previous research, showed some potentials for both teachers and students in connecting physical and digital worlds; making mathematical connections with real-world situations; bringing mathematics outside of classrooms; and integrating new technologies to create meaningful mathematical tasks. With our examples we explored opportunities to further integrate the two applied mathematical tools GeoGebra 3D and MathCityMap. Designing software integration could potentially offer an even more powerful tool for mathematics teaching in the outside. Certainly, there is a need for more extensive future research in this area, but in this paper, we wanted to showcase some initial opportunities with some tools and approaches that could offer new directions for mathematics teaching and learning.

References

Berdik, C. (2017). Kids Code Their Own 3D Creations with New Blocks-Based Design Program. *Tech Directions, 76(9)*, 23–24.

Carreira, S., & Baioa, A. M. (2015). Assessing the best staircase: Students' modeling based on experimentation with real objects. In *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 834–840). Prague, Czech Republic.

González, N. A. A. (2015). How to include augmented reality in descriptive geometry teaching. *Procedia Computer Science, 75*, 250–256.

Haas, B., Kreis, Y., & Lavicza, Z. (in review). Parent's perspectives: The use of Augmented Reality, digital and physical modelling in remote teaching in elementary school *Educational Studies in Mathematics*.

Hirstein, J. (2008). The impact of Zoltan Dienes on mathematics teaching in the United States. In B. Sriraman & Z. P. Dienes (Eds.), *Mathematics Education and the Legacy of Zoltan Paul Dienes* (pp. 107-111). Charlotte, NC: Information Age Publishing, Inc.

Lieban, D., & Lavicza, Z. (2019a). Dissecting a Cube as a Teaching Strategy for Enhancing Students' Spatial Reasoning: Combining Physical and Digital Resources. In *Bridges 2019 Conference Proceedings*, 319–326.

Lieban, D., & Lavicza, Z. (2019b). Instrumental genesis and heuristic strategies as frameworks to geometric modeling in connecting physical and digital environments. In *Eleventh Congress of the European Society for Research in Mathematics Education*.

Lieban, D., Ulbrich, E., Barreto, M., & Lavicza, Z. (2019). A new era of manipulatives: Making your own resources with 3D printing and other technologies. In *Eleventh Congress of the European Society for Research in Mathematics Education*.

Ludwig, M., & Jesberg, J. (2015). Using Mobile Technology to Provide Outdoor Modelling Tasks - The MathCityMap-Project. *Procedia - Social and Behavioral Sciences, 191*, 2776-2781.

MENJE (2011). Plan d'études, école fondamentale, numéro du courrier de l'éducation nationale spéciale.

Ng, O. (2017). Exploring the use of 3D Computer-Aided Design and 3D Printing for STEAM Learning in Mathematics. *Digit Exp Math Educ 3*, 257–263, https://doi-org.proxy.bnl.lu/10.1007/s40751-017-0036-x

Selter, C & Zannetin E. (2018). *Mathematik unterrichten in der Grundschule*. Seelze: Friedrich Verlag

Szucs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2013). Developmental dyscalculia is related to visuo-spatial memory and inhibition impairment. *Cortex, 49(10)*, 2674–2688. doi: 10.1016/j.cortex.2013.06.007

MATHEMATICAL MODELLING IN STEM EDUCATION: A MATH TRAIL USING LABSTAR™

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Abstract. *Regarding the current scene of STEM education, an effective discipline integration is a need. According to STEM: Integrated teaching framework, teachers use discipline specific methods for discipline integration. Mathematical modelling allows teachers to design integrated STEM activities, with a focus on mathematics discipline. In this paper we developed a math trail with mathematical modelling tasks at major historical venues of Istanbul. Students will collect and analyze data using LabStar™. We reported expert opinions about the tasks included in the math trail and prepared the final tasks for implementation.*

Key words: Mathematical modelling, math trails, outdoor education, STEM integrated teaching framework.

INTRODUCTION

The problems faced by society have altered as the pace of change in technology and science is accelerating. Problems that people face in today's ever-changing world are more multidisciplinary and their solution are required the integration of multiple STEM concepts (Wang, Moore, Roehring & Park, 2011). These problems and solution strategies that require STEM knowledge led the global recognition of the growing importance of STEM education for K-12 level that making students better problem solvers, innovators, logical thinkers, and technologically literate (Morrison, 2006).

Regarding the current scene of STEM education, an effective discipline integration is a need. "STEM education includes the knowledge, skills and beliefs that are collaboratively constructed at the intersection of more than one STEM subject area" (Corlu, Capraro, & Capraro, 2014). For K-12 educators, relating the disciplines with each other is still a major challenge. In literature, several approaches from disciplinary through to transdisciplinary are discussed for STEM Integration (e.g., Burke et al., 2014; Honey et al., 2014; Moore and Smith, 2014; Rennie et al., 2012; Vasquez, 2014/2015; Vasquez et al., 2013). However, teachers need general structures to guide them through discipline integration by applying STEM lesson plans with their students (English, 2016).

STEM Integrated teaching framework, in this regard, offers a methodological integration across disciplines. To employ methodological integration teachers, integrate discipline relevant methods such as project-based learning, scientific inquiry and mathematical modelling to teach other disciplines. For the STEM areas, integrated teaching framework relates scientific inquiry with science, computational thinking with technology, project-based learning with engineering and mathematical modelling with mathematics (Aşık, Doğança-Küçük, Helvacı, & Corlu, 2017; Corlu, 2017). Using these methods for discipline integration allows teachers to teach their main disciplines without losing its unique characteristics, depth, and rigor (National Research Council, 2011).

Mathematics is seen as difficult to integrate into a STEM lesson plan. Teachers usually use mathematics as a tool or as an algorithmic way if required to relate mathematics to other STEM disciplines (Wang, 2012). However, as in other STEM disciplines mathematics is

Yabas, D., Boyacı, H. S. & Corlu, M. S. (2020). Mathematical Modelling in STEM Education: A Math Trail using LABSTAR™. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 31-38). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.04

equally important to be integrated in a rigorous way to STEM lesson plans. Shaughnessy (2013), in a similar manner, indicated that mathematics in STEM should be made “transparent and explicit”. To guide teachers by integrating mathematics into their STEM lesson plans mathematical modelling in STEM integrated teaching framework comes forth.

Mathematical modelling is about translating real world phenomena into the language of mathematics. The mathematical modelling starts from a situation in real world, evolves into a real model and mathematical model respectively (Blum, 1993). Besides facilitating educators integrate mathematics into other STEM disciplines, mathematical modelling also provides more meaning to the mathematics teaching-learning activities (Blum, 1993). In Turkish context, within the new paradigm of mathematics curriculum, there is also emphasis on deep and rigorous learning by focusing on complex skills such as mathematical reasoning, problem solving and the ability to use mathematics in daily life (Ayas, Corlu, & Aydın, 2013). Mathematical modeling, in this sense, offers methods that teachers can design activities to develop their students’ complex skills.

A mathematical model allows to relate quantities in physical, social and everyday life using mathematical and statistical methods. Within mathematical modelling tasks technology is an essential facilitator to test various assumptions, explore consequences and compare predictions with data (Common Core State Standards Initiative, 2020). In Encyclopedia of Mathematics Education, mathematical modelling competency is defined as solving real-world problems using mathematics (Kaiser, 2014). These skills constitute identifying relevant questions, variables, relations or assumptions in a real-world situation, mathematizing of the situation and interpreting and validating of the solution (Ludwig & Xu, 2010).

In order to design an integrated STEM activity with rigorous mathematics we propose a math trail that cover mathematical modelling tasks where participants use an innovative just-in time data logging device LabStar™ for data collection and analysis.

DESCRIPTION OF THE MATH TRAIL

The math trail we designed covers selected historical places in İstanbul, a metropolitan city of Turkey. In each point at the math trail we developed mathematical modelling tasks for the students. For the math trail LabStar™ data logging device for doing just-in time measurements.

LabStar™: Just-in time data logging device

LabStar™ is a data collection device which collects continuous data through its sensors, namely, distance, ambient temperature, ambient humidity, air pressure, altitude, ambient light, acceleration, and sound intensity. Students can collect data anywhere about different variables, in line with the curriculum and teachers’ learning objectives. Besides data collection, LabStar ™ also enables students and teachers to represent, trim, analyze and interpret the data they collected via LabStar ™ mobile application. Students can proceed to analysis, examine the descriptive statistics, plot line of best fit and conduct bivariate analyses. These analyses create opportunities for teachers and students to describe data, discuss on graphical representations of data and create meaningful associations between representations and real-life situations. Collecting scientific data from real life and analyzing data by engaging in basic statistics also creates opportunities for interdisciplinary learning in STEM education contexts.

Historical Places of Istanbul Math Trail

The content and the activities of the math trail focus on mathematical modeling, data collection and analysis, by investigating phenomena about the scientific concepts such as light, altitude and pressure. Below are the target outcomes related with the main and other disciplines of STEM.

Mathematics: Formulate questions and collect, organize, and display relevant data to answer these questions (NCTM, p. 4); Analyze data, making inferences and predictions based on data, and understanding and using the basic concepts of probability (NCTM, p. 4), Investigate patterns of association in bivariate data (CCSSM) -Grade 8-, Make inferences and justify conclusions from sample surveys, experiments and observational studies (CCSSM) –High School-

Science: Process skills for inquiry: observing, explaining (hypothesizing), predicting, raising questions, planning, and conducting investigations, interpreting evidence, communicating (Harlen, 2000).

Technology: Use technology appropriately in gathering and interpreting data, Demonstrate the ways information acquisition and use technology applications to facilitate evaluation of work, both process and product

Tasks of the Math Trail

Task 1. Towers of İstanbul: Galata and Maiden Towers

1. Task basic data

Definition of task: Galata and Maiden Towers are two towers that watch over İstanbul. Maiden's Tower is located at sea level. However, Galata Tower is located above sea level. Your task is to investigate air pressure with respect to different altitudes and heights of Galata and Maiden Towers.

Position: **Galata Tower:** 41°1′32″ and 28°58′27″; Maiden Tower: 41°01′16.2″ and 29°00′15.3″

2. Stepped hints

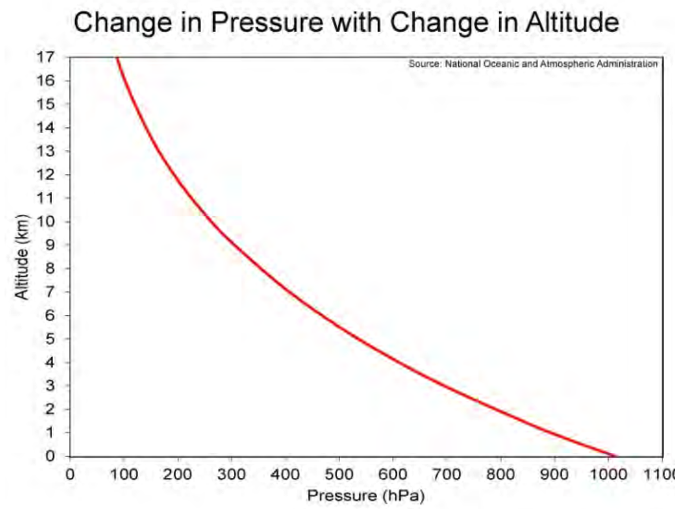
Hint 1. The atmospheric pressure at a certain location is related with the altitude and the temperature of the place. Measure the atmospheric pressure at different points of tower with LabStar.

Hint 2. How does the atmospheric pressure of Maiden's Tower change in different points at different heights of the tower? (e.g. entrance, top balcony). Is it possible to develop a mathematical model that relates altitude with atmospheric pressure?

Hint 3. Does the atmospheric pressure change at different times of the day? How can you design an experiment to find the answer using LabStar?

Hint 4. How can you design an experiment to test your model in Galata Tower? Is it possible to use the same model for estimating the atmospheric pressure at the top balcony of Galata Tower?

Hint 5. Using below graph for the pressure and altitude, how can you find the altitude of the Galata tower from the sea level? How can you design an experiment to find the answer using LabStar?



About places

Galata Tower: It is one of the oldest towers in the world. It was built by the Byzantine Emperor Anastasius in 528 as the Lighthouse. Its height from the ground to the end of its roof is 66.90 meters. Its weight is about 10,000 tons and its thick body is made of rubble stone. *Maiden's Tower:* The tower, which has a historical past dating back to 24 BC, was built on a small island where the Black Sea meets Marmara Sea. According to the rumor, it was named Maiden's Tower because the emperor imprisoned his daughter in the tower to protect his daughter, who was said to be killed by oracle due to the snakes.

From Grade: 7; *Tools:* LabStar, Paper-Pencil, Smartphone; *Tags:* Mathematics education, data analysis, statistics, altitude, atmospheric pressure

Task 2. The disctrict of tolerance: Balat

1. Task basic data

Definition of task: Like all districts of İstanbul, Balat also hosts many people of different religions and ethnicities. It contains many synagogues, mosques, and churches within itself. Each architectural building reflects their own cultural and aesthetic values of society. So, all differs from each other with their features. A researcher suggests such a hypothesis after visiting and observing different kind of architectural building that serve as worship places in Balat: -Churches are less bright than mosques and synagogues.

Please help him test his hypothesis by gathering data from Ahrida Sinagogue, Yavuz Sultan Selim Mosque and St. George's Cathedral respectively and analyzing it.

Position: Ahrida Sinagogue: 41.03278° and 28.94556°, Yavuz Sultan Selim Mosque: 41°01′35.6″ and 28°57′4.8″, St. George's Cathedral: 41°01′45″ and 28°57′07″

2. Stepped hints

Hint 1. To test his hypothesis, you need gather data related to amount of light. How would you design an inquiry to test the hypothesis? Are there any critical points while gathering data from each place? Please discuss which points are critical/where you should make your measurements when gathering data (position of LabStar, light etc.) to compare data from different places and write them in group notebook. After defining critical points, please measure the amount of light of synagogue, mosque, and cathedral, respectively.

Hint 2. Examine the data. How do you decide typical value representing the related data? Why? (mean, mode, median)

Hint 3. How can you compare the amount of light? Which statistics should you use? (mean, mode, median, or another statistical tests?)

3. About places

Ahrida Sinagogue: The synagogue, built in the early 15th century and named after the city of Ohrid, located in North Macedonia, where its founders migrated to Istanbul, is today the largest capacity synagogue in Istanbul. *Yavuz Sultan Selim Mosque:* The Yavuz Selim Mosque, is a 16th-century Ottoman imperial mosque located at the top of the 5th Hill of Istanbul. It was built on the hill closest to the Golden Horn. Its size and geographic position make it a familiar landmark on the Istanbul skyline. *St. George's Cathedral:* This church, which is connected to the Fener Greek Patriarchate of Istanbul, has been the center of Orthodoxy since the 6th century. It is the main church of most of today's Orthodox churches. The Greek Orthodox Patriarchate is located in the garden of this church, which was built in 1836.

From Grade: 7; *Tools:* LabStar, Paper-Pencil, Smartphone; *Tags:* Mathematics education, data analysis, statistics, light

Task 3. Basilica Cistern

1. Task basic data

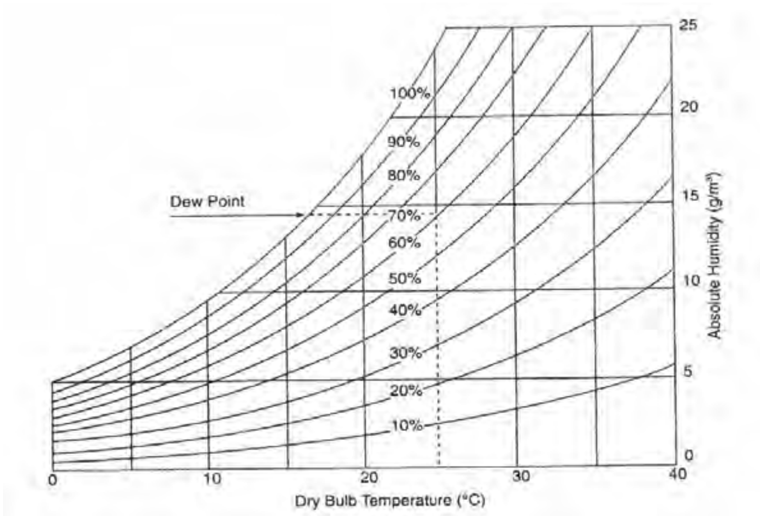
Definition of task: Basilica Cistern was used as a big underground reservoir in Byzantine and Ottoman periods. It is a very humid place to visit. Asthma patients can have a hard time to visit the Cistern for longer periods of time. Therefore, please calculate the amount of moisture in unit dry air in the cistern, in order to give information for the visitors of health risk groups. After your measurements you should decide whether the Cistern is safe for asthma patients in terms of moisture by comparing your data with scientific information from the studies about the amount of the moisture in closed areas and asthma patients.

Position: **41° 0′ 30.2004" and 28° 58′ 40.2060"**

2. Stepped hints

Hint 1. The amount of moisture in dry air is related to relative humidity and the temperature of the place. Measure the relative humidity and temperature with LabStar.

Hint 2. Using the graph below how can you find the amount of moisture in unit dry air (absolute humidity)?



Hint 3. Does the amount of moisture in unit dry air change in different points of the cistern? How can you design an investigation using LabStar to collect data about the amount of moisture in the air at different points in the cistern?

Hint 4. Does the amount of moisture in unit dry air change at different times of the day? How can you design an experiment to find the answer using LabStar?

3. Task meta-data

About this place: The cistern is 140 m long, and 70 m wide, and covers a rectangular area as a giant structure. Accessible with 52-step staircase, the Cistern shelters 336 columns, each of which is 9 m high. Erected at 4.80 m intervals from one another the columns are composed of 12 rows, each has 28 columns.

From Grade: 7; *Tools:* LabStar, Paper-Pencil, Smartphone; *Tags:* Mathematics education, data analysis, statistics, temperature, humidity.

Expert opinion about the Math Trail Tasks

Before implementing the math trail with the students, we took expert opinions from mathematics and science educators about the feasibility of the activities and their recommendations. Two mathematics and one science teachers, who have teaching experience above 15 years gave their opinions using the short evaluation form for each task. The form consisted of the questions below:

- Q1. How would you evaluate the age and grade level appropriateness of the task? (rating scale out of five points)
- Q2. Is the task appropriate for the development of students' mathematical modelling skills? (rating scale out of five points)
- Q3. Is the task clearly defined for teachers and students follow easily? (rating scale out of five points)
- Q4. What are your general opinions and suggestions about the task? (open-ended)

The responses of first three questions of each expert is presented in Table 1.

	Task1			Task 2			Task 3		
	Q1	Q2	Q3	Q1	Q2	Q3	Q1	Q2	Q3
Expert 1 (Mathematics Educator)	4	3	4	5	4	4	4	4	4
Expert 2 (Mathematics Educator)	4	4	4	5	4	5	4	5	4
Expert 3 (Science Educator)	5	5	4	5	5	4	5	5	4

Table 1: Expert Opinion about the Math Trail Tasks.

Table 1 shows that the experts found the tasks appropriate for the designated grade level. They made some recommendations about the science concepts to make the tasks more matched with the science curriculum objectives. For the development of mathematical modelling skills all experts revealed positive opinions. They found the tasks relevant for mathematical modelling skills and made some recommendations for the stepped hints to be more helpful by guiding the students make measurements and interpret data. The legibility of the tasks is evaluated as sufficient by all of the experts.

The answers for the open-ended question included recommendations for the tasks are focused on giving more detailed hints to students to help them pursue the task and make accurate measurements. The tasks were revised according to the suggestions of the experts.

References

Aşık, G., Küçük, Z. D., Helvacı, B., & Corlu, M. S. (2017). Integrated teaching project: A sustainable approach to teacher education. *Turkish Journal of Education, 6*(4), 200-215.

Ayas, A., Corlu, M. S., & Aydin, E. (2013). Mathematics and science assessment in the Turkish educational system: An overview. *Middle Grades Research Journal, 8*(2), 11-23.

Blum, W. (1993). Mathematical modelling in mathematics education and instruction. In T. Breiteig, I. Huntley & G. Kaiser-Messmer (Eds.), *Teaching and learning mathematics in context* (pp. 3-14), Chichester, Ellis Horwood Limited.

Burke, B. N., Reed, P. A., & Wells, J. G. (2014). Integrating technology and engineering in a STEM context. *Exemplary STEM programs: Designs for success*, 353-372.

Common Core State Standards Initiative. (2020). Standards for mathematical practice. Available online at http://www.corestandards.org/Math/Practice/

Corlu, M. S. (2017). STEM: Bütünleşik Öğretmenlik Çerçevesi [STEM: Integrated Teaching Framework]. In M. S. Corlu & E. Çallı (Eds.), *STEM Kuram ve Uygulamaları (pp. 1–10)*. İstanbul: Pusula.

Corlu, M. S., Capraro, R. M., & Capraro, M. M. (2014). Introducing STEM education: Implications for educating our teachers in the age of innovation. *Eğitim ve Bilim, 39*(171), 74-85.

English, L. D. (2016). STEM education K-12: Perspectives on integration. *International Journal of STEM Education*, *3*(1), 3.

Harlen, W. (2000). *The teaching of science in primary schools*. London: David Fulton Publishers.

Honey, M., Pearson, G., & Schweingruber, A. (2014). *STEM integration in K-12education: Status, prospects, and an agenda for research.* Washington: National Academies Press.

Kaiser, G. (2014). Mathematical modelling and applications in education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 396-404). NewYork, NY: Springer.

Ludwig, M., & Xu, B. (2010). A comparative study of modelling competencies among Chinese and German students. *Journal für Mathematik-Didaktik, 31*(1), 77-97.

Moore, T. J., & Smith, K. A. (2014). Advancing the state of the art of STEM integration. *Journal of STEM Education, 15*(1), 5–10.

Morrison, J. (2006). *TIES STEM education monograph series: Attributes of STEM education.* Teaching Institute for Essential Science, Baltimore, MD.

National Council of Teachers of Mathematics (NCTM) (2000). *Executive Summary: Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (2011). *Successful K-12 STEM education: Identifying effective approaches in science, technology, engineering, and mathematics.* National Academies Press.

Rennie, L., Wallace, J., & Venville, G. (2012). Exploring curriculum integration: Why integrate? In L. Rennie, G. Venville, & J. Wallace (Eds.), *Integrating science,technology, engineering, and mathematics* (pp. 1–11). New York: Routledge.

Shaughnessy, J. M. (2013). Mathematics in a STEM context. *Mathematics Teaching in the Middle School*, *18*(6), 324-324.

Vasquez, J. (2014/2015). STEM: beyond the acronym*. Educational Leadership*, *72*(4), 10-15.

Vasquez, J., Sneider, C., & Comer, M. (2013). *STEM lesson essentials, grades 3–8: Integrating science, technology, engineering, and mathematics*. Portsmouth, NH: Heinemann.

Wang, H. (2012). *A New era of science education: science teachers' perceptions and classroom practices of science, technology, engineering, and mathematics (STEM) integration*. Doctoral dissertation. Minnesota University, Minnesota.

Wang, H. H., Moore, T. J., Roehrig, G. H., & Park, M. S. (2011). STEM integration: Teacher perceptions and practice. *Journal of Pre-College Engineering Education Research, 1*(2), 2.

PAPERS

EXPLORING REAL WORLD ENVIRONMENTS USING POTENTIAL OF GEOGEBRA AR

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Abstract. *Two projects that we present in this paper are a proposal that allows students to open their eyes to the mathematical world around them, as if they were looking for a treasure of 2D or 3D geometric patterns that are waiting to be discovered by some curious person. Through the platform MathCityMap, a series of challenges will be found on the tour which include augmented reality that allow them to bring to life many objects of great cultural, historical or architectural importance. Attractive to any type of student, all the tours take mathematics out of the classroom and brings it closer to our everyday world through motivating and active learning, encouraging geometric thinking. In that sense, GeoGebra 3D with the augmented reality tool both facilitate the modelling of geometric real-world objects.*

Key words: GeoGebra 3D, GeoGebra AR, augmented reality, modelling.

INTRODUCTION

In this paper we report two projects aimed at middle school students and designed to explore and (re)discover the mathematical world. Intended to stimulate engagement and enthusiasm for maths through its connection with history, art or architecture contents, these projects facilitate integration and tailoring of multidisciplinary contents to the curricular and educational needs. The platform that we used here named MathCityMap (https://mathcitymap.eu) allows us to create the mathematical routes. The use of math routes has an extensive literature noted at (Jablonski et al., 2020; Richardson, 2004; Cahyono & Ludwig, 2019; Ludwig & Jablonski, 2019) and also many projects have been published in its repository. The novelty that we present here consists of modelling real-world architectural objects using GeoGebra 3D (https://www.geogebra.org) with its add-on augmented reality (AR) tool.

Goals

The goals of this paper are to experience mathematics easily outdoors, to promote geometric discovery and to connect with other subjects enhancing the science, technology, engineering, arts and mathematics (STEAM) learning through the use of digital manipulatives tools.

METHODOLOGY

Both projects are aimed at middle school students but it can be extended also to high school students by adapting the activities. They have in common to support the development of geometric thinking through the effectiveness of GeoGebra AR. For that purpose, curricular requirements are included in these projects which reinforce mathematical concepts from current or previous courses.

Ancochea, B. & Cárdenas, M.-I. (2020). Exploring Real World Environments using Potential of GeoGebra AR. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 41-46). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.05

All the students are organized in groups of 3-4 and each group interact in a collaborative way. Previously to the routes, they have to ensure that they have downloaded the GeoGebra 3D application in their mobiles or tablets, as well as one integrant of the group need to have the AR tool available on his device for the handling and adjustment of the geometrical models.

Concrete directions for a successful activity are given to students as follows:

1 To assign a role: a coordinator, a note/photo-taker and communicators, depending on the number of students.
2 To plan the route: the have to study the locations and strategies.
3 To identify and model geometric objects: explore the mathematical properties and try to solve the challenge.
4 To collect data and evidences of each locating point of the route: ensure to take notes, take a picture of the final product and connect math content with that learned in class.
5 To work collaboratively to create a presentation and a summarizing report.

Before the projects are approved, some teachers test the routes in order to adapt better the activities to the variety of students.

MathCityMap is an application that generates maps with activities and is multiplatform. It is a project of the University of Frankfurt in collaboration with other countries and universities.

GeoGebra is dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package. GeoGebra has become the leading provider of dynamic mathematics software, supporting STEAM education and innovations in teaching and learning worldwide.

Augmented reality (AR) (Muñoz, 2020) allows the user to mix real images with virtual images using electronic devices. The main features of AR are that combines real and virtual elements, it is interactive in real time and it is made in 3D.

There are different types of visualization:

Level 0. This is the lowest level and offers the minimum interaction with the physical world. The applications link the physical world with the virtual world by using barcodes and 2D codes (for example, QR codes).

Level 1. Applications use markers, 2D patterns that trigger the appearance of three-dimensional objects on them.

Level 2. Applications use images, objects or GPS coordinates to overlay virtual information.

Level 3. This is the highest level and would be covered by devices such as glasses or contact lenses that project information on what is being seen in real time.

GeoGebra performs an AR **level 3** through the mobile phone or the tablet by mixing real image with virtual image without the need of any marker.

Didactical competences and skills applied in the projects

- Mathematical competence: concepts used include: formulas, data, variables, three-dimensional coordinates and basic mathematical operations and statistics, reading and interpreting graphs, geometry and perspective, areas and volumes.
- Communicative competence: through a technical and communicative language the students will have to write reports and documents, write personal opinions, write texts of different typologies, exhibition and communication techniques, oral presentations, audiovisual and multimedia resources.

- Information processing and digital competence: by means of the new computer tools of the information and communication technologies linked to the development of the technical projects in all their phases. Students will acquire skills with graphic and three-dimensional design programs as well as video editing.
- Competence to learn how to learn: through the phases of the technological project they will have to obtain information, analyze it and select all the useful information related to their purpose to solve the challenge posed.
- Competence in knowledge and interaction in the physical world: this competence will be worked from the point of view of 3D modelling with augmented reality (volumes, faces, centroids, symmetry, proportionality, ratio aspect and rotations).
- Social and citizen competence: the designed routes are aimed at improving the cognitive, personal and social development of the students, as well as the impact of mobile learning in mathematics education.

Mathematical models applied in the projects

Theoretical foundations

- Recognize and classify polyhedrons and their elements.
- Develop the polyhedrons and obtain the surface and volume.
- Recognize, name and describe regular polyhedra.
- Solve geometrical problems involving the calculation of surfaces and volumes in bodies of revolution and polyhedra.

How do we mathematize the route?

- Referencing images: prior to the route and exploring geometric properties.
- Geolocating the tasks.
- Having a 3D surface repository.
- Generating didactic ideas.
- Using GeoGebra applets: create an ID and make it public.
- Giving augmented reality (AR) to the constructions.
- Creating new tools: it would simplify the constructions.

PROJECTS DESCRIPTION

The projects exposed here are designed to take place at the north of Barcelona in the region of Maresme coast which is rich in ruled geometry and historical architecture.

Project I: Mathematical walks at Maresme Sud

Our project would focus not only on geometrical objects founded around us, specially 3D as prisms, cones, cylinders, spheres and 2D examples of parallel and perpendicular lines, but also on probabilities, statistics, slopes and so on. We expect that the math walks will last one hour and a half. Our idea is to adapt the route and the questions after a first test walk and even later if pupils have new proposals because we consider that their contribution is basic in order to have a sort of *dynamic routes*.

It is headed by a group of teachers of Primary and Secondary schools at Maresme Sud, placed at the north of Barcelona that are registered in the Educational Resource Center (CRP) (https://serveiseducatius.xtec.cat/baixmaresme/categoria/crp). In coordination with that entity, teachers plan to prepare mathematical walks through different villages from Maresme Sud: El Masnou, Premià de Mar, Premià de Dalt, Vilassar de Dalt and Argentona. The starting point was a mathematical walk tested at the Mercat de Sant Antoni, Barcelona and surrounding objects (see figures 1 and 2). Departing from a GeoGebra construction previously done, students need to explore the model surfaces in parts or in total, using rotations, scaling and superposing the model with AR. Another very interesting proposed activity involved in the project is the research of different kind of surfaces: ruled surfaces, arched surfaces, surfaces of revolution. It is very easy to find many of them in many places, not only in churches or town councils but also in historical buildings. In that sense, GeoGebra is a very useful technique that allows us to draw a variety of surfaces for which there is no need to design complex mathematical models. Also, the planned activities include to add to GeoGebra some extra tools which can be helpful to design surfaces in a little number of steps.

Looking for symmetries, rotations, translations or other transformations is a very useful aim in a mathematical walk. But in addition, we suggest pupils to discover them in buildings or in nature. Studying art subjects contributes to the development of essential skills as it is well known and we will include some of them in the walks.

Figure 1: Examples of a real object modelled with GeoGebra 3D.

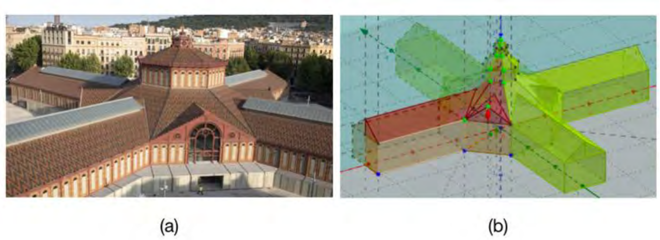

Figure 2: Examples of real object to model with GeoGebra 3D.

Project II: Mathematical walk exploring modernist fronts in Sant Pol de Mar with GeoGebra AR

This project invites students to enjoy a mathematical walk where the protagonists are the modernist fronts that, together with other geometric elements, allow to explore their symmetries, spatial distributions and to model some of their elements. It is located at Maresme Nord which covers the north area of Maresme and complement the geographical region of project I. Although there are only physical tools in MathCityMap platform like measuring tape, calculator or level, students also use digital tools included in GeoGebra as well as AR. The area occupied by the tour is 0.15 km^2 (see Fig. 3). Having a path with a total length of 0.8 km, the expected duration is about 2 hours and 40 minutes.

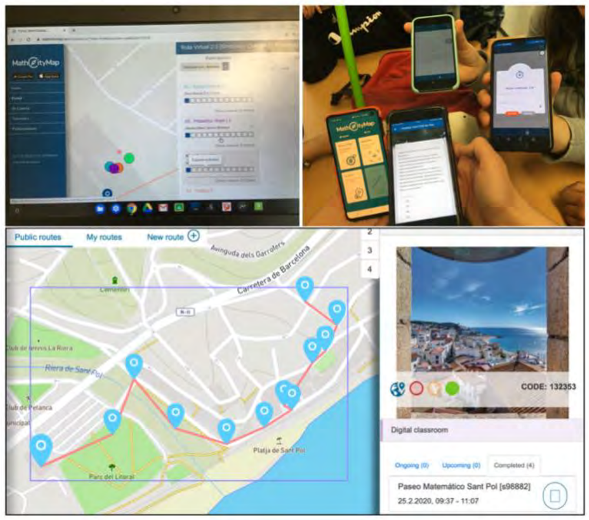

Figure 3: Images of the MathCityMap platform and the app installed in students' mobiles.

Figure 3 shows some images of the route. In the picture top-left can be observed a map with dots which shows students geolocated. In the picture top-right students prepare in groups the strategies for the walk. In the picture bottom-left is showed the path designed in MathCityMap platform and in the picture bottom-right is showed the representative picture of the all walk. The access code to do the walk from the platform is 1323552. Figure 4 shows a sample of the activity two, when students need to model a little house at a children park.

Evaluation of the project II

The project II was evaluated by the students through a Google form to measure the mathematical knowledge generated with the activities, data collection, analysis, interpretation and degree of satisfaction. A remarkable aspect of the survey is that 65% of the students answered in a positive way about the math learning contents improved. Summarizing, the 53% could solve the tasks in the planned time for the walk. The 49 % did not ask for any help. Finally, the 71% was agree that the math walk is interesting, the 12% was strongly agree whereas the 16% was undecided or disagree, which in general is a good result.

Figure 4: Activity 2 of the walk: (a) is the location and (b) is the GeoGebra model of one object from the Park.

CONCLUSIONS AND FUTURE WORK

In general, the objectives of both projects have been achieved enabling students to learn 3D geometry in a meaningful and efficient way. Project I which is still open, is evolving while adds a variety of models 2D and 3D to its collection that has been tested and are almost ready to apply in schools. This initiative involves schools from Maresme Sud that will create the future walks in a collaborative way. To accomplish that connection, MathCityMap platform will be useful while schools interact with each other. In project II, students found the experience very successful. However, some geometric models were very complex leading to a slow develop of the activity due to the limitation of the graphic card of their devices. Further, they came up with some suggestions for improve their routes and some students get encouraged to create their own activity and add it to the route. In the future, we are planning merge both projects and extend the mathematical routes around Catalunya by creating a large network with our own didactical stamp.

References

Jablonski, S., Lázaro del Pozo, C., Ludwig, M. & Recio, T. (2020) MathCityMap, paseos matemáticos a través de dispositivos móviles. *Uno Revista de Didáctica de las Matemáticas*, (087), 1/2020, S. 47-54.

Richardson, K.M. (2004) Designing math trails for the elementary school. *Teaching Children Mathematics*, 23(5), p. 269-272.

Cahyono, A. N. & Ludwig, M. (2019) Teaching and Learning Mathematics around the City Supported by the Use of Digital Technology. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(1), em1654. https://doi.org/10.29333/ejmste/99514

Ludwig, M., & Jablonski, S. (2019). Doing Math Modelling Outdoors - A Special Math Class Activity designed with MathCityMap. In: *5th International Conference on Higher Education Advances (HEAd'19) Universitat Politècnica de València*. p. 901-909

Muñoz J.L. (2020) Realidad Aumentada. http://www.mat.ucm.es/~joseluismunoz/avg/geo_aumentada/mobiliario_urbano.html

MATH TRAILS THROUGH DIGITAL TECHNOLOGY: AN EXPERIENCE WITH PRE-SERVICE TEACHERS

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Abstract. *This paper describes part of an on-going study that aims to understand the potential of digital technology in outdoors mathematics from the perspective of pre-service teachers. We followed a qualitative approach and data was collected through observation, two questionnaires and photographic records. The study involved forty-eight participants that used Math City Map to do a math trail in the city centre of Viana do Castelo. Results show that they valued the experience, highlighting the possibility of solving realistic problems, developing cooperative work, critical thinking and the establishment of mathematical connections. They found the app to be user friendly and motivating, mentioning its contribution for students' engagement through active learning, spatial orientation, autonomy and being more interactive then the paper version.*

Key words: Math trails; Problem solving; Mathematical connections; STEM education; Teacher training.

INTRODUCTION

This paper has a strong support on previous work developed by the authors in the scope of outdoor mathematics. Several studies conducted with pre-service teachers (e.g. Barbosa & Vale, 2016; Barbosa & Vale, 2018; Vale, Barbosa & Cabrita, 2019) have shown that the outdoors can be seen as a privileged educational context, which promotes positive attitudes and additional motivation for the study of mathematics. In particular, math trails have great potential for making more visible the connections between mathematics and everyday life, specifically the environment that surrounds us. These studies focused mainly on a particular detail of the math trails and that was task design, approaching different aspects of problem posing, using a *mathematical eye* to formulate tasks that highlight connections with daily life. Being part of the Consortium of the Project *Math Trails in School, Curriculum and Educational Environments in Europe* (MaSCE[3] - part of the Erasmus+ Programme, Key Action 2 – Strategic Partnerships under the number: 2019-1-DE03-KA201-060118), gave us the opportunity to contact with a different approach to math trails, adding the possibility to resort to digital technology. The use of Math City Map (MCM), a project of the working group MATIS I (IDMI, Goethe- Universität Frankfurt) in cooperation with Stiftung Rechnen, has been reported as having a positive impact in supporting teachers and students in the process of teaching and learning mathematics outside the classroom (e.g. Cahyono & Ludwig, 2019; Ludwig & Jablonski, 2019). Having the conviction that these approaches are extremely relevant in mathematical education and also in the development of certain skills expected from students in the 21st century, it is our purpose in this study to understand the potential of digital technology in outdoors mathematics from the perspective of pre-service teachers.

THEORETICAL FRAMEWORK

One of the core ideas of this paper is that of Math Trail. Hence, is pertinent to begin by delimiting this concept. We consider a math trail to be a sequence of tasks along a pre-

Barbosa, A. & Vale, I. (2020). Math Trails through Digital Technology: An Experience with Pre-Service Teachers. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 47-54). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.06

planned route (with beginning and end), composed of a set of stops in which students solve mathematical tasks in the environment that surrounds them (Vale, Barbosa & Cabrita, 2019, adapted from Cross, 1997). This is a context that offers rich learning experiences to the participants, with the advantage of enabling the exploration of mathematical concepts stated in the curricular guidelines, aspect that can be seen as an advantage in the teachers' perspective (e.g. Barbosa & Vale, 2018; Vale, Barbosa & Cabrita, 2019). By experiencing a math trail, the participants can use and apply mathematical knowledge learned in school and also mobilize informal daily life knowledge. Beyond this possibility there is a wide range of skills summoned by outdoor education like problem solving, critical thinking, collaboration, communication, reasoning or establishing connections. For all the stated reasons, we must consider that it is important to complement the work developed inside the classroom with experiences in the outdoors, allowing students to discover and interpret the world beyond those four walls and accepting that education can take place in different contexts (Kenderov et al., 2009).

In a math trail the participants come into contact with realistic problems that highlight the usefulness of mathematics, but more than that amplify the possibility of establishing connections between mathematics and reality. This feature can be a game changer in inducing positive attitudes towards this discipline (e.g. Bonotto, 2001; Borromeo-Ferri, 2010), relying specially on curiosity, motivation and interest. Beyond solving realistic problems, in this context we must not forget the influence produced by movement in students' attitudes. Thinking and learning 'are not just in the head'; on the contrary, the body plays a decisive role in the entire intellectual process, from the first to the last years of our lives. Students who move, either in the classroom or in the outdoors, can learn, regardless of their activity, more effectively than those in typically sedentary classrooms (Hannaford, 2005). Alongside cognitive engagement, math trails bring into the table two other dimensions: physical and social engagement (Hannaford, 2005). The interaction between these dimensions, facilitated by a math trail, is in line with active learning, known by committing students to the learning process, hence promoting positive attitudes towards mathematics (e.g. Vale & Barbosa, 2018).

Nowadays, mobile devices are fully integrated in our daily lives and, consequently, in the lives of students from very young ages. Teachers should be more aware of this fact and try to keep up with this trend incorporating resources of this nature into their practices. In addition to following the development and needs of contemporary society, it is also important to state that mobile devices are becoming a tool with great potential in both classrooms and outdoor learning, enhancing students' learning and allowing the access to important information in different places and in a more aesthetic manner (Sung, Chang & Liu, 2016). In general, the influence of technology and the immediate availability of information inevitably have been recently shaping the ideas and skills to be developed by students as we move along the 21st century. Moving beyond the mere use of technology to a perspective of integration with other areas of knowledge, the STEM fields (Science, Technology, Engineering and Mathematics) are being highlighted, both in curricula and in literature, as an interconnected approach that brings opportunities for students to be engaged in an active learning perspective, solving realistic problems (NCTM, 2014, 2018). Refocusing the discussion on mobile devices, it is important to state that the diversity of learning opportunities offered by this type of technology (e.g. portability, allowing immediate learning and quick access to information, motivation, facilitating communication between the teacher and the students) can make STEM education more interesting and enjoyable for students, widening the possibilities for engagement in STEM subjects, inside but also outside the classroom (e.g. Kärkkäinen & Vincent-Lancrin, 2013). The extension of the classroom to the outdoors is facilitated by the portability and wireless functionality of the mobile devices, which presents students with a more authentic and appropriate context (Cahyono & Ludwig, 2019). Digital technology can help develop a deeper understanding of mathematics, acting as a mind tool that facilitates inquiry, decision making, reflection, reasoning, problem solving and collaboration (Fessakis, Karta & Kozas, 2018).

METHODOLOGY

This is an on-going study that follows an interpretative qualitative methodology (Erickson, 1996). The participants are forty-eight students of an undergraduate teacher training course in primary education (6-12 years old). These pre-service teachers attend a unit course on Mathematics Education that acts as the context for the development of the study. Knowing that, so far, the participants did not have significant experiences working mathematics outside the classroom, we chose to start with an activity of this nature. Initially they completed a questionnaire (Questionnaire I) that aimed to access their perceptions about the teaching and learning of mathematics outside the classroom and also about the use of technology in that type of context. Then they had the opportunity to do a math trail using Math City Map (MCM), which was designed by the researchers to be used in the historical centre of the city of Viana do Castelo, Portugal. The pre-service teachers worked in groups of 3 and 4. They attributed the responsibility of the use of the app/smartphone to one of the elements of the group, while the others were in charge of the measurements, calculations and registers. After doing the trail they completed a second questionnaire (Questionnaire II), applied with the purposed to analyse eventual changes on the perceptions of the participants about outdoor mathematics and the use of technology, specifically the MCM app.

Data was collected in a holistic, descriptive and interpretive manner and included observations (of the pre-service teachers doing the math trail), questionnaires, photographs and written productions (solutions of the tasks). The latter were not used in this specific study. The researchers accompanied the participants during the trail, a fact that facilitated the accomplishment of the observation, allowing the access to reactions, comments, questions and attitudes. Since we had forty-eight participants, to maximize the observation, we chose to divide the group in half and do the math trail with each group separately. The questionnaires contained mainly open-ended questions, so that the content analysis focused on finding categories of responses regarding the perceptions evidenced by the participants, which were crossed with the evidences collected through the observation.

MAIN RESULTS AND DISCUSSION

Starting by analysing the results of Questionnaire I, we concluded that 91% of the participants considered that it is possible to teach and learn mathematics outside the classroom. The examples presented varied between: tasks related to real life situations; counting activities; money related tasks; shopping activities; games; competitions; clubs;

field trips; observing architecture/artwork/shapes in the outdoors; finding mathematics in nature, like patterns/shapes; doing a trail/peddy paper. 87% of the participants revealed that they have never had a mathematics class outdoors, which in a certain way may explain the general and vague ideas they had about how to do it. So, this fact indicates that, in order to incorporate certain methodologies in their future practices, pre-service teachers have to experience them first. As for technology knowledge, 60% of the participants stated that they did not know any digital resources to explore mathematics outdoors. The 40% that admitted knowing resources used for this purpose mentioned digital games, apps and robots, but none of the examples given allowed the exploration of the surrounding environment, they only had a playful strand.

Before going to the city centre to do the math trail with MCM, the participants had a brief session about the use of Math City Map. They came into contact with the main features of the app, to get acquainted before the activity, and downloaded the trail to the smartphones. Then the researchers accompanied them to the location of the trail and supervised the activity, which facilitated the observation of certain aspects. Regarding the use of the app, we can say that they didn't show noteworthy difficulties. They found it to be very intuitive and were extremely autonomous throughout the trail. The gamification feature was an extra motivating factor: on one hand it caused excitement when the solution was correct; and implied greater care before the introduction of the answers, which was reflected on several situations where the participants tried to make sure of the validity of the answer discussing it within their group. The dynamics of the math trail using MCM naturally promoted collaborative work, in each group, dividing responsibilities (e.g. smartphone; measurement; recording data; calculations), or even among different groups cooperating with the same goal in mind.

Throughout the mathematical trail, participants went through iconic points of the city of Viana do Castelo, having the opportunity to solve tasks centered on different mathematical contents (e.g. geometric transformations, patterns, measurements, estimates, areas, volumes, direct proportionality, visualization, counting). This dynamic allowed them to use a different lens, exploring the environment through a mathematical eye, but, at the same time, they could know better the city where they live in, observing more closely the elements directly worked on. In figures 1 and 2 we can observe different moments of the trail implementation that illustrate the pre-service teachers' work.

Figure 1: Pre-service teachers using Math City Map.

Figure 2: Pre-service teachers using Math City Map.

Throughout the trail it was possible to witness reactions and comments from the pre-service teachers that we think are relevant and must be emphasized because they reveal engagement: the trail gave them the opportunity to get to know better certain aspects of the city, related to historical and architectural features that they did not know of; many expressed interest in using the app with their future students; we identified a generalized satisfaction throughout the activity; they valued the need to move around, opposed to the sedentary work traditionally developed inside the classroom.

After experiencing the math trail with MCM, the pre-service teachers filled Questionnaire II. From the analysis of the results we were able to conclude that all the participants recognized the importance of teaching and learning outside the classroom, especially as a way to complement the formal educational context. Contrary to the results obtained through Questionnaire I, they were all convinced, with no exception, that teaching and learning mathematics outside the classroom is possible, showing that some of these pre-service teachers changed their opinion about this issue. Those who already though that this strategy was a possibility, stated it with even more emphasis, admitting that the experience exceed their expectations. We found several arguments supporting these ideas: follows the principles of active learning, promoting intellectual, social and physical engagement; learning is more meaningful for the students because they are directly involved; increases motivation and enthusiasm; helps understand the usefulness of mathematics, realizing its application in real life problems; allows to increase the knowledge of the cultural and natural heritage; facilitates collaborative work and helps develop communication skills, as well as critical thinking; it can lead to the use of technology.

The majority of these pre-service teachers expressed that they liked to solve all of the tasks presented along the trail, which is consistent with the observed motivation and enthusiasm. The tasks pointed as favourites corresponded to those considered to be the most challenging or the ones that presented information/curiosities/historical aspects about certain elements of the city that they did no know about. On the other hand, the least favourites were the ones that involved too many steps during the solution process, which led them to make some mistakes or to find the task to be too exhausting.

In this questionnaire the participants also commented on the use of MCM and its features. From the users/students perspective they highlighted as potentialities: the possibility to use curricular contents in real life situations; being user friendly and easy to understand, promoting autonomy; facilitating cooperation; helps to get to know the local environment;

develops spatial orientation; being more practical and interactive than the paper version; the possibility of getting immediate feedback; and the gamification feature. As for the teachers' perspective, the participants mentioned as potentialities: the possibility to design tasks adapted to the local environment and publishing them; addressing different mathematical contents and promoting interdisciplinary tasks; a way to diversify educational contexts; allows the teacher to supervise and accompany the work developed by the groups, due to the autonomy it provides the user. As limitations of the app, these pre-service teachers only referred to the possible lack of access to Wi-Fi, the fact that students of younger ages normally don't have smartphones and, in terms of the tasks, the limitation of the answer formats to either a value or multiple choice.

CONCLUSION

Based on previous studies developed with pre-service teachers (e.g. Barbosa & Vale, 2016; Barbosa & Vale, 2018; Vale & Barbosa, 2018; Vale, Barbosa & Cabrita, 2019) we had already concluded that designing and implementing math trails can promote positive attitudes towards mathematics and help gain a broader view of the connections we may establish with the surrounding environment. Math trails make mathematics come alive engaging the participants cognitively, emotionally and physically, which is why they can be associated with active learning. This type of experience enhances the "mathematical eye" of the trail designers as well as of the trail users (e.g. Vale, Barbosa & Cabrita, 2019), bringing out the usefulness and applications of mathematics.

This study focused only on the perspective of the trail user and not the designer. We intended to understand the potential of the MCM app in outdoor education from the point of view of pre-service teachers. Globally they valued the math trail experience as a meaningful pathway to engage students in realistic problem solving, that presents a diversity of opportunities for the establishment of connections between mathematics and other content areas, as well as with real life (e.g. Bonotto, 2001; Borromeo-Ferri, 2010). Active learning was also pointed out by the participants as a fundamental trait in a math trail, allowing intellectual, physical and social engagement, whose interaction normally generates positive attitudes (e.g. Hannaford, 2005; Vale & Barbosa, 2018). Math City Map was used as the means to present and execute the trail. This was the additional dimension of this study, trying to perceive its impact. These pre-service teachers valued the use of the app, finding it user friendly and motivating, especially due to the gamification feature. They also mentioned as positive its contribution for developing spatial orientation (moving with the help of the GPS and needing to recognize their position in space), cooperation (through group work and task division), students' autonomy and being more practical and interactive than the paper version. The only limitations recognized by the participants were related to constraints like the possible absence of Wi-Fi or smartphones (for example when working with students of younger ages) and also the limited possibilities for answer formats.

To conclude, when implementing the math trail there was certainly an additional motivation associated to the digital and interactive features of the MCM app, which facilitated and made more interesting the exploration of the outdoors from a mathematical point of view (e.g. Cahyono & Ludwig, 2019). Being pre-service teachers, the participants other than going through this experience as users, they also had the opportunity to assess the potential of the strategy (math trail) and the resource (MCM app) and envision how they could, as teachers, implement it in the future. Recognizing the importance of keeping up with the technological development and society requirements they considered the possibility of integrating this resource, and the math trail strategy, in their practices.

References

Barbosa, A., & Vale, I. (2016). Math trails: meaningful mathematics outside the classroom with pre-service teachers. *Journal of the European Teacher Education Network, 12,* 49-63.

Barbosa, A., & Vale, I. (2018). Math Trails: a resource for teaching and learning. In Gitirana, V., Miyakawa, T., Rafalska, M., Soury-Lavergne, S., & Trouche, L. (Eds.), *Proceedings of the Re(s)sources 2018 International Conference* (pp. 183-186). Lyon: ENS de Lyon.

Bonotto, C. (2001). How to connect school mathematics with students' out-of-school knowledge. *Zentralblatt für Didaktik der Mathematik, 33*(3), 75-84.

Borromeo-Ferri, R. (2010). Estabelecendo conexões com a vida real na prática da aula de Matemática [Establishing connections with real like in the mathematics classroom practice]. *Educação e Matemática, 110*, 19-25.

Cahyono, A. N., & Ludwig, M. (2019). Teaching and Learning Mathematics around the City Supported by the Use of Digital Technology. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(1), 1-8.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 119-161). New York, NY: Macmillan.

Fessakis, G., Karta, P., & Kozas, K. (2018). Designing Math Trails for Enhanced by Mobile Learning Realistic Mathematics Education in Primary Education. *International Journal of Engineering Pedagogy, 8*(2), 49-63.

Hannaford, C. (2005). *Smart Moves: Why learning is not all in your head.* Salt Lake City: Great River Books.

Kärkkäinen, K., & Vincent-Lancrin, S. (2013). Sparking Innovation in STEM Education with Technology and Collaboration: A Case Study of the HP Catalyst Initiative, OECD Education Working Papers, No. 91, OECD Publishing.

Kenderov, P., Rejali, A., Bartolini Bussi, M., Pandelieva, V., Richter, K., Maschietto, M., Kadijevich, D., & Taylor, P. (2009). Challenges Beyond the Classroom—Sources and Organizational Issues. In E. Barbeau & P. Taylor (Eds.), *Challenging Mathematics In and Beyond the Classroom – New ICMI Study Series 12* (pp. 53-96). Springer.

Ludwig, M., & Jablonski, S. (2019). Doing Math Modelling Outdoors - A Special Math Class Activity designed with MathCityMap. In *5th International Conference on Higher Education Advances (HEAd'19)* (pp. 901-909). Editorial Universitat Politecnica de Valencia: Valencia*.*

National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM) (2018). *Building STEM Education on a Sound Mathematical Foundation*. Retrieved from: https://www.nctm.org/Standards-and-Positions/Position-Statements/Building-STEM-Education-on-a-Sound-Mathematical-Foundation/

Sung, Y., Chang, K, & Liu, T. (2016). The effects of integrating mobile devices with teaching and learning on students' learning performance: A meta-analysis and research synthesis. *Computers & Education, 94*, 252-275.

Vale, I., & Barbosa, A. (2018). Estratégias para promover uma aprendizagem ativa da matemática. [Strategies for promote an ative mathematical learning] In A. Rodrigues, A. Barbosa, A. Santiago, A. Domingos, C. Carvalho, C. Ventura, C. costa, H. Rocha, J. M. Matos, L. Serrazina, M. Almeida, P. Teixeira, R. Carvalho, R. Machado & S. Carreira (Eds.) *Livro de Atas do EIEM 2018* (pp. 517-520). Coimbra: Escola Superior de Educação do Instituto Politécnico de Coimbra.

Vale, I., Barbosa, A., & Cabrita, I. (2019). Mathematics outside the classroom: examples with preservice teachers. *Quaderni di Ricerca in Didacttica (Mathematics)*, *2*(3), 138-142.

MOBILE-SUPPORTED OUTDOOR LEARNING IN MATH CLASS: DRAFT OF AN EFFICACY STUDY ABOUT THE MATHCITYMAP APP

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Abstract. *Both, mobile learning and extracurricular learning, offer great potentials for mathematics education. An interesting approach which combines those two concepts is the math trail idea: Students work outside on mathematical problems related to real-existing objects. Thereby, the mathematical learning progress can be structured by the MathCityMap app. The app guides students to the task situations, shows the task and gives hints. The subsequent described study shall examine the possible impact of long-term mobile-supported outdoor learning on mathematical skills by using the MathCityMap app. Therefore, twenty classes of eighth graders (Hessian Gymnasium) will be accompanied in the school year 2020/21. While one experimental group works on math trails by using the MathCityMap app, a second experimental group will work on a paper-and-pencil math trail. Within five tests, the learning progress of both groups will be compared among each other's and with a control group.*

Key words: Efficacy Study, Extracurricular Learning, MathCityMap App, Math Trails, Mobile Learning

CONCEPTUAL FRAMEWORK

Mathematical education in school should enable students to acquire long-term mathematical competencies. Two promising didactical approaches – namely mobile and extracurricular learning – will be presented in this section. The mobile-supported outdoor learning, which is the intersection of both didactical concepts, will complete the theoretical introduction.

Learning in a digital world: Digital Media and Mobile Learning

Digital Media are defined as "media which save or transfer information by means of electronic devices and replay them in an iconic or symbolic representation" (Pallack, 2018, p. 28; translation by SB). In the educational field, their usage raises numerous expectations. For instance, Krauthausen (2012) mentions the higher level of student motivation, the possibility of playful and discovery-based learning as well as potentials for internal differentiation.

Drijvers, Boon and Van Reeuwijk (2010) name two functions of digital technology. They distinguish 'Do Mathematics' and 'Learn Mathematics'. 'Do Mathematics' means that the digital device takes over an activity which could be carried out by the student manually. 'Learn Mathematics' is split into the aspects 'Practicing Skills' (training function) and 'Developing Concepts', which focus on the development of mathematical understanding.

Mobile learning is characterized as the usage of smart devices like tablets or smartphones in an educational context (Park, 2011). The computing power and portability of these devices as well as the possibilities of wireless communication and digital tools offer great potential for both traditional classrooms and extracurricular learning (Sung, Chang, & Liu, 2016). Based on the named arguments, many expectations are linked to the idea of Bring Your Own Device (BYOD) which describes the usage of students' private mobile devices for educational purposes. The large availability of private mobile devices – in Germany 97

Barlovits, S. & Ludwig, M. (2020). Mobile-Supported Outdoor Learning in Math Class: Draft of an Efficacy Study about the MathCityMap App. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 55-62). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.07

percent of the 12 up to 19-years-old teenagers own a smartphone (Medienpädagogischer Forschungsverband Südwest, 2018) – indicates that students are well-trained in the handling of those devices. In contrast to the usage of foreign devices, BYOD can minimize operating problems (Schiefner-Rohs, 2017). However, some challenges inherit the concept of BYOD. Firstly, different operating systems like iOS or Android may lead to technical problems. Secondly, BYOD urgently calls for a boost of students' media competence. In total, BYOD offers many potentials for the educational field but has to be integrated with caution to take advantage of it (ibid.).

Learning in one's own environment: Extracurricular Learning

A unified definition of extracurricular learning does not exist, however all educational activities outside the classroom are subsumed under this term (Sauerborn & Brühne, 2014). Therefore, extracurricular learning takes place whenever students work outside the school on "an original learning topic under aimed pedagogical instruction" (ibid., p. 11; translation by SB). For this study the following definition will be applied:

> "Extracurricular learning […] means a recurrent, in the school routine integrated teaching concept. It includes the immediate, natural environment of the children (as learning environment) by teaching curricular contents and it is closely related to regular lessons in the classroom" (Sitter, 2019, p. 73; translation by SB).

Collecting first-hand experiences, working on real problems and integrating discovery learning are the most important benefits of extracurricular learning (Karpa, Lübbecke, & Adam, 2015). Even though those didactical concepts are important for students' mathematical education, a specific mathematical extracurricular learning concept is still missing. In addition, most research projects about extracurricular learning in mathematical didactics focus on Science Centers (Sitter, 2019).

A promising approach for mathematical extracurricular learning is the idea of math trails. A math trail is a route which consists of several place-bound math tasks, which treat mathematical questions about real existing objects (Cross, 1997). Hence, students can perceive their own environment from a mathematical perspective. Furthermore, students are able to apply their mathematical skills in many different situations which leads to a deeper mathematical knowledge (ibid.). Shoaf, Pollak and Schneider (2004) highlight the value of collaborating. Furthermore, they emphasis that people of all ages can walk and work on a math trail (ibid.).

Digital learning in the own environment: MathCityMap Math Trails

After analyzing the benefits of both, mobile and extracurricular learning, those two approaches will be combined in the term of mobile-supported outdoor learning. This term describes the usage of mobile devices to support learning processes outside the classroom.

The MathCityMap app combines the idea of math trails and technological potentials of smartphones (Ludwig & Jablonski, 2019). The app guides students by GPS navigation to the next task, shows hints and gives immediate feedback on the correctness of a student's solution. Depending on its quality, groups receive up to 100 points per task. The system also provides a sample solution. When using the MathCityMap app, students should work together in groups of three. Only one smartphone or tablet with the installed app (available and free of costs for iOS and Android) is necessary in each group. For that reason, the app could be used for BYOD-based math class (ibid.).

By using the MathCityMap app, the smartphone covers both functions of digital technology, which are described by Drijvers et al. (2010). The smartphone itself can take over the function 'Doing Mathematics', e.g. when the installed calculator is used. On the contrary, the MathCityMap app also fulfills the function of 'Learning Mathematics'. As the examination of interesting, real existing objects is the core of the MathCityMap idea, MathCityMap tasks are target to practice students' mathematical skills. Depending on the task formulation, working on MathCityMap task leads to a deeper understanding of mathematical concepts. The following figure illustrates how tasks presented by the MathCityMap app can take over the function 'learning mathematics' with its aspects of practice and concept development.

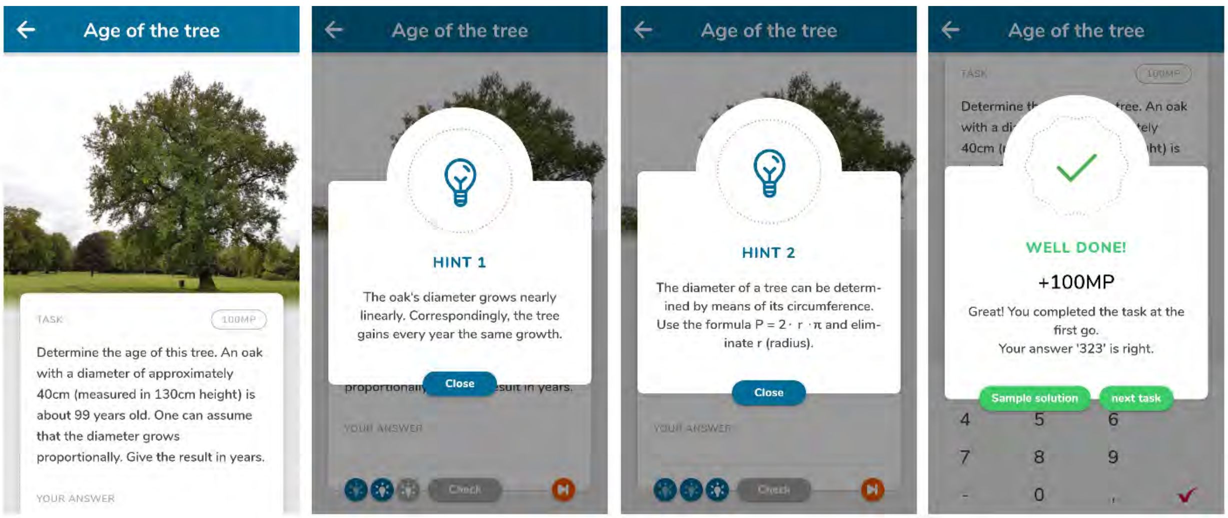

Figure 1: Presentation of math trail tasks by the MathCityMap app.

STATE OF THE ART

The MathCityMap app provides two useful tools, namely the gamification feature Leaderboard and the organizational feature Digital Classroom. Both tools will be described in the following. In addition, the work of Zender (2019), which should be extend by this study, will be summarized.

The optional feature Leaderboard, which lists the reached points for each group in a ranking, leads to a significant higher motivation of the students (Gurjanow, Olivera, Zender, Santos, & Ludwig, 2019). It also seems to increase the rate of completed tasks per hour. By using the feature Leaderboard, 3.4 tasks per hour were completed in average, whereas students only work on 2.8 tasks per hour, if no gamification element was used (ibid.). Furthermore, the Leaderboard feature reduces blind guessing significantly because the achievable number of points for solving the tasks decreases after the third wrong answer. Without gamification, 3.8 answers per tasks were entered into the MathCityMap system ($SD = 3.2$). If the Leaderboard is used, students input 2.0 answers in average ($SD = 2.0$; ibid.).

If the used mobile devices have internet access, teachers can retrace the students' progress of the math trail via the MathCityMap tool Digital Classroom. The feature provides a chat which enables teachers to communicate with all student group. In addition, it transmits the present location of each group to the teacher and indicates how many tasks were solved by the group. "In alignment with privacy and data protection, it allows the teacher to see the actions of the pupils during the maths trail activity" (ibid., 3), which could pave the way to an effective follow-up of the math trail.

According to Zender (2019), the usage of the MathCityMap app not only leads to a higher degree of motivation, but also to a better performance in mathematical tests. In a study with more than 500 German ninth graders, he could show that those students, which used the MathCityMap app for working on cylinder tasks, reached significantly better test results than the control group (Cohen's d = 0.8). This indicates a positive short-term effect of the mobile-supported outdoor learning with MathCityMap.

Zender (2019) also analyzed the long-term impact of math trails. In the follow-up test the experimental group could replicate the results of the post-test, whereas the results of the control group clearly decreased (d = -0.8). However, only 64 test persons (22 of the control group; 42 of the experimental group) have completed the follow-up test. Overall, those results suggest a positive long-term effect of extracurricular learning by using MathCityMap (ibid.). Nevertheless, a deeper investigation seems to be necessary. The below described study should close this research gap.

MOTIVATION AND RESEARCH QUESTIONS

The research project pursues the empirical target to analyze the short- and long-term learning progress of students in mathematics by using both mobile and extracurricular learning repeatedly. From a theoretical position, the study focusses on possibilities to embed the math trail method in regular school classes. The following research questions should be answered as a result of the study.

Embedding of MathCityMap math trails

- How could math trails be embedded in regular school lessons?

So far, math trails are often used and well-tested for the revision of learned content. Therefore, MathCityMap math trails usually cover a wide range of mathematical topics. In the framework of this study, theme-based math trails should be created. A math trail is defined as theme-based, if at least every second tasks refers to one special mathematical content.

The first theoretical outcome of the study will be an analysis of the Hessian curriculum and school books (eighth grade of the Hessian Gymnasium) to develop characteristic task types for theme-based math trails. Those theme-based trails will be created as a basis for the later empirical analysis of students' learning progress by using the math trail method.

- How an effective preparation and follow-up of math trails can be realised?

To stimulate and strengthen the outcome of extracurricular Learning, its embedding in the regular class is essential (Sitter, 2019). The necessary preparations for extracurricular learning include organizational, didactical and content-related parts. Sauerborn and Brühne (2014) name the scheduling of the activities as well as obtaining the essential approvals (parents and school management) as the main organizational challenges. The didactical preparation encompasses the topic at hand and the learning goal, while content-related parts refer to the introduction of the extracurricular learning in class. The follow-up is characterized by students' reflection about the extracurricular learning and by securing its main contents (ibid.).

Until now, a detailed conceptualization about preparations before and follow-up after a math trail is missing. For that reason, a second theoretical output of the study will be the development of such a concept.

Impact of MathCityMap math trails on students' learning progress

- Which impact has the usage of theme-based MathCityMap math trails in mathematics on the short-term learning progress of eighth graders?

In the described study of Zender (2019), ninth graders worked once or twice on a theme-based trail about cylinders. His research indicates a positive influence of MathCityMap on students' handling of applied math tasks. In the framework of this study, this observation shall be verified for eighth graders of the Hessian Gymnasium. However, the center of the study will be the following research question:

- Which impact has the long-term usage of the MathCityMap app in mathematics on the long-term learning progress of eighth graders?

The research project should not only replicate but extend the study of Zender (2019). Eighth graders of the Hessian Gymnasium shall work for one school year periodically on theme-based math trails using the MathCityMap app. The aim is to analyze the influence of mobile-supported outdoor learning using MathCityMap as an example.

METHODOLOGY

To analyze the long-term effect of repeated mobile-supported outdoor learning by using the MathCityMap app, approximately twenty classes will be accompanied in the school year 2020/21. The study includes one pre-test, four treatment phases and a final follow-up-test.

At the beginning of the school year 2020/21, the pre-test is carried out. The mathematical knowledge of the eighth graders (Hessian Gymnasium) is tested by past items of the German comparative study VERA-8. The test comprises a wide range of already learned mathematical topics from arithmetic, algebra, geometry and basic stochastical problems. Based on the results of the 90-minutes pre-test, the classes are divided into two experimental groups and one control groups (Table 1), so that all three groups possess a similar distribution of mathematical skill among the students.

Both experimental groups, consisting of ten respectively of five classes, work twice on theme-based MathCityMap math trails concerning four different topics. Whereas the Experimental Group I (EG I) uses the MathCityMap app including the app features hints and feedback, the Experimental Group II (EG II) works on the math trail with the 'classic' paper-and-pencil method. Thereby, both groups receive the same task formulations; once via digital media (EG I) and once by a printed trail guide (EG II).

The control group (CG), which consists of five classes, works on applied tasks inside the classroom. Those tasks are designed similarly to the math trail tasks including hints and sample solutions; however, the measured data are given in the task formulation. To allow the comparison of EG I and the control group, the tasks are presented by the MathCityMap app. In addition, a didactic analysis of the tasks presented to the students in the different settings (EG I: with app outdoors; CG: with app inside the classroom) seems to be necessary before starting the study.

We decided to dispense with further control groups to prevent an unnecessary expending of the study design. Several empirical studies (e.g. Reinhold, 2019) showed in the last years that the usage of digital media, respectively of apps, per se has no effect on the learning progress of students of the German Gymnasium. By implication, this indicates that the learning progress of students inside the classroom does not primarily depend on the media by which the content is presented. Therefore, we assume that our analysis of the control group can be easily transferred to the work on applied tasks which are given in a printed version.

Experimental Group I	10 classes	math trails	MathCityMap app
Experimental Group II	5 classes	math trails	printed trail guide
Control Group	5 classes	applied tasks	MathCityMap app

Table 1: Design of the experimental groups and the control group.

During the four treatment phases, each group receives two treatments of a double lesson each time. In total, the study includes treatments of eight double lessons plus pre- and post-test (Table 2). Every treatment is referred to one topic of the Hessian curriculum for the eighth grade. The four identified topics for eighth graders of the Hessian Gymnasium, which can be covered by theme-based math trails, are linear functions, interest calculation and basic stochastical problems. The fourth topic is referred to the Hessian comparison test Mathematikwettbewerb, which takes place at the end of the first half-year and includes arithmetic, algebraic, geometric and stochastic tasks.

2020/21 1st half-year			**2020/21** 2st half-year		**2021/22** 1st half-year
pre-test	two treatments á 90 min per topic				follow-up test
	linear functions	comparison test	interest calculation	stochastical problems	

Table 2: Timetable of the planned study in the school year 2020/21.

After completing a topic, a performance test of 45 minutes is conducted (Table 3). Each test consists of several applied tasks concerning the learning content. This allows us to identify the students' short-term learning effect on this topic. In addition, it enables us to compare the learning progress of the three groups.

Three months after the last treatment (beginning of school year 2021/22), a 90-minutes follow-up test is carried out. All items of the follow-up test are taken from the performance tests. The follow-up test has two functions. Firstly, this proceeding enables us to compare directly the students' results of the performance tests and the follow-up test. In other words, the repeated use of the test tasks documents the students' recollection of the four topics, i.e. their long-term learning effects. Secondly, a comparison of the pre-test and the final test makes it possible to analyze each student's learning progress – and thereby the absolute learning progress of each group, too.

Experimental Group I	two double lessons á 90 min	MathCityMap math trail	performance test á 45 min
Experimental Group II		‚classic' math trail	
Control Group		applied tasks	

Table 3: Procedure of the four treatments.

The two empirical research questions contain implicit the question after the influence of mathematical extracurricular learning, mobile learning as well as the impact of mobile-supported outdoor learning on students' learning progress.

To analyze the role the math trail method, we will compare the test results of EG I and the control group. As both groups work on similar applied tasks which are presented by the MathCityMap app, we are able to identify the variable 'math trail'. The impact of mobile learning can be examined by comparing the two experimental groups. Overall, we expect that this study design allows us to analyze how mobile-supported outdoor learning influences the mathematical learning progress of students.

EXPECTED RESULTS

The described study shall extend the work of Zender (2019), which shows a positive shot-term learning effect of MathCityMap math trails. It also indicates that the usage of MathCityMap math trails has a positive long-term influence on students' handling of applied math tasks. We assume that this learning effect is caused by first-hand experience and the students' necessity to collect data themselves. Therefore, we expect both a better short- and long-term test performance of the two experimental groups in comparison to the control group, which works inside the classroom on applied tasks. To formulate this hypothesis in regard to the conceptual framework, we assume that extracurricular learning by using the math trail method deepens and reinforce the learning experience.

From a theoretical perspective, it can be assumed that using the MathCityMap app (EG I) has several advantages compared to the paper-and-pencil trail guide (EG II), especially the possibility to call up stepped hints, to get an immediate feedback on the correctness of the solution or to show the sample solution (Ludwig & Jablonski, 2019). Both features can students help to structure and to organize their work progress and to validate their solution subsequently (ibid.). However, it remains to be seen whether the use of the MathCityMap app will lead to a better learning progress of EG I when comparing the two experimental groups. At least, previous studies on learning with the MathCityMap app suggest that using the app leads to a significant and lasting increase in the motivation of students working on math paths (Cahyono, 2018; Gurjanow et al., 2019).

References

Cahyono, A. N. (2018). *Learning Mathematics in a Mobile App-Supported Math Trail Environment*. Cham: Springer International.

Cross, R. (1997). Developing mathematics trails. *Mathematics Teaching,* (158), pp. 38-39.

Drijvers, P., Boon, P., & Van Reeuwijk, M. (2010). Algebra and Technology. In P. Drijvers, *Secondary Algebra Education. Revisiting Topics and Themes and Exploring the Unknown*. Rotterdam: Sense, pp. 179-202.

Gurjanow, I., Olivera, M., Zender, J., Santos, P., & Ludwig, M. (2019). Mathematics Trails: Shallow and Deep Gamification. *International Journal of Serious Games*, pp. 65-79.

Karpa, D., Lübbecke, G., & Bastian, A. (2015). Außerschulische Lernorte. Theoretische Grundlagen und praktische Beispiele [Extracurricular Learning. Theoretical background and practical examples]. *Schulpädagogik heute*, *6*(11), pp. 1-13.

Krauthausen, G. (2012). *Digitale Medien im Mathematikunterricht der Grundschule* [Digital Media in math class at primery school]. Berlin, Heidelberg: Springer-Verlag.

Ludwig, M., & Jablonski, S. (2019). Doing Math Modelling Outdoors. A Special Math Class Activity designed with MathCityMap. *5th International Conference on Higher Education Advances*, (pp. 901-909). Valencia, Spain: HEAd'19.

Medienpädagogischer Forschungsverband Südwest (mpfs) (2018). *JIM-Studie 2018. Jugend, Information, Medien. Basisuntersuchung zum Medienumgang 12- bis 19-Jähriger* [JIM study 2018. Teenagers, Information, Media. Basic exploration of the media use among 12- to 19-years-old]. Stuttgart.

Pallack, A. (2018). *Digitale Medien im Mathematikunterricht der Sekundarstufen I + II* [Digital Media in math class on secondary level]. Berlin: Springer Spektrum.

Park, Y. (2011). A pedagogical framework for mobile learning: Categorizing educational applications of mobile technologies into four types. *The international review of research in open and distributed learning, 12*(2), pp. 78-102.

Reinhold, F. (2019). *Wirksamkeit von Tablet-PCs bei der Entwicklung des Bruchzahlbegriffs aus mathematikdidaktischer und psychologischer Perspektive* [Effectiveness of tablets in the development of the fractional number concept from a mathematical-didactic and psychological perspective]. Wiesbaden: Springer Spektrum.

Sauerborn, P., & Brühne, T. (2014). *Didaktik des außerschulischen Lernens* [Didactic of Extracurricular Learning]. Baltmannsweiler: Schneider-Verlag Hohengehren.

Schiefner-Rohs, M. (2017). Smartphones machen keine neue Schule. Veränderungspotentiale digitaler Medien kritisch reflektieren [Smartphones do not lead to a new school. A critical reflection on potentials of change forced by digital media]. *Lernende Schule,* (79), pp. 25-27.

Shoaf, M. M., Pollak, H., & Schneider, J. (2004). *Math Trails.* The Consortium for Mathematics and Its Applications (COMAP).

Sitter, K. (2019). *Geometrische Körper an inner- und außerschulischen Lernorten. Der Einfluss des Protokollierens auf eine sichere Begriffsbildung* [Geometrical solids in school and in Extracurricular learning Environments. The impact of logging on conceptualisation]. Wiesbaden: Springer Spektrum.

Sung, Y.-T., Chang, K.-E., & Liu, T.-C. (2016). The effects of integrating mobile devices with teaching and learning on students' learning performance: A meta-analysis and research synthesis. *Computers & Education*, (94), pp. 252-275.

Zender, J. (2019). *Mathtrails in der Sekundarstufe I. Der Einsatz von MathCityMap bei Zylinderproblemen in der neunten Klasse* [Math trails on secondary level. The usage of MathCityMap for cylindrical problems in the ninth grade]. Münster: WTM-Verlag.

ANALYSIS OF STUDENT-TEACHER CHAT COMMUNICATION DURING OUTDOOR LEARNING WITHIN THE MCM DIGITAL CLASSROOM

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Abstract. *Mathematics trails offer students the possibility to apply their knowledge in authentic outdoor contexts. Organizing and carrying out such an activity is a challenge for teachers. The MathCityMap project digitizes the math trail idea and consists of a smartphone app for students to run math trails and a web portal for teachers to create math trails. Recently, a new feature called the 'digital classroom' was introduced, which allows teachers to track their students' progress in real-time and communicate with them via chat., offering the teachers the possibility to give synchronous feedback. In this article, we examine the use of the chat functionality against the theoretical background of challenges of outdoor learning environments and how technologies help to overcome them. A quantitative analysis of ten Digital Classrooms used in authentic settings shows that more than half (54.3%) of the messages sent in total during those sessions were about organizational aspects, whereas 26.4% contained didactical aspects. We therefore conclude, that the Digital Classrooms provides teachers with a valuable functionality for the conduction of math trails.*

Key words: math trails, mathematics, smartphone, app, mobile learning, learning analytics

INTRODUCTION

Mathematics in out-of-school situations is a proven concept for popularizing mathematics and offers many possibilities for use in the school context. As early as the 1980s, Blane and Clarke (1984) published a so-called math trail, which enabled interested individuals and families to take a tour in the city of Melbourne where mathematics could be discovered. For this purpose, the authors designed mathematical questions on interesting objects, which were intended to encourage visitors to discuss, calculate and develop questions. Ludwig and Jesberg (2015) presented the idea of combining the concept of math trails with the opportunities of modern smartphones in the MathCityMap (MCM) project. This article aims to identify potentials and challenges of math trails in general, and to examine the use of the chat functionality of the newly developed Digital Classroom.

THEORETICAL BACKGROUND

Benefits of outdoor learning

Sauerborn and Brühne (2009, p. 79) list the possibility to make first-hand experiences, illustrative tasks, increased interest and the possibility to make connections between subjects as theoretical benefits of outdoor learning. Numerous studies confirm these theoretical advantages. Falk and Dierking (1997, p. 211) report that about 96% of the participants have long lasting memories of excursions and could re-call details, such as activities or other participants, even years after the event. In a study with 60 twelve to fourteen years old, Wijers, Jonker and Drijvers (2010, p. 789) asked the students to construct parallelograms (also special cases of parallelograms, such as rectangles or squares) outside the classroom and with the help of a native smartphone app that uses GPS.

Baumann-Wehner, M., Gurjanow, I., Milicic, G. & Ludwig, M. (2020). Analysis of Student-Teacher Chat Communication during Outdoor Learning within the MCM Digital Classroom. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age* (pp. 63-70). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.08

The study authors report that the participants were highly motivated and learned about using the GPS-system, reading maps, and constructing parallelograms.

In a case study, Swedish researchers investigated the experience of regular, extracurricular learning of 14 students in lower and middle school (Fägerstam & Grothérus, 2018). Over a period of three years, one lesson per week in mathematics and German (third foreign language) was taught in the schoolyard. The following student quotes demonstrate that learning outdoors has triggered positive emotions in the students.

> I am not so good at maths, I don't know why; I must be born that way. But, sometimes it is fun. To be able to learn you need to think that something is fun, otherwise it doesn't work. With outdoor lessons you don't get bored and you learn easier and don't think it is quite as boring anymore. /…/ They [teachers] should do it [have outdoor lessons] because you can sense how students enjoy it and they pay more attention.
> Nelly, grade 9
> (Fägerstam & Grothérus, 2018, S. 383)

In this quote it is clear that the student finds it easier to learn mathematics when she enjoys it. This seems to be the case with out-of-school learning, which is why she advocates more outdoor teaching. Another student in the ninth-grade stresses that, unlike in the classroom, one can try out procedures practically outside (Fägerstam & Grothérus, 2018, p. 383). This statement speaks for the vividness of tasks in outdoor learning.

Mathematics Trails – Outdoor learning environment for mathematics classroom

Mathematics trails are one way to learn mathematics outside the classroom. According to Shoaf, Pollak and Schneider (2004, p. 6) a mathematics trail is "… a walk to discover mathematics". It can be almost anywhere and a so-called math trail guide "… points to places where walkers formulate, discuss, and solve interesting mathematical problems" (ibid.). In the context of mathematics education in schools, math trails offer opportunities for application of mathematics in real, authentic situations, as well as the modelling activities that precedes the calculations (Gurjanow, Oliveira, Zender, Santos & Ludwig, 2019). The major advantage of math trails over stationary learning facilities like mathematical museums or student labs is the spatial independence. Therefore, teachers may implement their own outdoor learning activities in the near surrounding of their school.

Methodologically, math trails are a form of learning at stations that takes place outside the classroom. Usually, students form groups of three and follow the math trail guide to work independently at the stations. To prevent task stations from getting too crowded, we recommend assigning every group a different starting position. The first empirical study on math trails was probably done by Kaur (1992). Kaur sent 20 students on a trail in the first attempt and developed a questionnaire with ten items in a pre-post design, which is to measure the attitude towards mathematics. It represents a significant difference: the students are aware of the mathematics around them have become more conscious and now see mathematics much more positively than before. Cahyono (2018) and Zender (2019) examined the influence of running math trails on the mathematical performance empirically (using a pre-post-test design). Both conclude that students could improve their scores significantly. In contrast to the Indonesian study (Cahyono, 2018), the significant results of the German study (Zender, 2019) are limited to students that ran a math trail twice and were part of the group of middle-achievers.

Challenges of mathematics trails

There are two main concerns that we will address in this article. The first issue arises from the spatial separation of the math trail tasks, which leads to a loss of overview for the teacher. Consequently, the teacher loses the ability to quickly make organizational adjustments or to support students in case they were stuck. Secondly, we know that learners with low prior knowledge are more likely to be overwhelmed by tasks that require them to take responsibility of their own learning process (Edelmann & Wittmann, 2012). Hence, the outdoor experience may turn into frustration and the students will not continue working on the math tasks.

THE MATHCITYMAP PROJECT (MCM)

The idea of MCM

During the last four years the working group MATIS I in Frankfurt develops a digital tool to support outdoor education with math trails (Ludwig & Jesberg, 2015). By means of the math trail method, students can explore their environment with a focus on mathematics. In the past, it was quite a challenge to create a math trail. The MathCityMap system provides a simplified, digital way of creating and sharing tasks and math trails.

Components of MCM

The MathCityMap system is a two component system that consists of a web portal and an app. The web portal (www.mathcitymap.eu) is a database that can be used to create tasks and trails. A trail is a collection of existing tasks. Those can be either private or public. All tasks are private by default. By request of the task owner, an expert review can be done. A task that fulfills the best practice aspects of MathCityMap will be published and made available for all users. Public tasks can be owned by another member of the worldwide community. The aspect of sharing one's work with others is one of the core features of the portal. The expert review system ensures the general quality of public tasks and trails.

The core feature of the MathCityMap app is giving access to math trails, which were created on the web portal. After downloading a trail to the mobile device, it is possible to run the math trail offline. The MathCityMap app guides students by GPS navigation to the next task, provides hints on demand (for each task the app displays up to three stepped hints) and gives an immediate feedback on the correctness of a students' solution. Depending on its quality, groups receive up to 100 points per task. Finally, the app also provides a sample solution, which can be viewed immediately after solving a task or giving wrong answers for at least six times. Providing a sample solution is mandatory for any author during the creating process of a task. This way, learners can always complete a task with a correct solution, even if they were unable to find the solution themselves.

The Digital Classroom

With the Digital Classroom, MathCityMap provides an educational environment that has three core features (see Figure 1). The GPS monitor shows the teacher where the students are and which tasks they are currently working on. It is also possible for the teacher to monitor the students' path and to instruct them, if they are walking into the wrong direction. It is one of the basic ideas of the Digital Classroom to reduce problems that occur in terms of organization. The chat function, which is meant to act as a channel for direct supervision, enables the teacher to provide instructions or a differentiated help for learners. In addition, students can have some of their interim results (e.g. measurements) validated by the teacher. Therefore, MathCityMap offers multiple feedback sources, as a combination of asynchronous (predefined hints) and synchronous (direct supervision) feedback. The third function is a log for process data which has the use of an e-portfolio. It is designed as an evaluation tool and contains information on the progress along the math trail for each participating small group. This includes, for example, the number of tasks processed so far, taken hints and the answers entered. The information obtained through the e-portfolio can be used for diagnosis and can be incorporated into further lesson planning. If for example

Figure 1: Left) The chat window. Middle) The tracking tool. Right) The event log.

RESEARCH INTEREST

The previous sections identified potentials and challenges of mathematics trails as a way to learn mathematics outside the classroom. To face these challenges, the MathCityMap Digital Classroom was developed. As described, one of the core features is the chat functionality. Since the introduction of the new feature in 2019, many data could be gathered in order to examine the use of the chat tool by teachers and their students and how they make use of it to deal with the above-mentioned challenges.

EVALUATION OF THE MCM DIGITAL CLASSROOM

Choice of Methodology

The number of messages sent during the conduction of a digital classroom by itself can be used as an indicator of the total usage and overall acceptance of the tool, but cannot give us further insights into the way and purpose it is used for. Furthermore, many Digital Classrooms are created for teacher trainings and as a test run by teachers themselves before employing it in their classes. The unfiltered data thus contains many unauthentic test cases. To answer the research interest about the way the Digital Classroom is used, a qualitative analysis of a selected number of Digital Classrooms used in authentic settings seems therefore preferable.

Data selection

All in all, since its introduction as a tool for MCM, the Digital Classroom has been used to create 417 sessions with at least two groups, sending in total 1.262 messages between the creator of the Digital Classroom and the participants. Out of those 417 sessions, 264 were created in English or German, and thus available for our analysis.

Furthermore, we wanted to analyze the communication between teachers and students from an authentic setting, as the Digital Classroom is also used during teacher trainings by MCM educators. Based on the title of the session, we could therefore again discard several sessions. Other criteria were the number of participating groups (at least five) and the duration of the session (at least an hour). Out of the remaining data, we chose the ten sessions with the most messages sent. Those ten sessions were created by ten different teachers, offering also a variety within this data sample.

Qualitative Analysis and categorization

Overall, 87 student groups participated in the selected Digital Classrooms, sending in total N = 368 messages. By assuming that a group consists of three students and a Digital Classroom is led by one teacher, we can estimate the total number of participants at about 261 students and 10 teachers. Since we do not collect personalized data within Digital Classrooms such as names, genders or age to comply with the European GDPR, the age of the students can only be estimated to be between 14 and 18 (grade eight to grade twelve) by the math trails they walked.

Theoretically, we expect the chat tool to be used for organizational or didactical purposes (see the challenges section above). Due to this reason, we conducted a qualitative analysis of the send messages and categorized them as (1) organizational, (2) didactical or (3) other. We categorized an exchange as organizational (1), if the content of the messages was about the course of the math trail, i.e. the time, problems and technical issues, meeting points and information regarding the leaderboard. When the context of the messages was about mathematical aspects regarding the tasks, we attributed them to the second category, as didactical (2). Everything else was sorted into the third category, other (3).

Results

Firstly, we want to give examples of messages sent between a teacher and a group of students about organizational aspects (1):

> Teacher: It is possible that the trail automatically closes at 12.00. If so: we meet at 12.15 at the subway station.

Here, the teacher uses the Digital Classroom to give information about the meeting time and place for when the session is over. Sometimes the tasks weren't solvable due to missing or shut off objects:

Group:	The well isn’t running.
Teacher (to Group):	Then you can’t solve the task. Pity.
Teacher (to all):	The well isn’t running. Skip that task.

After receiving the information, the teacher used the chat to remotely inform all groups to skip the corresponding task. The following exchange gives us insights on many different levels.

Teacher:	Are you cheating? You are very fast!
Group:	we are just running
Group:	We don’t have that much time left

Firstly, the teacher obviously used the event log of the Digital Classroom to have a closer look at the course and required time of the students to solve the tasks. Secondly, the responding group was highly motivated to solve the tasks as fast as possible, making them even run. Other messages we categorized under organizational aspects were messages sent to remind the students about the remaining time, to solve technical issues or to motivate the students by pointing out the current leaders when the gamification option was used during the session. Out of the 368 messages sent in total, the exchange of 200 messages (54.3%) was about organizational aspects of the trail

We want to give an excerpt of one of the longest exchanges in a didactical context.

Group:	Should we round off?????
Teacher:	Sorry for which task?
Group:	Solid of revolution.
Teacher:	yes, that is ok
Teacher:	but that shouldn’t matter for this task
Teacher:	What did you measure?
Group:	41cm
Teacher:	You should get a correct result with that. Did you consider the binomial theorem?
Group:	Yes
Teacher:	Then there has to be an arithmetical error. Did you integrate correctly?

In the end it turns out that the teacher made a sign error. In this case however, the students initiated the communication and used the Digital Classroom to get in touch with the teacher when they couldn’t solve the task anymore. This exchange in the chat tool resembles a lot a conversation taking place in a real classroom. 97 messages (26.4%) were sent in the context of mathematical aspects, i.e. the teachers giving additional information, correcting a task or the students requesting help during the solving process.

Salutations and other informal communication took up the remaining number of messages, accounting for a 19.3% of the sent messages.

Our findings are summarized in Table 1.

	Category 1 - Organizational	Category 2 - Didactical	Category 3 - Other	Total
Students	72	38	47	157
Teachers	128	59	24	211
Total	200	97	71	368
%	54.3%	26.4%	19.3%	

Table 1: Categorization of messages sent during ten trails.

DISCUSSION

As expected, the teachers used the chat tool foremost to make organizational adjustments or to give organizational information, as well as to support the groups individually on a didactical level during the solving process. Not surprisingly, the spatial separation and different, often unforeseeable situations outside of the rather predictable environment of the classroom lead to a higher need of commutation of organizational aspects. The lower number of messages with didactical content can be explained by the available hints inside the MathCityMap system to each of the tasks, resulting in less required support during the solving process. Furthermore, a math trail is usually run as group consisting of three students. Before consulting the teacher, the students therefore most likely already discussed the problem amongst themselves and helped each other. Especially the need of didactical support is thus already lower in comparison to the setting inside the classroom, when the students work on the tasks individually. The communication between a single student and the teacher could thus be again different.

A finer analysis taking also the party initiating the communication into consideration could give further insights in the way the communication takes place. We also cannot exclude the possibility that the teachers used other channels to get in touch with the students during the conduction of the math trail to give additional advices.

CONCLUSION AND OUTLOOK

As shown in our qualitative analysis, the Digital Classroom provides the teachers with the possibility to address the aforementioned challenges while conducting outdoor learning. Currently the transmission of pictures and audio files over the chat is in development, offering the students and teachers another way to exchange information fast and easily. As a follow up to the analysis conducted in this article, it could be studied how this new functionality yet again changes the way and nature of the communication between teachers and students. Using the event log of the Digital Classroom, further research regarding the solution process of students like the usage of hints, the required time for specific exercises and the use of the sample solution could be conducted. Based on the chronology of the events in combination with the communication, it would be possible to make further conclusions about the need and efficient nature of asynchronous support in the form of hints.

As already mentioned, the math trails are usually run by a group of students with different roles. An analysis of the communication between the students during the solving process would also be interesting.

In conclusion, the digitalization of the math trail idea using the possibilities of modern technology together with the features of the Digital Classroom inside MathCityMap offers teachers a meaningful way to re-gain control in an open and outdoor learning environment.

References

Blane, D.C., & Clarke, D. (1984). *A Mathematics Trail Around the City of Melbourne*. Monash Mathematics Education Centre, Monash University.

Cahyono, A. N. (2018). *Learning Mathematics in a Mobile App-Supported Math Trail Environment.* Springer International Publishing

Edelmann, W., & Wittmann, S. (2012). *Lernpsychologie*. Weinheim: Beltz Verlag.

Fägerstam, E., & Grothérus, A. (2018). Secondary School Students' Experience of Outdoor Learning: A Swedish Case Study. *Education, 138*(4), pp. 378-392.

Falk, J. H., & Dierking, L. D. (1997). School Field Trips: Assessing their long-term impact. *Curator*, *40 (3)*, 211-218. Accessed on March, 2nd 2020 at https://onlinelibrary.wiley.com/doi/abs/10.1111/j.2151-6952.1997.tb01304.x

Gurjanow, I., Oliveira, M., Zender, J., Santos, P. A., & Ludwig, M. (2019). Mathematics trails: Shallow and deep gamification*. International Journal of Serious Games, 6 (3)*, pp. 65–79.

Kaur, B. (1992). *Attitudinal outcomes from environmental activities*. Lecture at the 6th Annual Conference of the Educational Research Association. Singapur.

Ludwig, M., & Jesberg, J. (2015). Using Mobile Technology To Provide Outdoor Modelling Tasks - The MathCityMap-Project. *Procedia - Social and Behavioral Sciences*, 191, pp. 2776-2781.

Sauerborn, P., & Brühne, T. (2009). *Didaktik des außerschulischen Lernens*. Djurcic, Schorndorf: Schneider Verlag Hohengehren GmbH.

Shoaf, M., Pollak, H., & Schneider, J. (2004). *Math Trails*. Lexington: COMAP.

Wijers, M., Jonker, V., & Drijvers, P. (2010). MobileMath: Exploring mathematics outside the classroom. *ZDM: The international journal on mathematics education*, 42(7), 789–799. doi:10.1007/s11858-010-0276-3.

Zender, J. (2019). *Mathtrails in der Sekundarstufe I: Der Einsatz von MathCityMap bei Zylinderproblemen in der neunten Klasse.* Münster: WTM-Verlag

AUTOMATICALLY AUGMENTED REALITY FOR OUTDOOR MATHEMATICS

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Abstract. *In this short note we[1] describe a futuristic scenario in which mechanical intelligence could be used to visualize and discover new, hidden, geometric features in outdoor mathematics objects: i.e. on drawings, paintings, monuments, etc. Thus, we summarize some of the recently implemented commands for automated reasoning on top of GeoGebra, a free, popular dynamic geometry program, and how they could be used -with the help of some tools for the detection of geometric objects in images- for this purpose. On the other hand, we will refer to the MathCityMap project, that combines traditional math trails and new technology, through a web portal and a mobile application that guides users along a math trail, locating the position of the different tasks, giving feedback on the users answers and providing hints if they get stuck on some question. The potential connection of both devices (MCM, GeoGebra) will be described.*

Key words: augmented reality, outdoor mathematics, automated reasoning, GeoGebra, MathCityMap.

MATHCITYMAP

The MathCityMap (MCM[2]) project (Ludwig, Jesberg and Weiss, 2013) started around 2012 at the Goethe-University Frankfurt and aims to facilitate and improve, by exploiting the possibilities offered by the new technologies, the creation and development of outdoor mathematics trails. The MCM project has two main parts, a web portal and an application for smartphones, available over the iOS and Android systems.

The web portal allows registered users (for example, teachers) to create their own trails and tasks or to view and follow trails built by other users (public or shared within a restricted group). Task creation involves geolocating a concrete object, capturing some image of it, proposing the task itself, the possible answers, etc. and putting all this info into a form, creating in this way a trail document (with a title page, a map, and information about each task along the trail) that can be downloaded as a pdf or visualized in the smartphone screen.

The MCM portal intends to help the trail creator by automatizing some of these steps and by making available to the user a freely accessible repository of tasks and trails for their potential reuse.

On the other hand, the MCM app guides users (students, hikers, tourists…) along a math trail created with the web portal. The trail data can be downloaded to the smartphone, allowing then to walk along the trail without internet connection. Yet, with connection, the MCM app helps locating the position of the different tasks, gives feedback on the

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[2] www.mathcitymap.eu

Botana, F., Kovács, Z.. & Recio, T. (2020). Automatically Augmented Reality for Outdoor Mathematics. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 71-78). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.09

correctness of the answers proposed by the users and provides helpful hints if they get stuck on some question.

The Erasmus+ project *Mobile Mathematics Trails in Europe* (MoMaTrE[3]) uses the app MathCityMap as a tool for introducing a digital approach into the math walks realm, both within the context of mathematics dissemination and popularization or as a math education methodology.

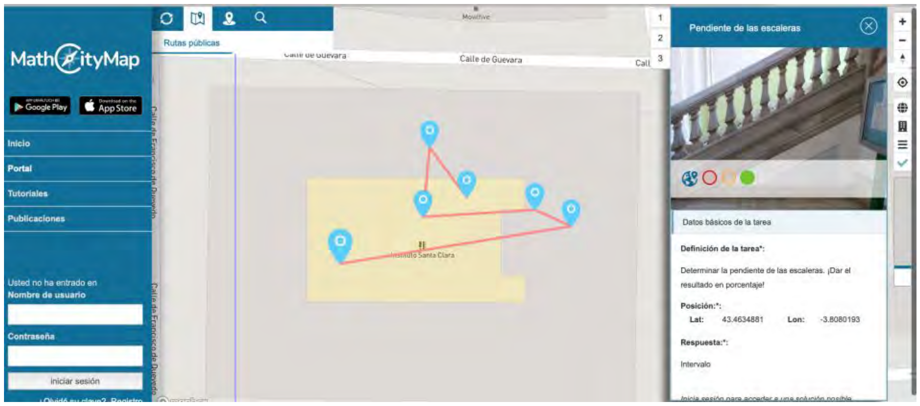

Figure 1: A task on MoMaTrE public route number 1065, from the MathCityMap portal[4].

Further details about MoMaTrE and MCM are available at their respective web pages (see footnotes 2 and 3). Figure 1 shows a precise task (determining the slope of some staircase inside a High School, the Instituto de Enseñanza Secundaria (IES) Santa Clara, in Santander, Spain, see footnote 4), as it appears in the MCM web portal.

MCM AND GEOGEBRA

Obviously, a good math route for performing outdoor mathematics has to meet some criteria (see Jablonski, Ludwig and Zender, 2018). In particular, one of them states that such trails should only address tasks that require the physical presence, facing the object that supports the proposed task, of the human solver.

But current technology makes quite fuzzy to determine, sometimes, what could be the real scope of the requirement, for true outdoor mathematics, of the *physical presence, facing the object* involved in the task. Does it allow the manipulation of visual data (such as screenshots) captured by the human solver *facing the object*? And, will the answer to this question be much different if the origin of the visual data comes, instead, from an image provided by the trail creator?

Consider, for example, the image of the staircase that appears in Figure 1, illustrating the featured question (finding the slope of the staircase). Clearly, the teacher who designed the task, included this picture just as a way to identify with precision the physical staircase inside the IES Santa Clara. But it could be also an image that could be pasted by the user on the graphic window of a dynamic geometry program such as GeoGebra[5], and then creating

[3] MoMaTrE, 2017-1-DE01-KA203-003577 www.momatre.eu

[4] https://mathcitymap.eu/es/portal-es/?view=trails&subview=public&id=1065

[5] Let us recall that GeoGebra is an open source, free, worldwide spread, dynamic geometry environment, available on computers, laptops, tablets, smartphones… of all kinds. See: www.geogebra.org

over the image some horizontal and vertical segments, estimating their sizes, etc. as illustrated in Figure 2, as a way of solving the task. Is this collaboration GeoGebra/MCM correct? Indeed, the list of admissible tools (measuring tape, carpenter rule, level, pocket calculator) to address this task does not include GeoGebra. Is this omission a crucial issue? Probably most of the students of grade 7 and over (the declared recipients of this task) have already a smartphone[6] at hand, and they will use it, anyway, for the calculator!

We think that outdoor mathematics tasks are, by definition, related to the mathematics of real life, and it does not seem very motivating to address real problems under some artificial constraints, such as limiting the use of technological tools –e.g. smartphones and apps, like GeoGebra-- that most people over 13 years old have already in their pocket.

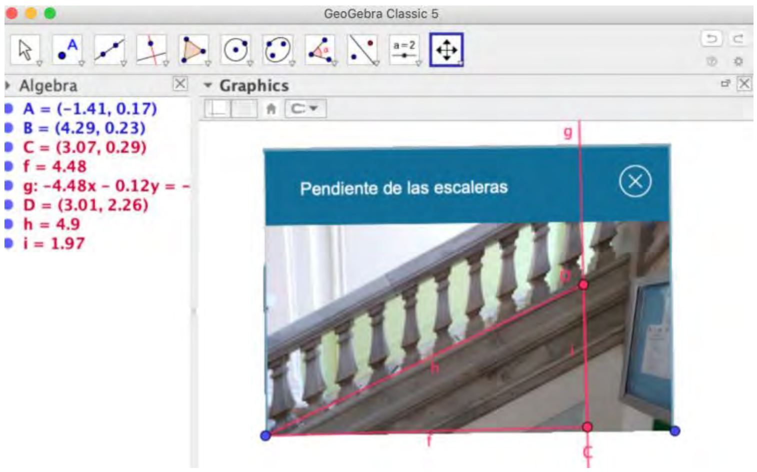

Figure 2: Computing the slope, on a photo, using GeoGebra.

These considerations concerning the relevance of allowing or forbidding the usage of GeoGebra to solve a proposed task, are, for sure, an interesting issue for debate. But it is not the main topic we would like to introduce in this paper. Rather, we would like to reflect in the next section on a related, but perhaps drastically different, scenario for the collaboration of GeoGebra with the MCM approach to math trails.

MCM AND THE AUTOMATED GEOMETER

Roughly speaking --as not all kinds of tasks are suitable for our proposal-- let us imagine a tourist, student, hiker… walking along an MCM route and stopping to address a specific task of geometric nature. At this point, he/she uses a smartphone to capture the image of the object involved in the task. Then, as we will enumerate in the "Sketch of the Algorithm" section, a mechanical protocol could be launched, starting with the obtained image, converting it as the input for GeoGebra's automated geometer features, and, finally, providing the user –in the live view of the camera-- with a rich collection of geometric properties holding on the considered object, that could be helpful to manipulate or to solve the task he/she is supposed to be dealing with.

[6] At least in Spain. See the 2019 report from the Spanish National Statistics Institute https://www.ine.es/prensa/tich_2019.pdf declaring the use of smartphones by 84% of the youngsters at age 13, and of 93.8% at age 15.

In the previous section we were considering, as the basic input for the collaboration MCM-GeoGebra, either the image of the task object, included in a pdf previously uploaded to the MCM app by the creator of the math trail (as the image of the staircase in Figures 1 and 2), or the image of the same object, but now captured by the user with a smartphone camera. And it was the user who had to handle this image to discover some geometric properties that are involved in the proposed task.

Here, the input is essentially the same, an image on a smartphone, but now we assume that, then, some pattern recognition tool automatically identifies its main geometric features. Next, this collection of geometric data (points, segments, lines, circles, etc.) is translated into a GeoGebra figure. Let us remark that we assume these first two steps (geometric features recognition and its conversion into a GeoGebra object) are done directly by the smartphone without any human intervention. We are not contributing here to the implementation and development of this –less mathematical and more software oriented-- part of the protocol, but we think, in the present state of the art, it is quite reasonable to expect it could be achieved quite soon.

Finally, GeoGebra automated reasoning tools -in particular, the tools we are developing for mechanically analyzing a given figure, deriving all the geometric properties and theorems that hold on it, see Botana, Kovács and Recio (2018)- start operating over the GeoGebra figure, finding automatically the solution to the proposed task or, at least, suggesting the user many features of the object that could help him/her solving it. This whole process is what we think could be labeled as a way to create an *automatically augmented reality* that could be quite useful as a future feature of MCM.

In what follows we will, next, summarize the automated reasoning tools already implemented in GeoGebra. Then, a more detailed description of the *automatically augmented reality* algorithm (see Botana, Kovács, Martínez-Sevilla and Recio (2019), for an extended version) we are proposing will be presented. Finally, the performance of such algorithm on a couple of imaginary examples (regarding the pavement decoration of some lookout point over the Sardinero Beach in Santander and, as a quite different example of outdoor mathematics, some enamel wood panel work by a Spanish artist) is described. A few reflections on the impact of this proposal are collected as a Conclusion.

A BRIEF ACCOUNT OF AUTOMATED REASONING TOOLS IN GEOGEBRA

By automatic proving of elementary geometry theorems, we refer to the theorem proving approach via computational algebraic geometry methods. The idea is to provide algorithms, using computer algebra methods, for confirming (or refuting) the truth of some given geometric statement. More precisely the goal is to decide whether a given statement is *generally* true or not, i.e. true except for some degenerate cases, to be algorithmically described.

The approach proceeds (see Kovács, Recio and Vélez (2019) for a detailed description of the involved mathematical context and algorithms) by translating geometric facts, hypotheses H and theses T into systems of polynomial equations, SH , ST, and then, translating geometric statements (H ⇒ T) as inclusion tests SH ⊆ ST between the solution sets of the corresponding systems of equations. Such inclusion tests are then elucidated by some computer algebra tools deciding if a polynomial f is or not an algebraic combination of some given collection of polynomials S, which is -approximately- a way to check whether the roots of f form a superset or not of the solution set of the system S = 0.

A closely related issue is that of the *automatic discovery* of theorems. Roughly speaking, automatic proving deals with establishing whether some statement holds true in most instances, while automatic discovery -in its most general conception- addresses the case of statements $H \Rightarrow T$ that are false in most relevant cases. In fact, it aims to automatically produce additional, necessary, hypotheses H' for the new statement $(H \wedge H') \Rightarrow T$ to be true. All these features are available since version 5 on GeoGebra, through the *Relation*, *Prove*, *ProveDetails* and *LocusEquation* commands. See Kovács, Recio and Vélez (2018) for an article including a kind of tutorial on these commands.

Currently, only an experimental, evolving, version of GeoGebra[7] incorporates the last novelties we are developing on automated deduction methods, including the mechanical statement of multiple geometric properties involving a given geometric object (say, one point) and its verification. Please visit https://github.com/kovzol/geogebra-discovery for updated information on GeoGebra automated reasoning tools.

SKETCH OF THE ALGORITHM

1. Input: Real world images captured by a smartphone (or similar), or hand drawings made by the user on a screen.

   Output: Information on the geometric properties of a mathematical model of the image/drawing.

The steps to perform are:

- Transform the image/drawing to a collection of traces (segments, (arcs of) circles, points, …) by using the classic Hough transform (Mukhopadhyay and Chaudhuri 2015) or other methods to detect geometric elements on an image.
- Conversion of the given traces into a collection of GeoGebra objects. To this end

  – conversion of the segments and arcs of circles, adding its extremes and eventually its intersection points,

  – estimation of their lengths/centers, radii (with three points), angles between segments, …

  – rounding up the estimated measures (lengths, angles, centers, radii, …), and

  – conversion of the above geometric elements into GeoGebra objects, together with their relations (alignment, parallelism, perpendicularity, ...). It should be noted that this conversion does not imply the figure reconstruction in GeoGebra, but just a list of GeoGebra objects and their relationships.
- Application of the automated reasoning tools in GeoGebra (both those related to the mechanical answer to user-stated questions (Kovács, Recio and Vélez 2018) and to the mechanical production of geometric statements (Botana, Kovács and Recio 2018)), giving as result a list of geometric properties.

[7] http://autgeo.online/geogebra-discovery/

- Interpretation of these properties in terms of the initial real-world objects.

A COUPLE OF EXAMPLES

An {8/2} polygon at the Sardinero Beach

We consider here an imaginary promenade[8] along the Santander Sardinero Beach, stopping at the lookout point (Mirador) at the end of the beach (see Figure 3), where the pavement is decorated with an (Schläfli symbol) {8/2} star polygon, based on an octogonal polygon (ABCDEFGH, Figure 4), with vertices connected every two of them.

Then the following task is presented: to find the ratios of segments c and d with respect to segment e, where segment c connects two consecutive spikes of the star, segment d is the base of the triangle in each of the star tips and segment e is just a side of the star. That is, the side "e" of the star, the side "d" of the internal octogon, the side "c" of the external octogon. See Figures 3 and 4.

Figure 3. A {8/2} star polygon on the Sardinero Beach Mirador.

We expect the walker has no specific measuring tools at hand, except, of course, a smartphone. In fact, using as measuring units the side of some tiles that form the pavement in the Mirador, it is easy to conclude that e=10, d=14, c=19 units, approximately. But what could be the mathematical relation among these segments? To get a hint, we consider opportune to launch our protocol, capturing the image of the star polygon, converting it into a GeoGebra construction and applying automatically the Relation tool, that yields the

[8] Inspired by Abad-Palazuelos et al. (2014).

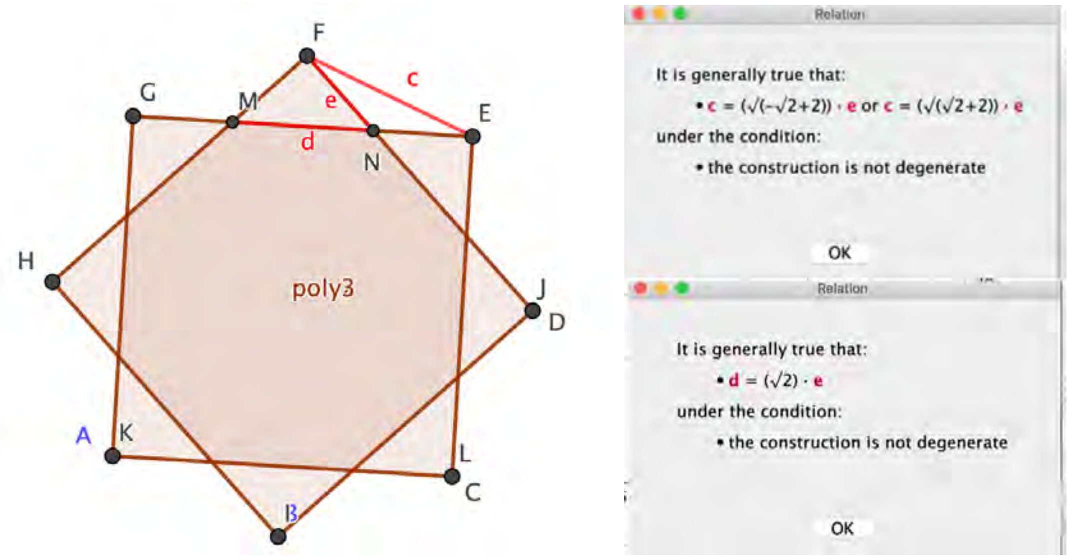

Figure 4. Left, the GeoGebra representation of the {8/2} star. Right, the output of the Relation(d,e) and Relation(c, e) commands.

output described in the Figure above, that is, ratio d/e is the square root of 2, so the measured estimation is quite correct. Same for the ratio c/e, that has two options, according to GeoGebra (for some technical reason, there are two possible constructions of an octogon), but one of them validates, again, that the guessed ratio is quite approximate. In conclusion, GeoGebra's Automated Geometer helps confirming and provides sound indications about the mathematical facts behind the scene.

A Delaunay triangulation on an enamel panel

INFINITE EYE I is a 2016 synthetic enamel on wood panel made by artist Okuda San Miguel[9]. Our approach (see Figure 5) to automatically analyzing this artwork considers, first, converting some characteristics of the image, for instance, the triangle vertices, into a GeoGebra list of points and, then, GeoGebra could attempt, through the automated reasoning tools, to discover if the artist creation is more or less close to some known mathematical triangulation. Here, since GeoGebra has a native command for Delaunay triangulations, we compared the result of applying such triangulation to the given vertices and concluded that it is quite close to the one imagined by Okuda (80% success: 50 triangles coincide, while other 10 do not). Is there any anthropological reason for such coincidence, that happens as well in many other Okuda works?

CONCLUSION

We have exemplified how GeoGebra, through its recently implemented commands for the automated analysis of geometric facts, could be used to study images, captured by smartphones, of different objects associated to MCM tasks along a math route. These commands could allow to automatically conjecture, prove and discover multiple geometric properties -not obvious at all in many cases- of the target objects. In this way GeoGebra

[9] http://okudasanmiguel.com

Figure 5. Is there a Delaunay triangulation on Okuda's INFINITE EYE I?

would mechanically provide a rich layer of extra information over reality, yielding an *automatically augmented reality* output, that could be relevant in multiple contexts, not just for math trails: e.g. helping visually impaired persons to learn about their environment.

References

Abad-Palazuelos, E., Barandica-Romo, B., Fuente-Somavilla, M. J., Gómez-Velarde, M. I., Martínez-Rosales, E. and Núñez-Castaín, A. (2014). *Santander, mirar y ver…matemáticas, arquitectura e historia.* Colección Divulgación Científica, 5. Editorial Universidad de Cantabria. Santander.

Botana, F., Kovács, Z., and Recio, T. (2018). Towards an automated geometer. *Lecture Notes in Computer Science*, 11110, 215–220.

Botana, F., Kovács, Z., Martínez-Sevilla, A. and Recio, T. (2019). Automatically Augmented Reality with GeoGebra. In: Theodosia Prodromou (Ed.), *Augmented Reality in Educational Settings*, 347-368. Brill-Sense. Leiden, Boston.

Jablonski, S., Ludwig, M., Zender, J. (2018). Task Quality vs. Task Quantity. A dialog-based review system to ensure a certain quality of tasks in the MathCityMap web community. In: H.-G. Weigand, A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, N. Grønbæk & J. Trgalova (Eds.). *Proceedings of the Fifth ERME Topic Conference (ETC 5) on Mathematics Education in the Digital Age (MEDA)*. 5-7 September 2018, 115-122. Copenhagen, Denmark.

Kovács, Z., Recio, T, and Vélez, M.P. (2018). Using automated reasoning tools in GeoGebra in the teaching and learning of proving in geometry. *International Journal of Technology in Mathematic Education*, 25, 33–50.

Kovács, Z., Recio, T, and Vélez, M.P. (2019). Detecting truth, just on parts. *Revista Matemática Complutense*, 32, 451–474.

Ludwig, M., Jesberg, J., Weiß, D. (2013). MathCityMap – eine faszinierende Belebung der Idee mathematischer Wanderpfade'. *Praxis der Mathematik* (53), 14-19.

Mukhopadhyay, P. and Chaudhuri, B.B. (2015). A survey of Hough transform. *Pattern Recognition*, 48, 993–1010.

THE NORWEGIAN STUDY MATH & THE CITY ON MOBILE LEARNING WITH MATH TRAILS

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Abstract. *The paper presents the design of a Norwegian study on the use of mobile math trails in mathematics teaching. Math trails take students on a guided tour through the city or the schools' surroundings where they solve different mathematical problems. The study aims at identifying students' contextual modelling and their use of digital devices when working with the trails. The study uses innovative methods from sociology to analyze students' mobile learning. The paper presents first results from the pilot study (which took place in Germany), since results of the main study are not yet available.*

Key words: Math trails, modelling, videography, mathematize, digital tools, mobile learning.

INTRODUCTION

Mathematics education research shows that the motivation for learning mathematics for students in Norway is mostly extrinsic and grounds only to a limited extent on interest and joy when learning the subject, especially when looking at students' transitions from primary to lower secondary level and in international comparison (Kaarstein & Nilsen, 2016). Mathematical modelling has now become part of the new core elements of the Norwegian mathematics curriculum (*Kjerneelementer*), but so far there are comparatively few worked out examples for Norwegian teachers to integrate modelling in teaching and researchers anticipate that teachers will struggle to work with modelling (Berget & Bolstad, 2019). Math trails can emphasize extracurricular and playful learning, which many teachers perceive as a useful complement to traditional school teaching patterns. Math trails exist since the 1980s as an out-of-school leisure activity for families and persons interested in mathematics (Blane & Clarke 1984; Shoaf, Pollack & Schneider, 2004). Although not a new idea, math trails have gained more attention recently in mathematics educational research, especially since math trails are included in teaching approaches in schools. Additionally, one can enhance math trails by including digital devices like smartphones or tablets, as done in the MathCityMap project (Ludwig & Jesberg, 2015; Cahyono & Ludwig, 2019) or with the help of Google Maps (Fessakis, Karta & Kozas, 2018). Tasks comprise estimating and measuring of variables, calculating and comparing sizes, areas and volumes and solving problems, thus carrying essential elements of mathematizing and mathematical modelling (Buchholtz, 2017). Corresponding research findings on students' learning when doing such kind of math trails are still scarce (Zender, Cahyono, Gurjanow & Ludwig, submitted), nonetheless because math trails originally were invented as a leisure activity for people interested in mathematics and have not been the object of systematic mathematics educational research. The study *Math & The City* and its pilot described in this paper is intended to contribute to fill this research gap by looking at students' mobile learning when working with math trails.

Buchholtz, N. (2020). The Norwegian Study Math & The City on Mobile Learning with Math Trails. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 79-86). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.10

MOBILE LEARNING WITH MATHTRAILS

Mobile learning is a comparatively young research field in educational research. Earlier definitions included the involvement of mobile devices in the learning process and the physically mobility of the learners as central and necessary characteristics of the educational concept (O'Malley et al., 2005). Recent definitions of mobile learning highlight the importance of the personalization of the learning content and its context-relatedness (Frohberg, 2008; Frohberg et al., 2009). Thus, the notion of mobile learning is overcoming more and more the boundaries between formal and informal learning contexts. Math trails are well suited as prototypes of settings for mobile learning. The context-relatedness can be acknowledged in both ways: the physical context of the learning environment has a clear relation to the learning content and co-determines it (Frohberg, 2008) and the socializing context can be acknowledged by the collaborative learning, in which situations, relationships and emotions can be linked with the learning experience in the environment (ibid.). Mobile end devices with their location-independence are ideal for extra-curricular learning environments and create a mediating context between the physical and the social environment, for example when multimedia and interactive apps are used (Buchholtz et al., 2019). If such a learning path in an app sequences several mathematical tasks and links them to different locations, we will call this a mobile math trail. Mobile math trails consist of tasks with respective sub-tasks, comprising different mathematical concepts. The tasks can be designed to ensure different activities related to mathematical modelling. Students have to understand the task in the mobile device, which can be presented on the mobile device in several different ways using multiple representations in order to enable access to the task. Furthermore, the students have to contextualize the task and identify relevant variables and quantities in the real objects that are related to the task-at-hand. Then they need to carry out measurements autonomously in groups. The tasks process mathematizations by a meaningful assignment of determined variables into a mathematical model. When solving modelling problems, students need to interpret their mathematical results and validate them with the help of the real object to get the answer. The technology supports learning in a way that students get immediate feedback on their calculations after entering their results in a corresponding digital device. When the feedback is supplemented by a systematic reward, Mobile Math trails can even contain elements of participation and gamification (Gurjanow et al., 2019).

THE STUDY MATH & THE CITY

Research objectives

The aim of the study *Math & The City* is to analyze students' mobile learning when working with math trails, which in this case means analyzing their interaction with the objects related to the different tasks on the trail, their use of the digital device during the math trail and their contextual modelling processes and strategies when estimating and taking measures. The study is funded by DIKU, the Norwegian Agency for International Cooperation and Quality Enhancement in Higher Education. The study follows a design-based research approach (Gravemeijer & Prediger, 2019) in which math trails are developed and subsequently evaluated. Exploratory qualitative research methods are used to analyze students' mobile learning and modelling activities.

The corresponding research questions of the study are:

RQ1) Do the tasks process contextual modelling processes and strategies when the students encounter the objects and solve the tasks?

Here we take localizing variables and sizes, measuring activities and data collecting, discussing and using respective formulas and heuristics as well as interpreting results in the context of the real objects as observables.

RQ2) Do the tasks enable mobile learning during the math trails?

Here we take the interactions with the mobile device, the interaction with the physical objects and the communication between the students as observables.

Task design

The *Actionbound* app used for the math trails (www.actionbound.com) is an app developed initially in the field of media education for the creation of digital learning paths (so-called Bounds). For the math trail design, the tasks in the project follow experience-based task criteria (cf. Buchholtz, 2017). The app offers different task formats like for example quizzes, free writing tasks, sorting and estimating. Time limits and solution intervals can be defined, so that the consideration of measurement errors or different modelling assumptions can be ensured. The process of mathematizing within the tasks takes place in comparatively small steps and the developed mathematical models are of low complexity. On the one hand, this supports independent mathematizing piece by piece and, on the other hand, this keeps the processing time of the tasks generally quite short. Fig. 1 shows an example of a task that is part of a math trail from the *Math & The City* pilot study, which took place in Hamburg (Germany).

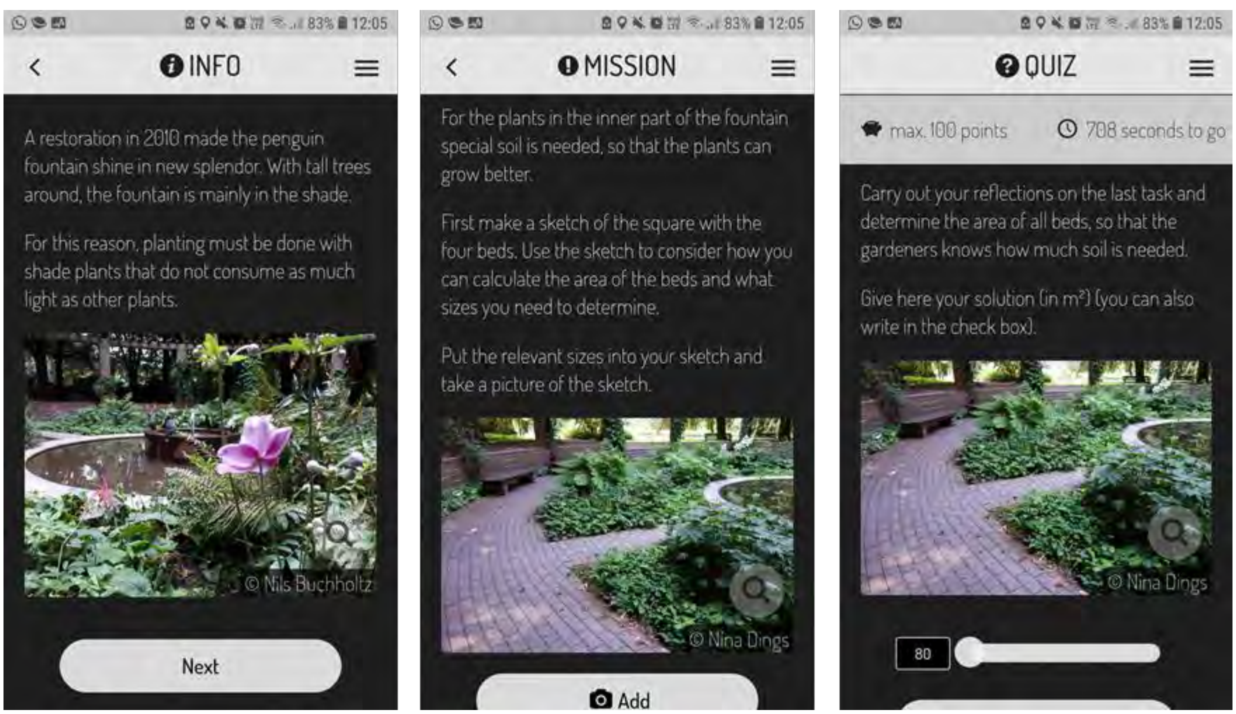

Figure 1: Task on area calculation.

The task is about the area calculation of four beds in a park. The tasks forces the participants to draw a sketch of the beds with the respective sizes and to make a photo upload of this sketch. Afterwards they have to do the calculations and determine the area of the beds in order to estimate the amount of soil. The mathematization here includes finding a proper way to calculate the area, which is limited by the fountain in the middle and the four footpaths. It bases mathematically on an annulus. The area of the four footpaths can be modelled by suitable rectangles, which are subtracted from the result. An alternative approach is to measure the edges of the beds and to calculate the differences of the area of circle sectors.

Data collection

When conducting the math trails with students, each group of students is equipped with a tape measure, something to write and a mobile end device (iPad), which is presenting the task in Actionbound and offers different answer options according to the task-design. The students submit all solutions to the tasks in the app. Each group of students is furthermore equipped with an action camera that records the progress of the math trail. The students themselves are responsible for recording the process at the individual stations of the math trail. In November and December 2019, the Oslo trails of the main study were carried out with two school classes of grade 11(*1P*) and their mathematics teachers. In each case five groups of three students each were equipped with cameras and iPads. Currently, the data is being evaluated, therefore only results of the pilot study in Hamburg can be presented in this paper.

Method for the evaluation of the math trails

For the evaluation of the math trails methods of qualitative explorative research are used. A method from urban city planning developed by Jean-Yves Petiteau in the 1970s can be applied, which origins in French sociology. The method of the narrative walk-in-real-time interview (*Méthode des Itinéraires*) was originally invented to collect and describe the subjective view of pedestrians in order to draw conclusions about urban planning (Petiteau & Pasquier, 2001; Miaux et al. 2010). Central to this method are specific city walks, where the researcher takes the passive role as being guided by the test person and interviews and records the test person while a photographer walks behind and takes pictures at each change in direction or emotional change. There are different forms of the method existing, depending on how determined or open the walk is planned and how familiar the test person is with the environment (Evans & Jones, 2011). The method has been applied in different research context, for example to collect data about the exploration of botanical gardens (Hitchins & Jones, 2004) or the perceptions of elderly people when it comes to road traffic (Franz, 2008). The method here is adapted for the math trails in order to gather information about the modelling activities and the subjective views about the different learning contexts of the tasks and the use of the digital device. The method contains a three-step procedure: First, the researcher informs the student groups about the math trail and explains the research method (pre-walk briefing). Second, during the itinerary – when the students are working with the math trail – the researcher walks together with the students. Even if the app determines the road of the math trail, the students take the active role during this phase; the researcher only assists and asks clarification questions about the mathematizing, the students' work with the mobile device and the context of the tasks. The students group describe their experience in "real time". They are recorded when working by action cameras, which they wear on their body and which makes the data collection minimally invasive. Third, after the trail, the researcher asks the student groups about their strategies in different situations and their reflection about the math trail as a learning opportunity (post-walk interview). The data of the video-material and the interviews are analyzed with the help of qualitative content analysis (Mayring, 2014).

FIRST RESULTS OF THE PILOT STUDY

The pilot study for *Math & The City* was conducted in Hamburg in summer 2019. Due to the summer vacations and short preparation time, the mobile math trail could unfortunately not be conducted with students. But in order to test the application of the method and the functionality of the app, the math trail could be conducted with two volunteer participants A. (36, male) and N. (37, female) that worked on a math trail on circle calculation (grade 9). They got an introduction in using the mobile device and the app. Both indicated in the pre-walk briefing that they had "not worked with circle calculations since school", but they were "motivated to try out the math trail". The paper presents first results on how a task processed contextual modelling when A. and N. solved it (RQ1).

One of the task they were working on during the trail is about the area calculation of four beds in a park (cf. Fig. 1). The video-material shows how A. and N. worked collaboratively on the task and planned their mathematization in order to get to a solution. First, they related the task properties to the real object and made a sketch with the relevant sizes (Fig. 2 left; Fig. 4 left) The following scene was categorized as *object-related planning*, this category contains elements of relating the task-at hand from the mobile device to the real object, often combined with deictic gestures:

> A.: Well that's what it says. Draw a sketch. […] This circle, so to speak. […] So, you've got an outer circle here with the beds, [*points to beds*] [...] and then an inner circle like that [*points to fountain*] and then.
>
> N.: Yeah. [*draws*] There you go, outer circle.
>
> A.: Mhm.

Figure 2: Object-related planning and data collection.

After that, A. figured out a calculation approach, while N. was already taking the first respective measures at the fountain. A. then helped N. with taking the measures. The following scene was categorized as *joint data collection*, involving a.o. using the tape measure in several ways (cf. Fig. 2 right).

A.: How do you calculate that now? […] So, you still have to…[*holds the tape measure*]

N.: Hold still! Don't move!

A.: 3 meters. It's almost exactly 3 meters. That's 3 meters.

N.: Until there [*pointing*]? […]

A.: So then, do here from the edge [*holding the tape measure at the edge of the fountain*]. And run to the outside. And you know where to go, don't you? How much is that?

N.: 4.20 meters. […] Yeah, plus 3 meters.

A.: These are […] 7.20 meters is the radius.

They agreed on a mathematization approach, where they first calculated the area of the greater disk with the radius, then subtracting the area of the smaller disk of the fountain in the middle of the beds. The following scene (Fig. 3 left) was categorized as *working mathematically*, it was observable that in this scene, the real objects were of less importance and A. and N. were mainly working with their sketch and used the mobile device as calculator.

A.: We actually have to calculate the inner circle here first, because we have to subtract it.

N.: Yes. And that's 3.50 meters.

A.: Nah, this can't be now. I am […] that it was at least 3 meters. […]

N.: No, 3.50 meters. We still have to include the stone slab.

A.: We said […] A equals pi times r squared. So, let's start with the inner circle again [*writes*]. Get the [calculator]. You calculate. […] Please calculate 3 meters … 3.5 times 3.5.

N.: [*enters the numbers*] 12.25.

A.: Good. Then now we have pi times 12.25. […] And the you calculate now 3.14 times 12.25.

N.: 38.465 [*A. writes*].

Figure 3: Working mathematically and additional data collection.

They proceeded calculating the area of the outer disk and subtracted the area of the inner disk (Fig. 4 right). A third step in their mathematization was to measure the area of one rectangle-shaped footpath, multiply it by four and subtract this area from the result (Fig. 3

right; Fig. 4 right). Small mistakes with the equal sign are observable. However, the answer they gave in the app was considered as correct.

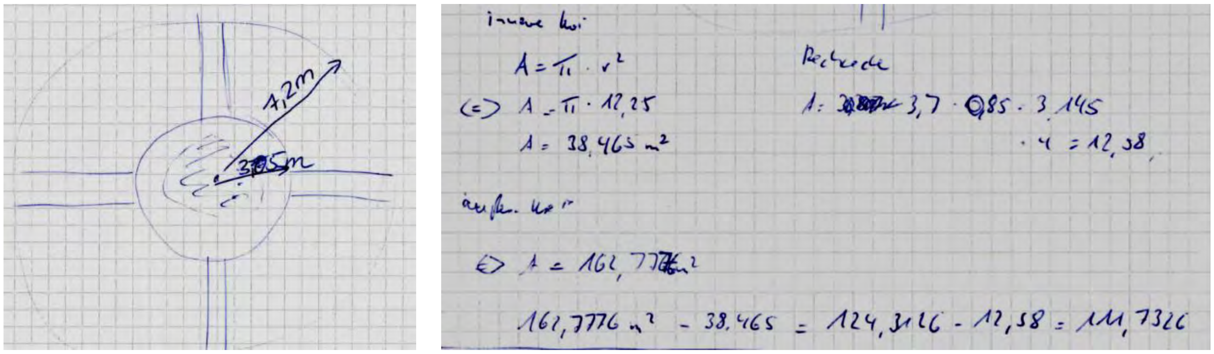

Figure 4: Solution approach from A. and N.

CONCLUSION

From the results of the small pilot study with only one case, we cannot derive in-depth knowledge about different contextual modelling processes and strategies and mobile learning with math trails. Here, the data evaluation of the main study will provide results in a more systematic way. On the video recordings, A. and N.'s approach to the task solution can be clearly seen in connection with the real objects of the math trail, so that the recordings with the Norwegian students will lead to deeper insights into contextual modelling processes and strategies. Furthermore, videography also takes into account the social context of (mobile) learning. Not only could we analyze the interaction between A. and N. when coordinating the mathematical activities and working with the mobile device. Additional digital screenshots and the accessible solutions entered in the Actionbound app can provide further insights into the interaction of the participants and the digital medium.

References

Berget, I. & Bolstad, O. (2019). Perspektiv på matematisk modellering i Kunnskapsløftet og Fagfornyinga. Nordisk tidsskrift for utdanning og praksis, 13(1), 83–97.

Blane, D.C. & Clarke, D. (1984). *A Mathematics Trail Around the City of Melbourne*. Melbourne: Monash Mathematics Education Center, Monash University.

Buchholtz, N. (2017). How teachers can promote Mathematising by means of Mathematical City Walks. In G. A. Stillman, W. Blum & G. Kaiser (Eds.), *Mathematical Modelling and Applications - Crossing and Researching Boundaries in Mathematics Education* (pp. 49–58). Cham: Springer.

Buchholtz, N., Drexler, J. & Vorhölter, K. (2019). Mathtrails digital unterstützen – Chancen und Grenzen mobilen Lernens im Mathematikunterricht. In G. Pinkernell & F. Schacht (Hrsg.), *Digitalisierung fachbezogen gestalten. Herbsttagung vom 28. bis 29. September 2018 an der Universität Duisburg-Essen. Arbeitskreis Mathematikunterricht und digitale Werkzeuge in der GDM* (S. 11–22). Hildesheim: Franzbecker.

Cahyono, A.N. & Ludwig, M. (2019). Teaching and Learning Mathematics around the City Supported by the Use of Digital Technology. *Eurasia Journal of Mathematics, Science and Technology Education, 15*(1), em1654.

Evans, J. & Jones, P. (2011). The walking interview: Methodology, mobility and place. *Applied Geography, 31*(2), 849–858.

Fessakis, G., Karta, P. & Kozas, K. (2018). The Math Trail as a Learning Activity Model for M-Learning Enhanced Realistic Mathematics Education: A Case Study in Primary Education. In M.E. Auer, D. Guralnick & I. Simonics (Eds.), *Teaching and Learning in a Digital World. Proceedings of the 20th International Conference on Interactive Collaborative Learning – Volume 1* (pp. 323–332). Cham: Springer.

Franz, M. (2008). *Ältere Menschen an städtischen Hauptverkehrsstraßen*. Diplomarbeit an der Fakultät Raumplanung, Technische Universität Dortmund.

Frohberg, D. (2008). *Mobile learning*. Department of Informatics, University of Zurich, Zurich.

Frohberg, D., Göth, C. & Schwabe, G. (2009). Mobile Learning projects – a critical analysis of the state of the art. *Journal of Computer Assisted Learning, 25*(4), 307–331.

Gravemeijer, K. & Prediger, S. (2019). Topic-Specific Design Research: An Introduction. In G. Kaiser & N. Presmeg (Eds.), *Compendium for Early Career Researchers in Mathematics Education* (pp. 33–57). Cham: Springer.

Gurjanow, I., Oliveira, M., Zender, J., Santos, P. A., & Ludwig, M. (2019). Shallow and deep gamification in mathematics trails. In M. Gentile, M. Allegra & H. Söbke (Eds.), *Games and learning Alliance - 2019: 7th International conference, GALA 2018, Palermo, Italy, December 5-7, 2018, Proceedings* (pp. 364–374). Springer International Publishing.

Hitchins, R., & Jones, V. (2004). Living with plants and the exploration of botanicalencounter with human geographic research practice. *Ethics, Place and Environment, 7*, 3–18.

Kaarstein, H. & Nilsen, T. (2016). Motivasjon. In O.K. Bergem, H. Kaarstein & T. Nilsen (Eds.), *Vi kann lykkes i realfag. Resultater og analyser fra TIMSS 2015* (pp. 63–77). Oslo: Universitetsforlaget.

Ludwig, M. & Jesberg, J. (2015). Using Mobile technologiy to provide Outdoor Modelling Tasks - The MathCityMap-Project. *Procedia – Social and Behavioral Sciences, 191*, 2776–2781.

Mayring, P. (2014). *Qualitative content analysis: theoretical foundation, basic procedures and software solution*. Klagenfurt. http://nbn-resolving.de/urn:nbn:de:0168-ssoar-395173.

Miaux, S., Drouin, L, Morencz, P., Paquin, S. & Jacquemin, C. (2010). Making the narrative walk-in-real-time methodology relevant for public health intervention: Towards an integrative approach. *Health & Place, 16*(6), 1166–1173.

O'Malley, C., Vavoula,, G., Glew, J., Taylor, J., Sharples, M et al. (2005). *Guidelines for learning/teaching/tutoring in a mobile environment*. hal-00696244.

Petiteau, J.Y & Pasquier, E. (2001). La méthode des itinéraires: récits et parcours. In M. Grosjean, J.-P. Thibaud (Eds.), *L'espace urbain en methods* (pp. 63–77). Marseille: Parenthèses.

Shoaf, M.M., Pollack, H., & Schneider, J. (2004). *Math Trails*. Lexington: COMAP.

Zender, J., Cahyono, A.N., Gurjanow, I. & Ludwig, M. (submitted). *New approaches in the research on mathematics trails with technology*. Paper presented at the WERA 10 Focal meeting in Tokyo, 5.-8. August 2019, University of Tokyo.

GET OUT INSIDE: PROGRAMMING TOYS 2.0 TO ESCAPE THE ISLAND

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Abstract. *Escaping from an island is the context used to promote the resolution of geometric problems that involve the programming of toys 2.0 in a collaborative environment. In this article, we characterize the signs produced in this experience considering the Theory of Semiotic Mediation and we analyse its impact on learning from the student's perspective. Thus, artifact signs, pivot signs and mathematical signs were cyclically mobilized and in different directions depending on the nature of the challenge. Of the students surveyed in the final questionnaire, 72.2% preferred this experience to explore geometric content over that explored with dynamic geometry software.*

Key words: Escape Island, toys 2.0, problem solving, Theory of Semiotic Mediation, signs.

INTRODUTION

A variety of resources can be used in the current digital age to promote meaningful practices inside and outside the classroom. Experiences using both manipulative and digital artifacts, conceived to enhance the building of knowledge show the potential of integration towards the promotion of significative learning (Faggiano, Montone, & Mariotti, 2018; Maschietto & Soury-Lavergne, 2013).

To design this learning environments, STEM Education could help teachers think how to combine Science, Technology, Engineering and Mathematics to "pose, ponder, and solve problems that are real world and authentic to our worlds" (MacDonald, Danaia, & Murphy, 2020, p. 5). Taking advantages of technological artifacts to learn mathematics outside the classroom is a challenge but also an opportunity to promote connections to the real world.

Having as background a study, centered on Mathematics, which aims to investigate the influence of collaborative programming activities on the development of the abilities to solve and pose geometric problems, critical thinking, creativity and collaboration, in this article, we focused on the analysis of the results of one of the tasks that combines problem solving in geometric contexts through the programming of toys 2.0.

In this context, we have set three analysis guiding questions: How do the students think? What kind of signs are they mobilizing? What difficulties do they manifest?

TOYS 2.0

Originating in Turtle geometry - "computational style of geometry" (Papert. S, 1980, p. 55), several devices are currently available on the market (Figure 1), often called robots that allow the creation of learning environments that combine solving geometric problems through graphic or tangible programming (Figure 1).

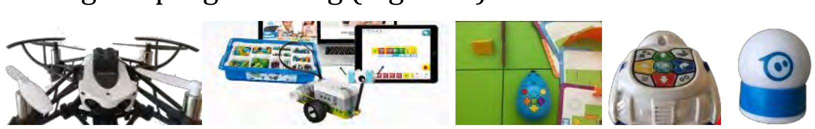

Figure 1: Parrot Mambo, Lego Wedo 2.0, Robot Mouse, Mind Designer Robot, Sphero 2.0.

Cunha, E., Cabrita, I. & Fonseca, L. (2020). Get out Inside: Programming Toys 2.0 to Escape the Island. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 87-94). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.11

But what is a robot? Simon (2017) tried to define it so he asked some of the area leading researchers. For Anca Dragan, from the University of Berkeley, “a robot is a physically embodied artificially intelligent agent that can take actions that have effects on the physical world,” (Simon, 2017, para. 5). This definition is corroborated by Kate Darling, from the MIT Media Lab. while emphasizing that there is no good universal definition, she considers that a robot “would probably be a physical machine that's usually programmable by a computer that can perform tasks autonomously or automatically by itself” (Simon, 2017, para. 6). Therefore, the incorporation of artificial intelligence is evident. A *drone* will only be considered a robot if it is autonomous if it is not directly controlled by user actions.

Being a robot cannot be the common property of all devices presented in Figure 1, because, for example, a *drone* may or may not be, depending on its usage. So, what is the common property? They are ‘toys’! ‘Toys 2.0.’ for bringing an *upgrade* over traditional toys: the possibility of being programmed.

THEORY OF SEMIOTIC MEDIATION

Inspired by a Vygotskian approach, shared signs are generated within the social use of artifacts to the accomplishment of a task, involving mediator and mediates. Learning effectiveness depends on the analysis of the artifact's semiotic potential. From this analysis a double semiotic link is established:

> on the one hand, personal meanings are related to the use of the artifact, in particular in relation to the aim of accomplishing the task; on the other hand, mathematical meanings may be related to the artifact and its use. (Bussi & Mariotti, 2008, p. 754)

According to Bussi and Mariotti (2008) we have three categories of signs: artefact signs, pivot signs and mathematical signs. The artefact signs refer to the use of the artefact to carry out the task; the mathematical signs are those that emerge from the mathematical context; and Pivot signs are those which promote the passage between artifact and the mathematical context.

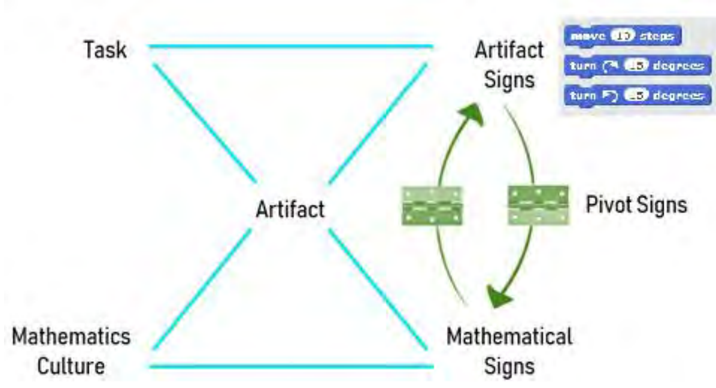

Figure 2: Artefacts and signs in programming tasks, adapted from Bussi and Mariotti (2008, p. 757).

However, when the tasks involve programming toys 2.0, we must consider we are moving from mathematical signs to the artefact signs since programming requires previous mobilization of mathematical signs (from the student’s mathematical culture which may or may not be valid). When the toys are tested, new signs are produced: pivot signs, emerging from reflecting about the mistakes made, and new mathematical signs. We can move from artifact signs to mathematical signs and vice versa (Cunha, Cabrita, & Fonseca, 2019). For this reason, the original scheme of Bussi and Mariotti (2008, p. 757) was adapted by placing the arrows in both directions (Figure 2).

METHOD

In 2018-2019, this study involved 41 pre-service teachers of two 3rd year classes of Technologies in Mathematics Education of the Basic Education course at Viana do Castelo Higher Education School. The Escape Island was part of a series of original tasks and experiences (Figure 3) using Toys 2.0 which were organized in a schedule set to ensure that all students would go through every tasks and experiences.

Each group filmed and photographed each experience to use in the collaborative reflection of each task.

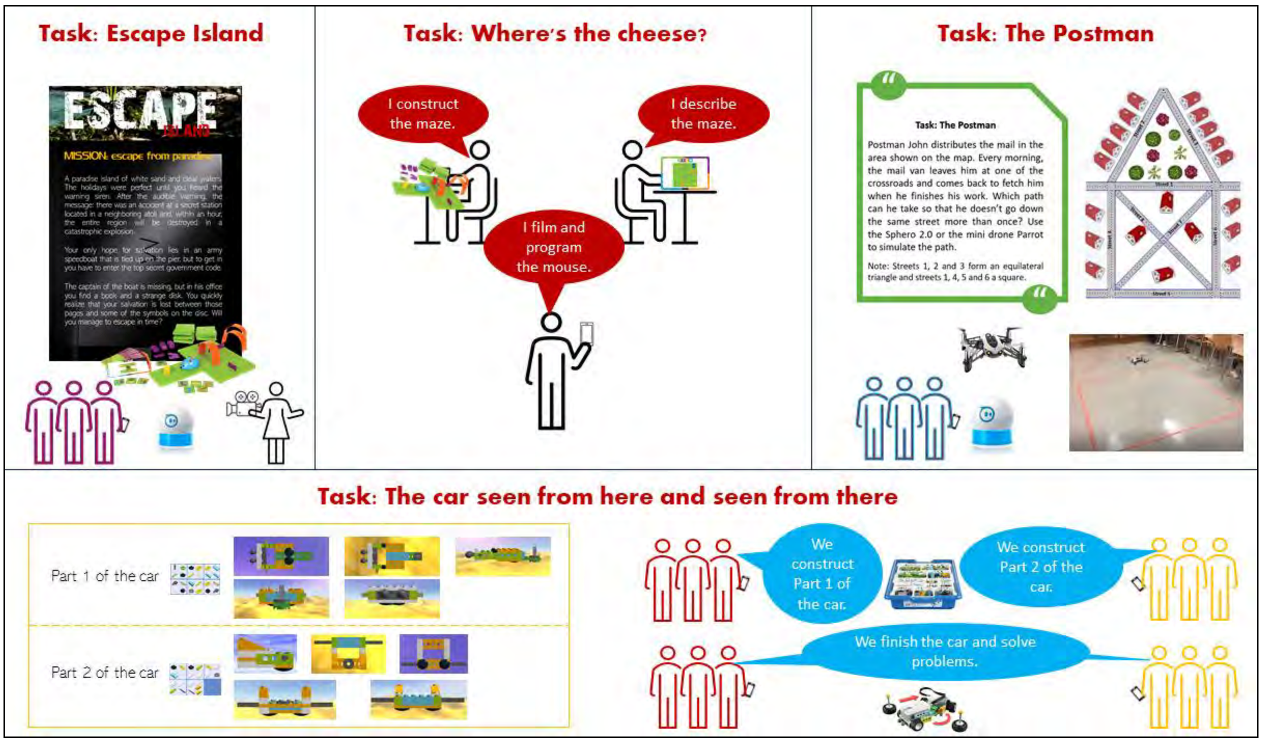

Figure 3: Set of tasks that took place simultaneously.

In this article the analysis will be focused on the task Escape Island. So, in another room, which we identified as the captain's office, in groups of three or four, students had one hour to find a code which would allow them to “escape” an island. To find it, they had to discover hidden clues and solve mysterious puzzles, filled with logical and mathematical problems.

To engage students, the story was presented through the Book of Rules, shown in Figure 4. After that, the students went into the captain's office and found a computer, a tablet, a decoder disk and an emergency manual (Figure 5).

When reading the first page of the Emergency Manual (Figure 6), students realized that unveiling the first clue was related to page 2 of the manual (Figure 7). The symbol at the top of this page was also on the computer at the captain's desk. When clicked, the message showed "Which is the code?" (Figure 8).


ESCAPE ISLAND

MISSION: escape from paradise

A paradise island of white sand and clear waters. The holidays were perfect until you heard the warning siren. After the audible warning, the message: there was an accident at a secret station located in a neighboring atoll and, within an hour, the entire region will be destroyed in a catastrophic explosion.

Your only hope for salvation lies in an army speedboat that is tied up on the pier, but to get in you have to enter the top secret government code.

The captain of the boat is missing, but in his office you find a book and a strange disk. You quickly realize that your salvation is lost between those pages and some of the symbols on the disc. Will you manage to escape in time?

1

Figure 4: Book rules.

Figure 5: Captain's office.


EMERGENCY MANUAL

If you are reading this message, there was a catastrophic accident in the US-X7 base laboratories.
To escape the island, a state-of-the-art speedboat is at your disposal, but it is locked with a security protocol deactivation code, used to ensure that only authorized members of the project will escape alive.
You have at your disposal a computer, a tablet, 2.0 devices to program, a decoder disc and this emergency manual.
The road to security starts on page 2. From then on, you will have to decode the puzzles. Take care of all the details and good luck.

1

Figure 6: Page 1 of the manual.

Figure 7: Page 2 of the manual.

Figure 8: Captain’s computer.

The link between clues required students to, without using Scratch, interpret what would be drawn by the cat when the code was activated. When inserted into the computer (the number “3”), it delivered the message: "Code accepted. You have found the first digit required to open the suitcase. Examine the Enigma under the computer".

Enigma A (Figure 9), which was hidden under the computer, gave two clues to discover a three-digit code, required to open the secret equipment bag. The code´s hundreds was number 3 and the tens digit would only be available if they removed the blue ball from the model (Figure 10) by programming the Sphero 2.0 (they had to program and use Sphero since all the balls on the model were trapped and would “explode” if touched by something else). To perform this task they had an iPad, which had the Tynker app installed, and some critical information on page 6 of the manual giving them two important clues (Figure 11): the data that the garden on the model was shaped like an isosceles triangle; and the measure of one of its angles.

When the blue ball was pushed out of the model, students could open it. They found the number 2 and the Enigma B card (Figure 12). They had to follow the cat while it moved according to the Scratch code found on page 5 of the manual (Figure 13), to discover three symbols which were to be inserted on the decoder disk (Figure 14), recycled from “Exit: The Game – The Forgotten Island (Brand & Brand, 2017) (note that only the decoder disk was recycled from this game and that all the tasks of Escape Island are original). Decoding the disk, they found the number 7.

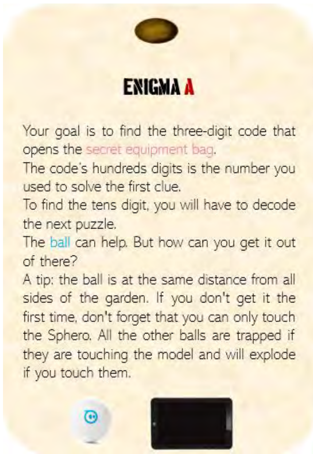

Figure 9: Puzzle A.

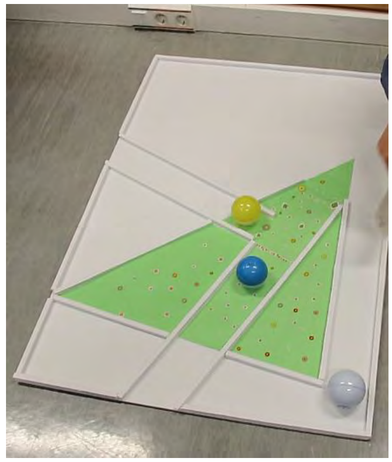

Figure 10: Model of the garden.

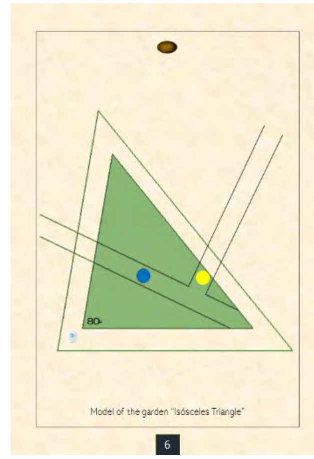

Figure 11: Page 6 of the manual.

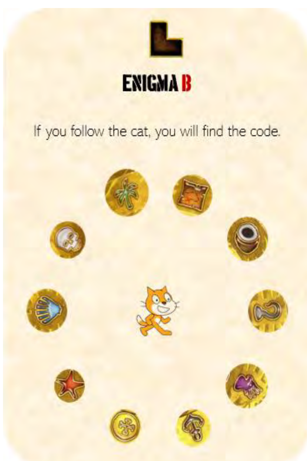

Figure 12: Puzzle B.

Figure 13: Page 5 of the manual.

Figure 14: Decoder disk.

So, the code 327 allowed them to open the secret equipment bag. Inside the bag, was the Enigma C card (Figure 15), the robot mouse kit and three of the decoder disk’s symbols.

After reading the card, the students had to search the classroom to find the black box shown in Figure 16 (it had the same symbol as the Enigma C card). The box had two holes and they had to use the phone's flashlight to illuminate the interior. Then, one of the members of the group observed what was inside (Figure 17) and gave instructions so that his colleagues could use the robot mouse kit parts to build the maze on the classroom floor. The student who looked inside the box was also able to program the path the mouse should run and he knew where to place it on the maze. Knowing where the mouse would be placed was crucial because they could then use its position as reference to interpret the coordinates associated to the decoders disk symbols (these coordinates were shown on page 4 of the manual). They could then place the symbols on the maze and the order by which they were touched by the mouse (when activated), gave them the code to insert on the decoder disk. The coded resulted on “18”, which was the number needed to complete the blank spaces on page 3 of the manual. Therefore, "318" was the code that unlocked the escape launch.

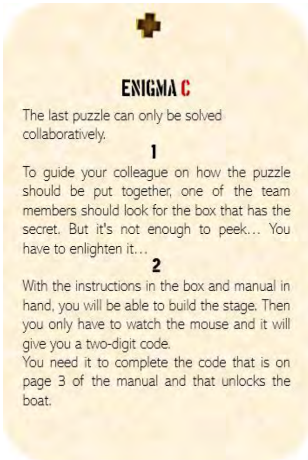

Figure 15: Puzzle C.

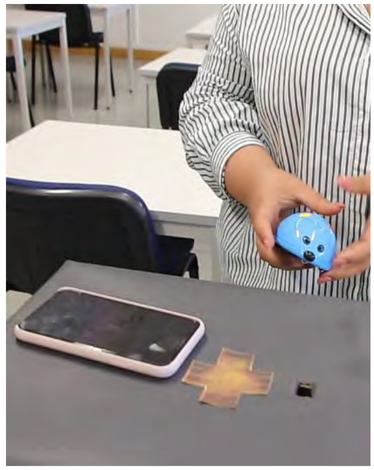

Figure 16: Box that has the secret.

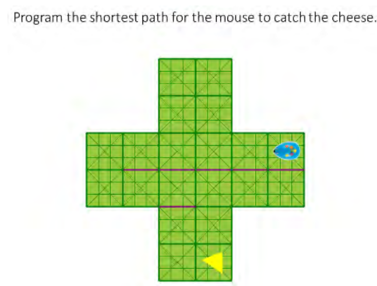

Figure 17: Secret inside the box.

RESULTS

To solve the first clue, students had to interpret the code shown on Figure 6 and insert it in the computer. Most began by drawing on paper the path which would be drawn by the cat but only one group did it successfully. The remaining groups drew figures as show on Figure 18 and had to try to simulate the cat's path (impersonating it and performing its moves). This experience showed us that all students started from the digital artefact signs – Scratch – and all of them could interpret what they meant. But, when they went on to the drawing, although the correct usage of the mathematical signs was obvious (for instance, what "90º angle" meant), they quickly lost track of where the pencil was pointing to, leading to several mistakes. Only when recurring to the pivot sign impersonation, using their own body, they could represent the symbol "three".

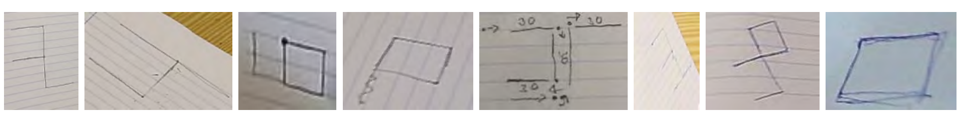

Figure 18: First try examples.

The next enigma demanded that the students programmed Sphero 2.0 to push the blue ball out of the maze. To do it they could use the Tynker app.

To solve this puzzle, they had to mobilize mathematical signs so that they could set a strategy, then move to mobilizing artefact signs and test it. Only in a second phase did most of the groups realize that they were rotating the exterior angle of the triangle and that the blue ball was at the incenter of the triangle. The angle to rotate was 155 degrees, being (180-25). We could see the mobilization of pivot signs, as gestures, among the passage from mathematical signs to the artefact signs, in a cycle of experiences which ended when the ball was finally removed from the garden model.

When they opened the ball, they found "number 2" and the Enigma B card. In this puzzle, they had to follow the cat by using the code available on page 5 of the manual. The interaction was like the one registered on the first experience. The students started from the artefact and used their fingers to follow the symbols (pivot sign and mathematical sign). Most solved this problem by estimation, failing to recognize that the symbols were all at the same distance from the cat, so each of the angles in the centre was 36 degrees. For example, when they had to rotate 108 degrees, some groups placed their fingers perpendicularly to show what was the next symbol to place on the decoder disk. One mistake that continued to occur was the fact that they had forgotten where the cat was looking when trying to find the next symbol.

When they opened the suitcase, they found Enigma C. First, they had to search the classroom until they found a box with two holes. One of the holes had to be enlightened by one's cell phone flashlight so that they could look through the remaining hole and see what was inside. Within was the picture of a maze shaped like a cross which was to be reproduced on the classrooms floor. To do so, the student who was looking inside the box had to guide his colleagues on how to assemble the maze correctly. Communication and collaboration were two of the skills mostly highlighted during this task. It was interesting to see that the students frequently used pivot signs, as "pointing", hoping their colleagues would understand what they were trying to say. The information was not always accurate – mathematical signs – which difficulted the assembly of the maze. In the end, the decoding disk symbols were to be placed on the maze using as reference the coordinates were the mouse was to be placed. Some groups had lots of trouble to perform this task.

According to Bussi and Mariotti (2008, p. 753) "gestures, drawings, or words may be the different semiotic means used to produce these signs, the production of which may be spontaneous or explicitly required by the task itself.". However, we should note that, in the programing tasks, we observed gestures, drawings or words being used to mediate the passage from the artifact sign to the mathematical sign but also to mediate the passage from the mathematical sign to the artifact sign (in a trial and error cycle which took place until the correct answer was found).

In this curricular unit's final quiz, the students were questioned on which was the classroom experience they thought had a bigger impact on acquiring concepts and mobilizing strategies to solve problems (between guided concept exploration on a dynamic geometry software and the Escape Island). 18 students answered this question: 13 (72,2%) stated it was the Escape Island; 3 (16,6%) chose the guided concept exploration on a dynamic geometry software; and 2 (11,1%) did not give a conclusive answer. To justify their answer, 6 focused the ludic, challenging or gaming side of it; 1 the motivational side of it; 3 the importance of wondering about which strategy to use; 1 the mobilization of skills you don't normally use; and 5 focused the remembering/mobilizing/deepening of mathematical concepts. We highlight the answer given by a student which embodies several mentioned aspects and shows the impact this task had to her in the mediation process:

> SA: Without a doubt, Escape Island had more impact on me. First, because it was something totally different from everything we had ever done, not for the toys, but for all the context and mystery that hung in the room. Afterwards, all challenges required us to mobilize knowledge that we had acquired in previous classes. In Sphero's challenge, it was even a little shameful to take so long to reach the solution and realize that the ball was in the incenter, because my group no longer remembered what the incenter was. But the mistakes we made and the wrong things we said will make sure I will never forget that the incenter is the intersection point of the bisectors.

CONCLUSION

The Escape Island is one of the tasks of a study that intends to analyse the influence of programming activities to develop skills such as problem solving, critical thinking, creativity and collaboration. However, in this paper, we focus only on one of the tasks, the Escape Island challenge, analyzing how students thought, the signs they mobilized, the difficulties which emerged and the students' opinions about the experience.

On challenges which began with artifact signs, namely programming sings, the students were able to correctly interpret the mathematical sign associated with each command. However, when they used a pencil to draw the path set by the commands, they quickly lost track and most of the groups had to use the pivot sign impersonation, using their own body to follow the instructions correctly (without losing track). On the puzzle which demanded they removed a ball from the model, the students began by activating mathematical signs but, before transforming them into programming sings, most of them used their own bodies to simulate the movement they expected the ball would make. After this experimentation, they tried programming and, in a cycle of several attempts, they confirmed if the initial conceptions were correct, mobilizing new knowledge and reflecting critically on their mathematical culture. It should also be noted that in the last challenge when communicating the steps to build the maze, students tended to point instead of communicating using mathematical signs.

Finally, the students presented evidence that the experience was significant and remarkable for their learning.

References

Brand, I., & Brand, M. (2017). Exit: the game – the forgotten island. Devir.

Bussi, M. G. B., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom Artifacts and signs after a Vygotskian perspective. In L. English (Ed.), *Handbook of International Research in Mathematics Education* (pp. 746–783). Routledge. Retrieved from http://www.cfem.asso.fr/actualites/archives/bartolini-mariotti_handbook

Cunha, E., Cabrita, I., & Fonseca, L. (2019). Graphic programming artefacts in the development of geometric skills. In *CIEAEM71* (pp. 164–170). Braga, Portugal.

Faggiano, E., Montone, A., & Mariotti, M. A. (2018). Synergy between manipulative and digital artefacts: a teaching experiment on axial symmetry at primary school. *International Journal of Mathematical Education in Science and Technology*, *49*(8), 1165–1180. https://doi.org/10.1080/0020739X.2018.1449908

MacDonald, A., Danaia, L., & Murphy, S. (Eds.). (2020). *STEM Education Across the Learning Continuum: Early Childhood to Senior Secondary*. Singapore: Springer Singapore. https://doi.org/10.1007/978-981-15-2821-7

Maschietto, M., & Soury-Lavergne, S. (2013). Designing a duo of material and digital artifacts: the pascaline and Cabri Elem e-books in primary school mathematics. *ZDM*, *45*(7), 959–971. https://doi.org/10.1007/s11858-013-0533-3

Papert. S. (1980). *Mindstorms Childern computer and powerful ideas*. New York: Basic Books, Inc. Retrieved from http://dl.acm.org/citation.cfm?id=1095592

Simon, M. (2017). What Is a Robot? | WIRED. Retrieved October 29, 2018, from https://www.wired.com/story/what-is-a-robot/

LEARNING MATH OUTDOORS: GRAPH THEORY USING MAPS

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Abstract. *In this paper, we present planning for a learning trajectory to teach a graph theory rule, specifically that underlying Euler's Königsberg Bridges problem, using a design research approach. With a Realistic Mathematics Education approach and contextualizing the learning trajectory in the embodied cognition theoretical framework, we designed and tested some tasks to be performed outdoors and with the use of the students' body and movement. Preliminary results show how students connected the outdoor activities with other tasks that required a higher level of abstraction, understanding some mathematical properties, referring to the experience they lived in first person. These results motivate further investigations on this topic.*

Embodied; RME; discrete mathematics; Graph theory; Eulerian paths; maps; navigation.

INTRODUCTION

Mathematics and Computer Science (CS) educators have been analyzing the possibility of using graph theory in a lower grade school setting for quite some time now. In some educational systems, it is considered almost a part of the traditional curriculum (e.g. Niman, 1975), while in others it begins to appear with some single activities proposed to the students as "extracurricular" (Ferrarello & Mammana, 2018). However, some factors as the distance from a traditional school curriculum, a lack of teacher training on the subject and a general uncertainty by teachers about trying new topics in a mathematics area they are not familiar with (Gaio & Di Paola, 2018), has not made it possible for these discrete mathematics topics to enter the school world with a shared acknowledgement yet. On the other level, the abstraction capability, with abstract thought as opposed to mental patterns, is considered in the literature proper to children only over 11 or 12 years of age (Piaget, as in Lister, 2011). Can any task stimulate the ability to abstract, at least partial, already in primary school age?

Purpose for this paper is to describe the implementation of a learning trajectory (as in Clements & Sarama, 2004) about a graph theory topic, specifically Eulerian paths, involving activities and tasks to be performed outdoor and in person, with the use of maps and navigation skills, combined with some paper and pencil tasks. The design of the learning trajectory follows a design research approach (Plomp & Nieveen, 2007) and the principles of Realistic Mathematics Education (RME, Gravemeijer, 1994) instruction theory. In this prospect, outdoor education in a suitable context gives a strength to RME, as living the situation in person makes the activity much more related to a valuable experience.

We will describe the theoretical framework and theories used as a background, then proceed with a description of the learning trajectory implemented and draw some conclusions from the data collected. The goal is to show how such an outdoor didactical approach can lead to a better understanding of the problem, a more realistic view of the mathematics involved and a deeper discovery of the mathematical rule by the students, grounded in their physical experience of the real world with their bodies.

Gaio, A., Branchetti, L. & Capone, R. (2020). Learning Math Outdoors: Graph Theory using Maps. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 95-102). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.12

Literature review

As mentioned, graph theory and discrete mathematics in general, has not to be so popular in the school world, especially at lower school grades (Gaio & Di Paola, 2018). Therefore, the need to produce a teaching-learning proposal which can be innovative and ease the approach to this area of mathematics. Even though we think it can be really interesting to present students with this kind of mathematics, which is a strong branch of mathematics and CS in modern research, the main reason for choosing the topic lies in the fact that it really can provide suitable math problem (e.g. CS Unplugged project, Bell et al., 2009) to enhance general problem solving abilities and real world mathematics. Also, activities in discrete mathematics “allow a kind of new beginning for students and teachers” (Goldin, 2004). Nonroutine problems are used to elicit mathematical reasoning processes and raise interest in both teachers and students, with new opportunities for mathematical discovery (ibid, 2004).

The replacement of algorithmic models with an emphasizing of the students’ own construction of knowledge (Jonsson et al., 2014) is stressed and, also, students will be engaged in activities for which “they have to ‘struggle’ (in a productive meaning of the term) with important mathematics” (ibid., 2014). It is in this sense suggested that children's creative development and a wide range of skills can be learned in an authentic context such as the outdoors (Beard & Wilson, 2006).

Finally, curriculum recommendations and guidelines support this approach; see for example the European Union recently published recommendations (EU Council, 2018) where mathematical competence is “the ability to develop and apply mathematical thinking and insight in order to solve a range of problems in everyday situations”.

THEORETICAL FRAMEWORK

As a theoretical framework for our research we choose that of embodied cognition (Lakoff & Núñez, 2000), since it states that the comprehension of abstract mathematical concepts is rooted in sensory-motor experiences and in interaction with the environment and the world (ibid., 2000). The conceptualization of abstract concepts, using “ideas and modes of reasoning grounded in the sensory-motor system” is called *conceptual metaphor* (ibid., 2000). Mathematical activity which is embodied, can result in a better understanding of abstract concepts (De Freitas & Sinclair, 2014) and also, cognition can be strongly linked to and actually “based in perception and action, and it is grounded in the physical environment” (Alibali & Nathan, 2012).

Such views can be related to some key aspects of the Realistic Mathematics Education (RME, Gravemeijer, 1994) instruction theory. The development of RME was all started by a Hans Freudenthal idea, that mathematics should be considered as a human activity (1991). From this significant idea, the principles of the RME theory was then formalized: guided reinvention, didactical phenomenology, and emergent models.

RME is an instructional approach that aims at bridging a gap between abstract mathematical concepts and the real world; mathematics is seen as a human activity and is therefore seen as connected to reality. Students are the actors of their learning and, guided by teachers and educators, enhance a process of reinvention of the mathematical process (ibid., 1994).

METHODOLOGY

For the design of the learning trajectory, the design research framework has been used (Plomp & Nieveen, 2007). Educational design research is the systematic study of designing, developing and evaluating educational interventions (such as programs, teaching-learning strategies and materials, products and systems) as solutions for complex problems in educational practice, which also aims at advancing our knowledge about the characteristics of these interventions and the processes of designing and developing them (Plomp & Nieveen, 2007).

The subject of discrete mathematics and graph theory and our aim seems to be a suitable environment to use such an approach. According to Clements and Sarama (2004), learning trajectories are constituted by three parts: (i) a specific mathematical goal, (ii) a path along which the children develop to reach that goal and (iii) a set of instructional activities that help move along that path. This structure looks simple yet complete to give educators a good understanding of both the math and the children concept development.

In our learning sequence, the specific goal (i) is to find out a rule to define if a given graph has or not a Eulerian path. A Eulerian path is a trail in a finite graph that visits every vertex exactly once. Apart from this, a mathematical goal we have in mind in the research is also a general improvement of collaborative problem solving and argumentative skills in the students.

The path (ii) to reach the goal is described as the developmental progression of thinking and learning in the children's understanding of the problem and of the general rule. We move from an informal understanding of the problem, to some tentative generalization of an odd/even rule, to the final formulation of a mathematical rule, appropriate for the age range of the students.

The set of instructional activities (iii) is described in the following paragraphs, presenting the sequence of tasks we used, after a double process of refinement in cycles of instructions, as in the principles of design research, going from a preliminary teaching experiment to an hypothetical learning trajectory and finally to the learning trajectory which follows.

Tasks of the learning trajectory

The preliminary teaching experiment and the learning trajectories here described were implemented in 4^{th}, 5^{th} and 6^{th} grade classes in Italy, involving a total of 19 classes and around 380 students, during the school years 2016/2017, 2017/2018 and 2018/2019. The final version of the learning trajectory, which is presented, has been implemented in 8 of these, five 6^{th} grade and three 5^{th} grade classes, with a total of around 175 students involved in the research.

The tasks proposed are about Eulerian paths in a graph, connected with the well-known Königsberg bridges problem. The town is crossed by the Pregel river, with two islands in the middle. This makes Königsberg divided on two islands and two sides, interconnected by seven bridges. Is it possible to find a path crossing all seven bridges exactly once? The problem is impossible, but a proof for it lies in a property of certain graphs to have an even degree for every vertex (i.e. an even number of edges attached to it), except for at most two vertices which are the starting and the ending one of the paths. Examples of Eulerian paths tasks can also be drawing problems such as: “can you draw a given graph, without taking the pen off the paper, and without repeating the same vertex more than once?”.

In our first teaching experiments, this kind of activity was implemented, trying to guide the students from particular cases towards a generalization of the “even degree rule”, our (i) specific mathematical goal. The paper and pencil activities worked with some of the students, but resulted, in a majority of cases to be more abstract and not adherent to a problem student could refer to. “Why can we not jump from one vertex to another and do we have to stick the pen on the paper?” or “Why can’t we pretend to cross the same bridge twice?” are some examples of reactions to the proposals. Therefrom, the need to try a different way and change the context and way of execution of these tasks. The connection was made with the possibility of moving in the space, recreating graphs in the city streets. We used a map for orienteering, a navigation outdoor sport requiring to find the best way between different control points, usually in a given order, interpreting the map and finding the right locations and routes. The use of such a detailed map allows students to work on particular topographical skills as map symbols and on their navigation ability.

We finally designed the tasks in the learning trajectory, mixing the outdoor and the paper and pencil tasks, to consider both the informal approach of the outdoor activities and the possibilities of later formalizing the concept in the more abstract paper requests.

Task (1) is used as a teaser for the next activities. Namely, the Königsberg bridges problem, as described above, is presented to students, who are let free to try to find a solution, which they obviously cannot. No big discussion is done on why it does not work, but the next tasks will guide us towards the solution.

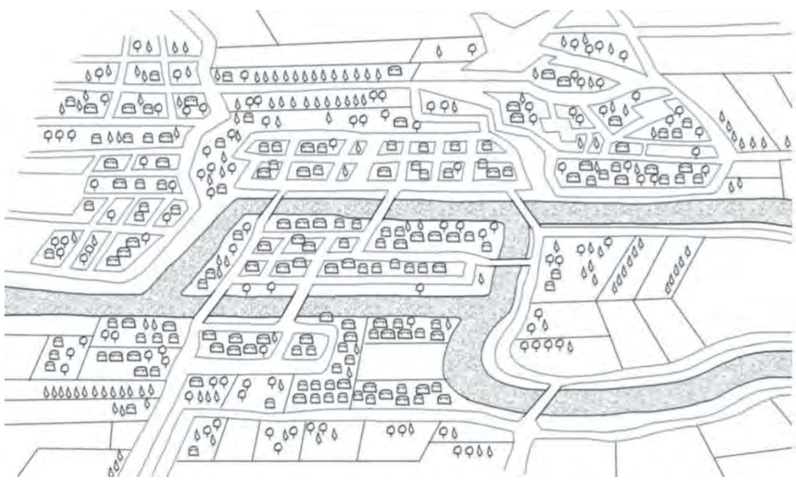

Figure 1: Task (1), the Königsberg bridges problem.

Task (2) is proposed in the classroom as a worksheet but refers to a problem contextualized in the real world. The story is that of a tourist guide, that wants to show people around, starting from and arriving at the city train station (but any other point would be fine), visiting all the roads marked in red (see Fig.2) only and only once – not to have clients seeing the same thing twice and therefore making a longer tour than needed. Is it possible to plan such a tour? Same question was posed in different situations, with different answers, as in the picture.

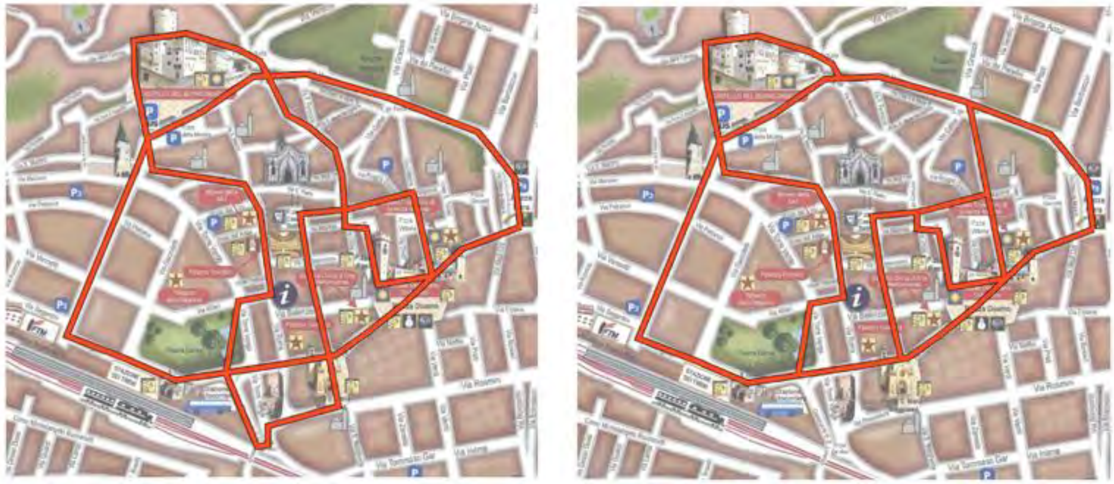

Figure 2: Task (2), with the two different situations.

Task (3) brings basically the same problem in an outdoor setting. After an introduction to orienteering maps and how to read them, the students are given a path to follow on one map of the city center (see Fig.3); the task is assigned to groups of 3 to 5 students. The goal is to follow a path which covers all the purple lines, but only once for each line, starting in the point A. "Where will the path end?" After the execution, all students which complied with the instructions will meet in point D.

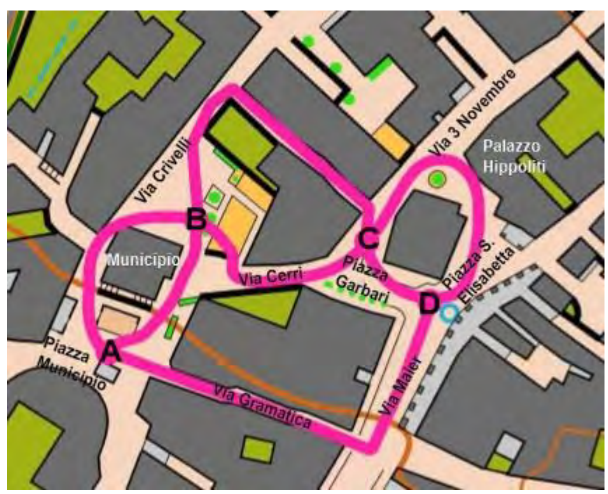

Figure 3: an example of Task (3), on a city orienteering map, with some streets highlighted.

Task (4) was a reflection, back all together, about the differences of paths taken by the various groups and what they had in common, like the end point D. "Why are we (educators) so sure that the ending point must be there?"

Task (5) is now a series of classic paper and pencil problems (as in Fig.3) about drawing a figure without taking the pen off the paper, and without re-tracing the same line more than once. Students are asked to find out which ones are possible to do, and which ones are not. Later, a discussion with the class to try to find a general rule is done, with the goal of observing some regularity about even and odd degree of the vertices of the graphs.

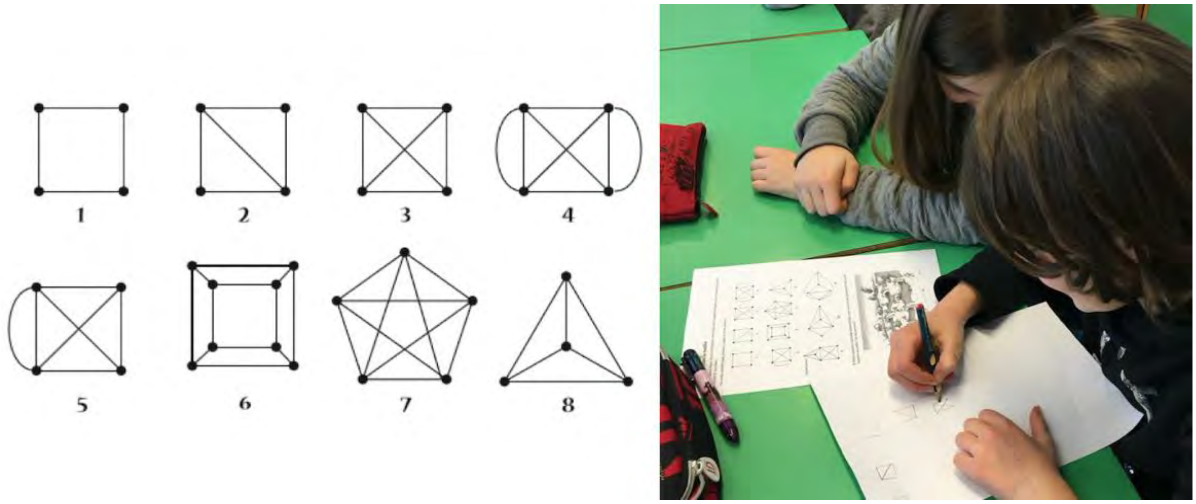

Figure 4: students involved in the paper and pencil activity, Task (4).

After agreeing on a rule that is working for each graph, in Task (6) we get back to the Königsberg bridges problem with the task of modelling it as a graph. Students, in groups, are let free to explore this task, which involves a good abstraction capability and proved not to be as easy as thought. The graph obtained allows students to prove that the Königsberg bridges problem has no solution, having all the vertices an odd degree.

Evidences from the experiments

Some observations emerged as evidence from the teaching experiments conducted. The relation between the different tasks came clear into the student's mind while performing them, even without the teachers telling them so. While working on task (5) for example we got some reactions as Lucia's one: "Wait, this is the same we were doing outside, it is like when we went around the city", referring to task (3), or Michele, during a discussion in drawing conclusions and a rule from task (5): "So this is why you were sure we would have met at the same end-point [point D in Fig. 2], there cannot be any other way!".

On some occurrences, the outdoor and RME activity worked in fostering some ideas related to the real world and movement and grounded in the use of the sensory-motor system. We observed that the outdoor activity made the students reflect in a different way than usual.

Task (3) proved to be an alternative to doing the same task on a paper. A new dimension came into play here, as showed by some of the students' reflections in task (4).

During a discussion of what happened in task (3), some students realized mathematical properties which were not obvious before (members of two different groups discussing, called 1 and 2):

Giulia (1):	We met here at point B, but we were going two different ways then.
Martin (2):	Yeah, we went from A to B, therefrom to C and then D, all the way back and then again to D. It was the easiest way.
Francesca (2):	Maybe…there was not only one way to do this, we took two different paths. Or…was your path correct? Did you follow the rules?
Giulia (1):	Yes, we did. But we found a different solution. Do you think it is correct?
Teacher:	Why didn't you just follow each other, then?

Marco (2): We could not, at that point. They [the other group] took the road from B to C, but we already used that one, so, no…

Andrea (2): Still, we all met at point D at the end, so it has to be possible that there are different solutions, but they all end up there. Why is this?

Here, the physical aspects of moving, and meeting others, leads to what will later become a conceptualization of math properties, necessary to find a general solution for the problem. In a situation elicited by the teacher discussion, for example asking why they did not follow each other until the end, when meeting in one common point, Marco answer: "we could not, at that point. They [the other group] took the road from C to D, but we already used that one, so, no…" shows that he is relating what happened in the outdoor setting with a later mathematics reflection. Finally, from a teacher interview, reflecting on the outdoor activity, the impression that outdoor learning gives a better sense of reality and generate interest and motivation in the children, together with a greater sense of responsibility, was pointed out.

DISCUSSION AND CONCLUSIONS

The changes done in the research process, placing both outdoor and practical tasks in our learning trajectory, proved to add some new challenges and to foster a better understanding of the mathematical problem. The outdoor task proved to be the best saved in the students' mind, with continuous referrals to it in the successive tasks, surely in the dimension of a positive idea of mathematics but also giving us a hint that this kind of mathematics is going towards a process of interiorization of the activities done, as RME suggests. Moreover, graph theory is a source of nice topics for classroom proposals, but sometimes requires a good deal of abstraction activity in the students.

To fill this gap, as can be seen from the evidences illustrated, principles of RME, together with body and sensory-motor experience and interaction with others, allowing students to perform what Lakoff & Núñez call metaphorical thought, through their spatial experience and concrete results obtained. Students are bringing into the mathematical setting observation they have from the real-world experience, beginning a path to "comprehend the abstract in terms of the concrete", called *conceptual metaphor*, and in this way relating their experience to the abstract mathematical problem of the graph theory rule.

This process of reconstructing the mathematical rule in a practical approach, moving in an outdoor setting and interacting with the environment and with peers in the same situation, seems to be more meaningful and the sense-making process more convincing to children, and surely worth some further research in the area.

References

Alibali, M. W., & Nathan, M. J. (2012). Embodiment in mathematics teaching and learning: Evidence from learners' and teachers' gestures. *Journal of the learning sciences*, *21*(2), 247-286.

Beard, C. M., & Wilson, J. P. (2006). *Experiential learning: A best practice handbook for educators and trainers*. Kogan Page Publishers.

Bell, T., Alexander, J., Freeman, I., & Grimley, M. (2009). Computer science unplugged: School students doing real computing without computers. *The New Zealand Journal of Applied Computing and Information Technology*, *13*(1), 20-29.

Clements, D. H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical thinking and learning*, *6*(2), 81-89.

Council Recommendation of 22 May 2018 on key competences for lifelong learning (Text with EEA relevance.) *Official Journal of the European Union*, ST/9009/2018/INIT; OJ C 189, 4.6.2018, p. 1–13.

De Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. Cambridge University Press.

Ferrarello, D., & Mammana, M. F. (2018). Graph theory in primary, middle, and high school. In *Teaching and learning discrete mathematics worldwide: Curriculum and research* (pp. 183-200). Springer, Cham.

Freudenthal, H. (1991). *Revisiting mathematics education: China Lectures*. Dordrecht, Netherlands: Springer Netherlands.

Gaio, A., & Di Paola, B. (2018). Discrete Mathematics in Lower School Grades? Situation and Possibilities in Italy. In *Teaching and Learning Discrete Mathematics Worldwide: Curriculum and Research* (pp. 41-51). Springer, Cham.

Goldin, G. A. (2004). Problem solving heuristics, affect, and discrete mathematics. *ZDM*, *36*(2), 56-60.

Gravemeijer, K. P. E. (1994). *Developing realistic mathematics education*, doctoral thesis. Utrecht, The Netherlands, CD-ß Press.

Jonsson, B., Norqvist, M., Liljekvist, Y., & Lithner, J. (2014). Learning mathematics through algorithmic and creative reasoning. The Journal of Mathematical Behavior, 36, 20-32.

Lakoff, G. & Núñez, R. E. (2000). *Where Mathematics Comes From. How The Embodied Mind Brings Mathematics Into Being*. New York: Basic Books.

Lister, R. (2011, January). Concrete and other neo-Piagetian forms of reasoning in the novice programmer. In *Proceedings of the Thirteenth Australasian Computing Education Conference*-Volume 114 (pp. 9-18). Australian Computer Society, Inc..

Niman, J. (1975). Graph theory in the elementary school. *Educational studies in mathematics*, 351-373.

Plomp, T., & Nieveen, N. (2007, November). An introduction to educational design research. In *Proceedings of the seminar conducted at the East China Normal University, Shanghai (PR China)* (Vol. 23).

MATHCITYMAP – POPULARIZING MATHEMATICS AROUND THE GLOBE WITH MATHS TRAILS AND SMARTPHONE

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Abstract. *For already over 40 years mathematics trails have been used in order to improve the attitude towards mathematics. With the availability of mobile devices, a new approach came into sight for mathematics trails and thus the MathCityMap project was founded. The projects started by reconstructing the workflow to create and walk a maths trail but then moved on to use the potential of mobile devices to create a completely new maths trail experience by elements of gamification, automatic feedback and communication. This paper describes the efforts of the project, evaluations and continuous development along with the needs of its users. The experiences, which are shared in the paper, may be useful for other initiatives to popularize mathematics.*

Key words: Maths trails, popularization, digital learning environment, MathCityMap

INTRODUCTION

Mathematics is a subject that polarizes students. Kollosche (2018) examined the perception of mathematics in an explorative study with 199 grade nine students. The results indicate that a big group of learners (about two-thirds) has a negative attitude towards mathematics, which is reflected by statements like “despair”, “stress”, “demotivation”, “depression”, “anxiety”, “exhaustion” and “headache” (Kollosche, 2018, p. 255). In particular, the students complained about a lack of emotions and discussions and furthermore find it hard to make sense of the learned content (Kollosche, 2018, p. 255). These findings might explain why mathematics is not the most popular topic among adults either. Various activities have been carried out around the world in order to improve the perception of mathematics and even the ICMI has held a whole conference solely on the topic of popularization of mathematics in Leeds 1989. For that conference Henry O. Pollak invited Dudley Blane, a mathematics educator from the Monash University, whom he had med during the ICME-5 in Adelaide (Pollak 2019, personal communication). On the ICMI topic conference in Leeds Blane (1989) reported on his activities in Australia to popularize mathematics with great success: He blazed maths trails. His first attempt was during a week of mathematics in offering an activity for families to discover the mathematics around them (Blane & Clarke 1984). Things went viral and suddenly mathematics trails have been blazed all over Australia and people like Edmund Hillary or the Governor-General of Australia started to support this kind of activities (Blane & Jaworski, 1989, Blane 1989). The idea of maths trails, originally intended for schools in the UK (Lumb, 1980), spread worldwide and even the ICME-7 in Quebec had an official maths trail during the conference as an activity for the participants (Gaulin, 1994). Eric Muller picked up the topic of mathematics trails for Canada and created the first deep gamified maths trails: The Welland Canal Math Trail (Muller, 1993a) and The Niagara Falls Math Trail (Muller, 1993b). The second one was printed like a newspaper, with measuring tape at the side of every page and handed out to tourists visiting the Niagara Falls as a sightseeing activity (see figure 1).

Gurjanow, I., Zender, J. & Ludwig, M. (2020). MathCityMap – Popularizing Mathematics around the Globe with Math Trails and Smartphone. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age* (pp. 103-110). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.13

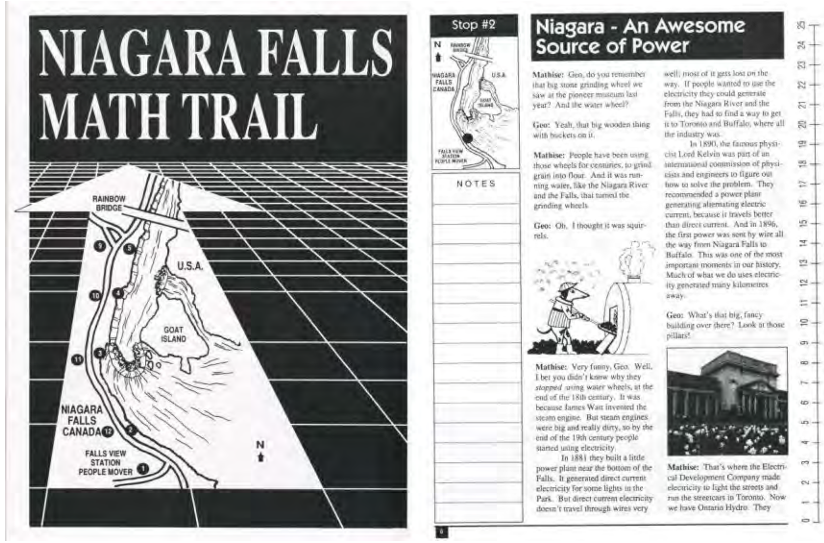

Figure 1: Two pages of the Niagara Falls Math Trail

As mentioned above, Pollak invited Blane to the topic conference in Leeds, where the word was spread, but furthermore, Pollak organized a teacher training in the USA in 1990 to which he also invited Blane (Pollak 2019, personal communication). Kay Toliver attended that training (Toliver, 2000) and developed a new way of going on a maths trail. She emphazised the idea of students creating the trail tasks instead of following an already set up trail (Toliver, 1993). A TV show was produced: “Good morning Miss Toliver” and she also got the founding for the first maths trail web portal www.nationalmathtrail.org[1]. Her work had an impact in the development of maths trails in the USA.

Meanwhile, the technological development has led to a massive spread of smartphones[2] around the globe that could be beneficial to the maths trail idea. The possibilities of a new technological approach in maths trails were introduced at ICME-12 (Jesberg & Ludwig, 2012) and the first results were presented on ICME-13 (Zender & Ludwig, 2016): the MathCityMap Project.

MATHCITYMAP AND THE POPULARIZATION OF MATHEMATICS

The MathCityMap (MCM) project combines the opportunities of a smartphone with the didactical ideas of mathematics trails. According to Shoaf, Pollak and Schneider (2004, p. 6) a “(…) mathematics trail is a walk to discover mathematics” and can be almost anywhere. The benefits of doing mathematics outdoors are based on the use of mathematical thinking in a carefree non-threatening environment (Shoaf, Pollak & Schneider, 2004, p. 5). Additionally, discussions about interesting problems and phenomena in small groups help to promote a positive stance towards mathematics and thus help popularize it. Originally, a real or a written guide helps to discover interesting mathematics in the local area. The MCM app for smartphones (available for Android and iOS) contains more than 1000 electronic maths trail guides from all over the world. Once downloaded, a maths trail can

[1] the site is archived here: https://web.archive.org/web/20090704044821/http://www.nationalmathtrail.org/
[2] in 2019 nearly 6 billion people have a mobile device, 4 billion of them have an internet connection (GSM Association, 2019)

be used even without an active internet connection. Furthermore, the app does not solely display the image and text of a task, but also offers dynamic hints, an automated answer validation and various modes of gamification (see figure 2).

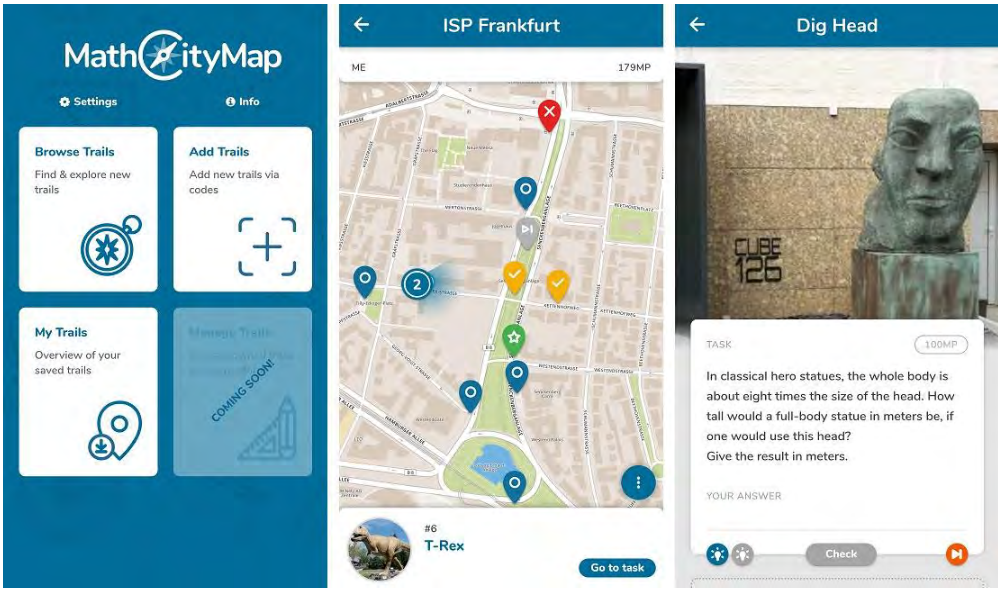

Figure 2: The MathCityMap Application for Smartphones.

Besides the MCM app, the MCM web portal (https://mathcitymap.eu) offers a handy tool for authors to create their own mathematics trails (see figure 3). Users can create tasks and trails by filling in a form to provide necessary data like the position of the object, an image, a task description, a sample solution as well as hints. Published contents are visible to all visitors of the web portal and can be included in own trails. This way the project tries to encourage people to blaze their trails, reuse task ideas, combine public tasks with own tasks to create new trails. A community of maths trailers has been established and is constantly growing.

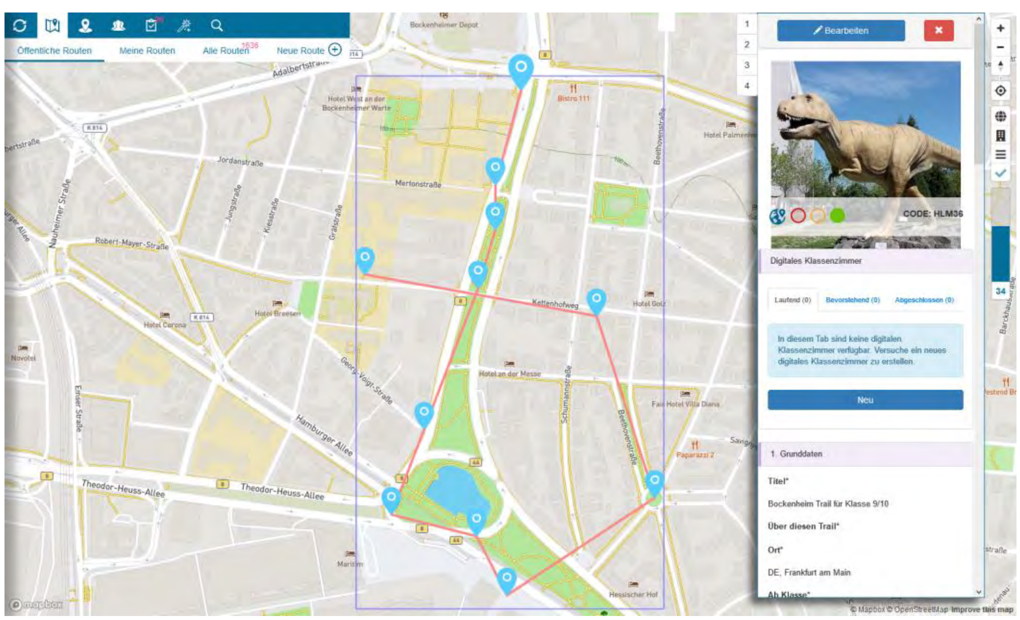

Figure 3: The MathCityMap Webportal.

Since the launch of the MCM project in March 2016, the team members have been strongly convinced that going on a maths trail can help popularize mathematics. Nevertheless, to establish a community of maths trailers, the idea of MCM itself had to be spread amongst the potential users. Team members took part in international conferences like ICME, CERME, EARCOME, ICTMA as well as national conferences and actively presented the idea of mathematics trails and the use of technology to a scientific audience. In addition, many teacher trainings that focused on the real implementation of maths trail in classes have been held. Gurjanow, Ludwig and Zender (2017) conducted the first survey among MCM users in 2017 to discover weaknesses of the system and learn more about the wishes the active users have. One major finding was the lack of material (e.g. best practice examples) that lead to not using the system. As a measure, we introduced a weekly-published article called “Task of the Week” that presented a best practice task, which can easily be implemented at other places. Also, a social media (in our case Twitter) strategy was developed and implemented. In 2018, the second user survey revealed that the taken measures were successful, since a lack of material was not reported anymore and the community growth increased (Gurjanow, Zender & Ludwig, 2018). In 2017, an Erasmus+ program was won with the help of seven European partners, who also support the idea of doing mathematics outdoors using modern technology. During the MoMaTrE[3] project the didactical ideas and the MCM software have been further developed. The app and the web portal have been translated into ten other languages. On the one hand, a professional software company is part of the consortium to pace the technological developments in that field. Bugs must be fixed fast to not lose users. In addition, new features require fast programming to get them in time. On the other hand, the Spanish Association of mathematics teachers (FESPM) is also part of the consortium and plays a big role in the dissemination of the project.

Annually user surveys and direct contact to the users during teacher trainings enable the MCM project team to have a feeling for what is the next step to take. Continuous development is one of the most important parts of the project philosophy. It is of vital importance to listen to the wishes and suggestions of the users.

Moreover, high standards are also held against users. The rules for publication are transparent and can be found in the web portal. Before a task will be made available to the public, it has to meet certain technical requirements and to pass a manual review process. This process ensures that published tasks are of high didactical quality and at the same time leads to a small professional development of the participating teachers (Jablonski, Ludwig & Zender, 2018).

New technical developments in maths trails

The first attempt of the MathCityMap Project was to digitalize mathematics trails. The next step was to create a benefit from using ICT. The first advantages were the automatic feedback and the hints the users could get from the app. In further development, the idea of gamification were picked up again for MCM (Gurjanow, Oliveira, Zender, Santos, & Ludwig, 2019). Shallow gamification came in form of scores and a leaderboard; the deep gamification came as a narrative for the trail. At the moment the trail blazer can choose a

[3] Mobile Math Trails in Europe: http://momatre.eu

pirate setting and the trail is completely changed into a pirate-style story of a treasure hunt (see figure 4). More themes are under construction and will be published within 2020.

Another new technological development is the digital classroom of MCM. It digitally represents the student groups of a classroom that are on a math trail with the MCM app in real time. Core features are the path tool, the chat and the event log. The path tool displays the positions of the participants on a map. The chat enables teachers to communicate with their students while they are solving math trail tasks to support their solution process or to organize the trail session. The event log keeps track of actions that students performed inside the MCM app, such as taking hints, solving tasks etc. The log can be used to evaluate the math trail in the following math lessons.

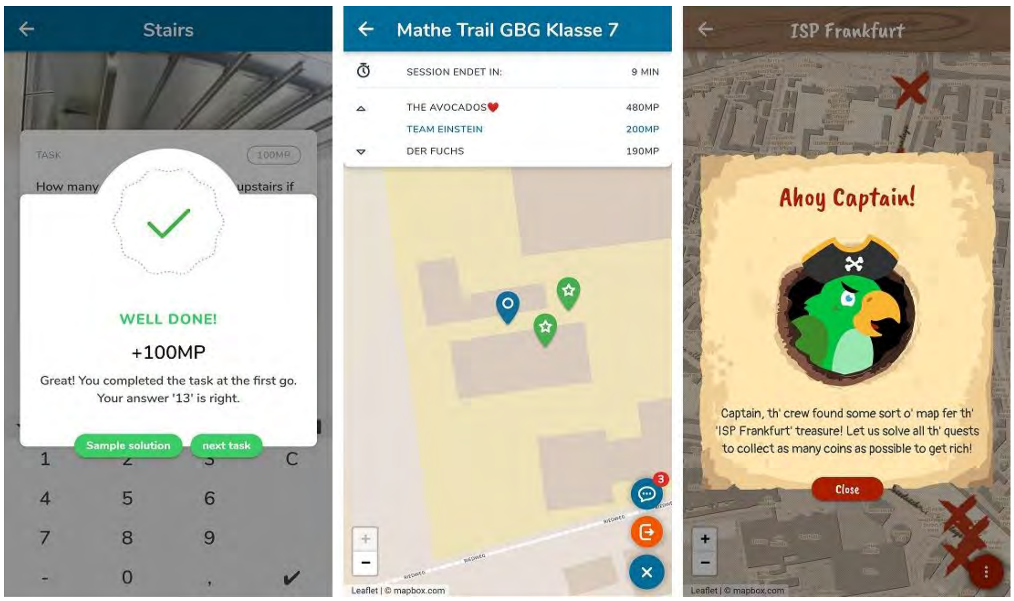

Figure 4: Shallow and Deep Gamification in the MathCityMap App.

The intention of the digital classroom is to provide the teacher a helping tool to organize a math trail as well as to regain control in the outdoor learning setting. Since the launch of the digital classroom in 2019, more than 400 sessions have been run with a peak of over 70 sessions in the month of June (see figure 5). The average number of groups per session is about 8.5, which leads us to an estimated number of 10500 students that were part of a digital classroom in 2019, since a group consists of three students.

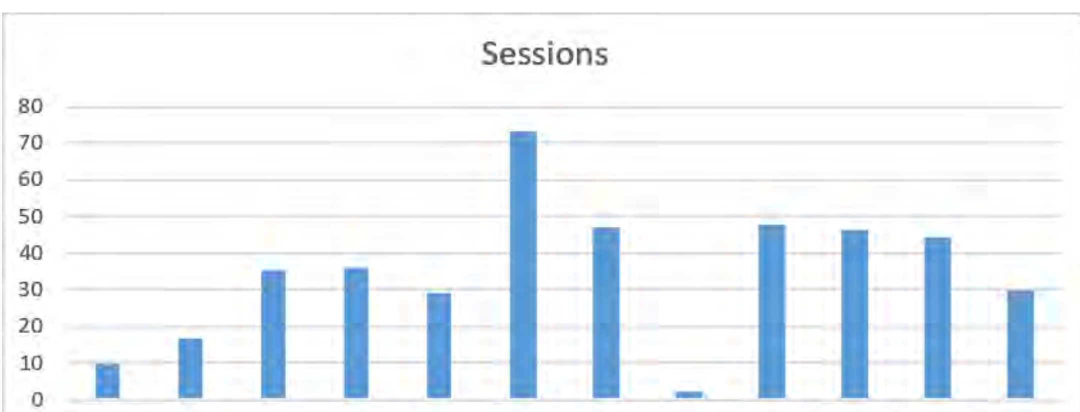

Figure 5: Monthly Number of Sessions in 2019 from January to December.

RESULTS

Summarizing the efforts, the MCM team gave mathematics trails a modern face by using the smartphone and web 2.0 technologies. A European network of mathematics education professionals who share the fascination of outdoor mathematics was established. The combination of the maths trail idea, a solid technical realization and a network of professionals lead to a growing number of users (see figure 6). In September 2019 (three years after the launch), there are over 3000 maths trails authors that created almost 10000 tasks world-wide. Over 440 mathematics trails have been reviewed by mathematics educators and can be freely used by everyone. The MCM app has been downloaded 25000 times. The growth of users is mainly focused on Europe so far, due to the MoMaTrE project. Nevertheless, a solid base has been established in Indonesia and South Africa and by the strong commitment of the Spanish teacher association, some decent first signs can be seen in Latin America.

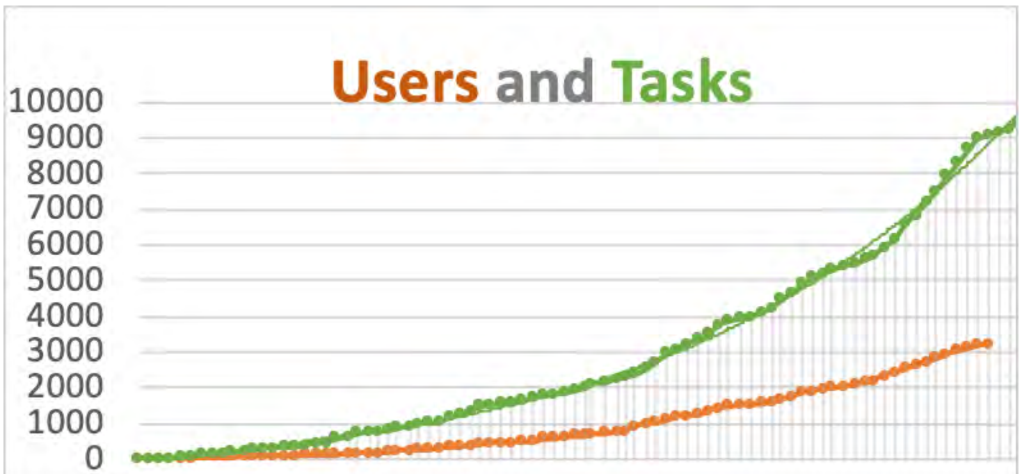

Figure 6: Number of Users and Tasks at the MCM web portal, from 1.3.16 to 01.09.19.

The MCM system could also win some reasonable prices in Germany, which mainly pushed the awareness and dissemination of the maths trail idea. The *Landmarks in the land of ideas* price was won in 2019[4], after the team was announced *Mathemacher des Monats* from the DMV in 2018[5]. In cooperation with the *Stiftung Rechnen*, a program called *Mathe.Entdecker* was founded[6]. The aim of the program is to provide high quality maths trails for families at chosen locations. For example, trails were created at the stock exchange in Stuttgart, at a science summer event in Kappeln (Northern Germany), at *Remstal Gartenschau 2019* in Schwäbisch Gmünd or in the center of Frankfurt am Main.

Finally yet importantly, some voices of the users on the question, why they use MathCityMap:

[4] https://land-der-ideen.de/en/project/mathcitymap-mcm-3896

[5] https://www.mathematik.de/des-monats/2350-mathcitymap-ist-mathemacher-des-monats-juni-2018

[6] http://stiftungrechnen.de/mehr-erleben/matheentdecker/

> It helped to authenticate mathematics learning; Brought a lot of fun and life to mathematics learning", "It's interesting, exciting, challenging", "To connect mathematics with the environment", "to make student more active and love to study math", "To sensitize teachers that mathematics can be done anywhere. To show them how real-life examples from mathematics has a place in the curriculum. To foster interest among teachers.

The next step would be to get mathematics trails into the school curriculum, like it had been in Ireland (Government Publications 1999).

References

Blane, D. (1989). Mathematics trails. *ICMI Papers on The Popularization of Mathematics*. Leeds, UK

Blane, D.C. & Clarke, D. (1984). *A Mathematics Trail Around the City of Melbourne*, Monash Mathematics Education Centre, Monash University.

Blane, D., & Jaworski, J. (1989). Mathematics on the Move. *IMA Bulletin* 25, 114–116.

Gaulin, C. (Ed.). (1994). *Proceedings of the 7th International Congress on Mathematical Education*. Sainte-Foy: Presses de l'Université Laval.

Government Publications. (1999). *Mathematics – Teacher Guidelines*. Dublin, Ireland.

GSM Association. (2019). *The Mobile Economy 2019*. GSMA Head Office. United Kingdom.

Gurjanow, I., Ludwig, M., & Zender, J. (2018). Aids and Obstacles in the use of ICT - Two surveys amongst MCM Users. *Proceedings of the ICMI-EARCOME8*. Taipei (Taiwan).

Gurjanow, I., Oliveira, M., Zender, J., Santos, P. A., & Ludwig, M. (2019). Mathematics trails: Shallow and deep gamification. *International Journal of Serious Games*, 6 (3), 65–79.

Gurjanow, I., Zender, J., & Ludwig, M. (2017). What influences in-service and student teachers use of MathCityMap? In T. Dooley, & G. Gueudet, *Proceedings of the tenth congress of the european society for research in mathematics education* (CERME 10) (p. 2366-2374). Dublin, Ireland.

Jablonski, S., Ludwig, M., Zender, J. (2018). Task Quality vs. Task Quantity. A dialog-based review system to ensure a certain quality of tasks in the MathCityMap web community. In: H.-G. Weigand, A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, N. Grønbæk & J. Trgalova (Eds.). *Proceedings of the Fifth ERME Topic Conference (ETC 5) on Mathematics Education in the Digital Age (MEDA) 5-7 September 2018*, Copenhagen, Denmark, p. 115-122.

Jesberg, J. & Ludwig, M. (2012). Mathcitymap - make mathematical experiences in out-of-school activities using mobile technology. *Talk on the 12th International Conference on Mathematics Education*. Seoul.

Kollosche, D. (2018). Soziale Dimensionen der Wahrnehmung von Mathematik durch Schüler. In: G. Nickel , M. Helmerich, R. Krömer, K. Lengnink & M. Rathgeb (Eds.). *Mathematik und Gesellschaft. Springer Spektrum, Wiesbaden*, 249-258.

Lumb, D. (1980). Mathematics trails in newcastle. *Mathematics in School*, 9 (2), 5–5.

Muller, E. (1993a). Welland Canal Math Trail. Retrieved from http://spartan.ac.brocku.ca/~emuller/mathtrail/wcmt/wcmtstart.html

Muller, E. (1993b). *Niagara Falls Math Trail*. Ontario, Canada.

Shoaf, M., Pollak, H. & Schneider, J. (2004). *Math Trails*. Lexington: COMAP.

Toliver, K. (1993). The Kay Toliver Mathematics Program. *The Journal of Negro Education*, 62 (1), 35–46.

Toliver, K. (2000). *E-Conference Transcript with Kay Toliver April 27, 2000*. New York: Bell Atlantic Technology Education Center.

Zender, J. & Ludwig, M. (2016). Mathcitymap – A new approach of an old concept. *Talk on the 13th International Conference on Mathematics Education*. Hamburg.

DEVELOPMENT OF AN INTENSIVE STUDY PROGRAMME ON OUTDOOR MATHEMATICS TEACHING WITH DIGITAL TOOLS

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Abstract. *Within the Erasmus+ project "Mobile Math Trails in Europe" (MoMaTrE), a two-week Intensive Study Programme was conducted in March 2019. 30 university students from the field of mathematics education from Germany, France, Portugal and Slovakia got to know outdoor education and the "MathCityMap" system for creating mobile math trails from a student's and teacher's perspective. With a main focus on task design and task review, the students tested their tasks with school classes and made authentic outdoor learning experiences. The following paper presents the aims, contents and schedule of the Intensive Study Programme with special focus on the MathCityMap system. Further, it analyses the evaluation among the students and gives insight into their achievements and experiences with respect to the programme's aim.*

Key words: outdoor learning, math trails, MathCityMap, Intensive Study Programme

THE INTENSIVE STUDY PROGRAMME IN THE CONTEXT OF THE MOMATRE AND MATHCITYMAP PROJECT

Authentic tasks should be a noticeable part of mathematics school lessons (e.g. Borromeo Ferri, Greefrath & Kaiser, 2013). Following the definition by Vos (2015), an authentic task should be created in an "out-of-school origin" and needs a "certification" (Vos, 2015, p. 108). Mathematics tasks are frequently proceeded inside the classroom with help of a picture and/or textual information. Such a mathematical task refers to an authentic object, but is in many cases adapted to the educational context. Here, the authenticity in the sense of an out-of-school origin and a certification is obviously (at least partly) not guaranteed. Leaving the classroom for mathematics education can play an important role in the implementation of authentic mathematics tasks. One option of outdoor mathematics is to do so called "math trails". A math trail is a walk where one can discuss and solve mathematical tasks (Shoaf, Pollack & Schneider, 2004). To solve a task, it is absolutely mandatory to leave the classroom because mathematical interaction with the task object at the object's location is required (Ludwig & Jesberg, 2015).

Originally, math trails were not created with the intention to teach mathematics. Further, they were solved solely with paper and pencil. The concept of math trails is nowadays enriched with possibilities of mobile devices to automatically give feedback and allow guidance throughout the trail. Hereby, the idea is led into an educational setting (Gurjanow, Jablonski, Ludwig & Zender, 2019). Despite positive findings regarding the impact on long-term learning and the motivation of learners (Cahyono, 2018), (prospective) teachers raise concerns regarding the organization and implementation, especially due to a lack of experience with this teaching method. This requires theoretical and empirical considerations, which are provided within the Erasmus+ project "Mobile Math Trails in Europe" (MoMaTrE, www.momatre.eu).

The MoMaTrE project started in 2017 with a project duration of three years. The consortium contains seven partners:

- University Constantin the Philosopher Nitra (Slovakia)

Jablonski, S. & Ludwig, M. (2020). Development of an Intensive Study Programme on Outdoor Mathematics Teaching with Digital Tools. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 111-118). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.14

- University Claude Bernard Lyon 1 (France)
- Federation of Mathematics Teachers Societies (Spain)
- Institute Superior of Engineering Porto (Portugal)
- University of Lisbon (Portugal)
- Autentek GmbH Berlin (Germany)
- Goethe University Frankfurt (Germany)

On the one hand, the project provides material and digital tools for teachers to easily create outdoor mathematics activities in their mathematics classes. On the other hand, it supports lecturers to create courses for university students in the field of educational studies in order to teach them how to enrich their future classes with mobile outdoor mathematics activities.

During the project, the ideas and advantages of outdoor mathematics are disseminated through research, workshops, articles and events. One of those events was an Intensive Study Programme which took place in March 2019 at Goethe University in Frankfurt. During two weeks, 30 university students from the field of mathematics education of the partner institutions in France, Germany, Portugal and Slovakia got to know outdoor education by means of digital tools with a special focus on the MathCityMap system.

The MathCityMap System

MathCityMap is a two-component system for realizing out-of-school mathematics learning using digital tools. One component is a web portal (www.mathcitymap.eu) aimed at teachers and authors who can access public tasks and create their own tasks. A task consists of a title, task image, GPS position, solution to validate, sample solution, hints, grade level, and keywords. The system is explicitly adapted to the needs of mathematics tasks. For example, the solution format "interval" tolerates small deviations in measurement and modelling tasks. Furthermore, MathCityMap offers the "Task Wizard", a catalog of pre-prepared and didactically elaborated tasks for frequently occurring questions, e.g. the slope of a handrail in percentage or degrees.

Figure 1: A math trail in the MathCityMap web portal and in the MathCityMap app.

Combining multiple tasks creates a so-called math trail (see Figure 1). A math trail can be enriched with gamification elements such as points or a pirate narrative in which all tasks get embedded. The trail with its features is created in the web portal and then started on the second component – the smartphone app ("MathCityMap" for Android and iOS). The app has – from a student's perspective – the following main features: support of navigation, presentation of the task and the task object, providing previously entered hints, and validation of the solution.

Through these features, it supports the students in their autonomous learning and the teacher in the preparation and organization of a math trail. Especially the "Digital Classroom" gives the teacher the chance to follow the students' working progress and the opportunity to support them individually in a chat while running a trail (for more information see Ludwig, Baumann-Wehner, Gurjanow & Jablonski, 2019).

CONTENTS AND SCHEDULE OF THE INTENSIVE STUDY PROGRAMME

The Intensive Study Programme's underlying aim is that university students of mathematics education get in touch with MathCityMap as an innovative theory-based approach of teaching outdoor mathematics supported by technology. The Intensive Study Programme took place at Goethe University from 18th till 30th March 2019. Each MoMaTrE partner educating university students in the field of mathematics education chose participants for the Intensive Programme with regard to motivational aspects and previously made teaching experiences.

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Opening and Organization	Lectures: Philosophy and Learning Environments	Lecture: MathCityMap Route	Excursion Mathematikum Gießen	Task Review in Groups	School Visits	Student Activity I	Lecture: Typical Errors in Task Design	Student Activity II	Math Trail Competition
Paper and Pencil Math Trail	Lecture: Task Analysis	Task Creation in Groups					Revise Trail		Final Presentation
Lecture: Outdoor Education	MathCityMap Math Trail		Task Creation in Groups	Feedback on the Tasks	Testing of the Final Trail	Lectures: Augmented Reality and Gamification	Work on Final Report	Work on Final Report	Closing Ceremony
Lecture: MathCityMap	Reflection	Introduction of the MathCityMap Web Portal		Rating of the Tasks	Prepare Student Activity I	Prepare Student Activity II			

Figure 2: Schedule of the Intensive Study Programme.

During the first week (see Figure 2, Day 1 till Day 5), the students learnt about the concept of mathematics trails and how they are embedded in mathematics education (in particular: education outside the classroom). The students received a lot of input from experts from the MoMaTrE consortium as well as invited external experts from the field of outdoor education. The students attended different lectures within the topics of Outdoor Learning, Task Design, Mobile Learning and Additional Features:

- Reasons for Outdoor Education and its Implementation (Outdoor Learning)
- Creating rich learning Environments (Outdoor Learning)
- Analysis and preliminary Analysis of Tasks (Task Design)
- The MathCityMap system (Task Design, Mobile Learning, Outdoor Learning)
- From a mathematical Trail to a MathCityMap Route (Mobile Learning, Outdoor Learning)

- Typical Mistakes when creating MathCityMap Tasks (Task Design, Mobile Learning, Outdoor Learning)
- Gamification in outdoor Education (Additional Features)
- Augmenting MathCityMap for advanced Trails (Additional Features)

Further, they did a traditional math trail without technical equipment and made a first-hand experience of walking a math trail with the MathCityMap app.

After a change in perspective, the students became task creators of their own. In international groups, they worked out tasks for school students on different levels, grades and topics. Firstly, they searched for task ideas in Frankfurt's Old Town. Afterwards, the students worked with the MathCityMap web portal to implement their tasks and create a new math trail for a field test with real school classes. During this phase, the students also learnt about the MathCityMap task design criteria (Jablonski, Ludwig & Zender, 2018), e.g.:

- Uniqueness (the object must be clear)
- Attendance (a task can only be solved at the location of the object)
- Activity (the task solver should be mathematically active)
- Multiple solutions (the task should be solvable in various ways)
- Reality (the task should have meaningful relevance and not appear too artificial)

In preparation for this test, the students conducted a peer review and finally created the tested math trails, both for lower and higher secondary students (see Figure 3).

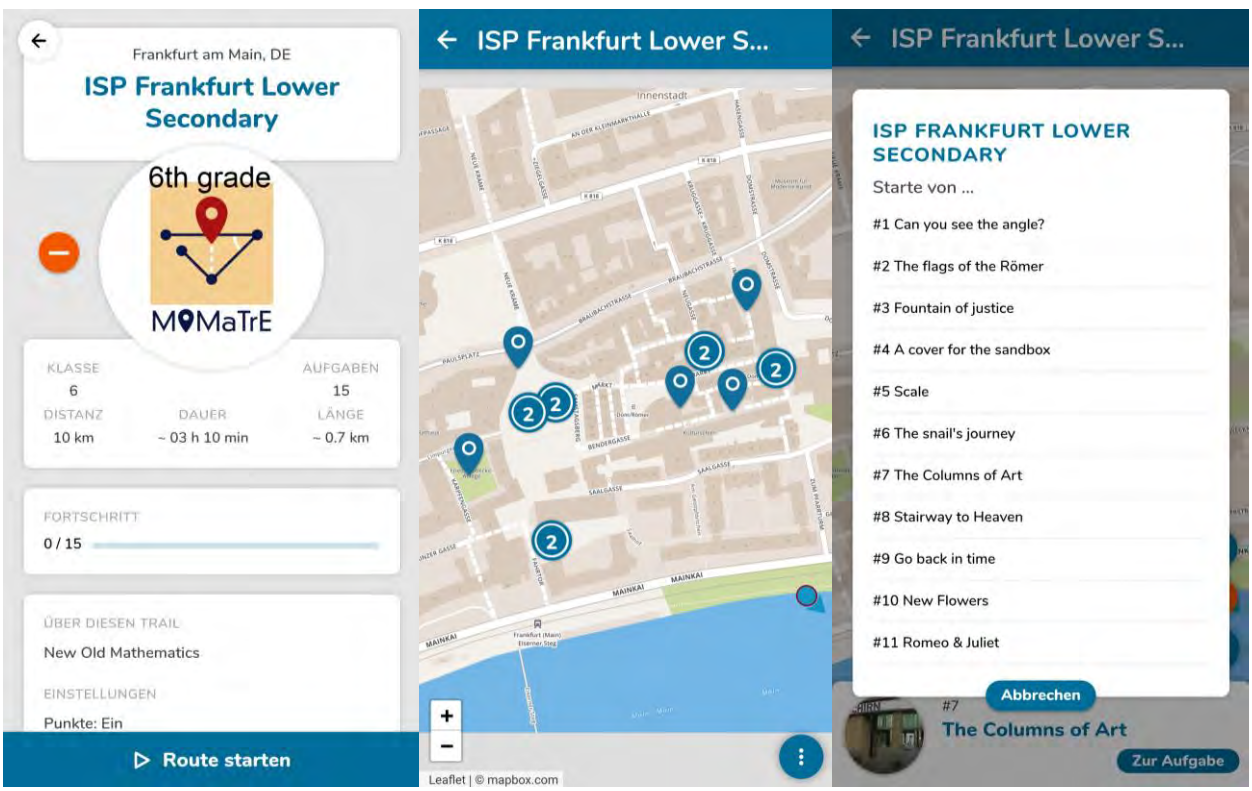

Figure 3: Screenshots of the Final "ISP Frankfurt Lower Secondary" Trail.

During the second week (see Figure 2, Day 6 till Day 10), lower secondary and higher secondary school students joined the course on two different days to do the math trails. These two field experiments were the highlights of the course, since the students experienced their work and themselves in an authentic context. There were two major tasks to be completed by the university students during the field tests:

- Evaluate if their created tasks work as intended, find and implement modifications to improve the tasks
- Make observations according to their previously defined observation focus

Afterwards, the trail events were reflected upon and summarized in the final report and a final presentation of the students. This report was based on daily tasks that the students had to upload every day including short reports on their individual impressions on lectures, activities and events. Further, the students presented and analyzed their individually created tasks and wrote a global reflection on the Intensive Study Programme.

EVALUATION OF THE INTENSIVE STUDY PROGRAMME

At the end of the Intensive Study Programme, the participating students filled in a questionnaire. Their responses as well as their final reflections in their reports are the basis for the evaluation of the Intensive Study Programme. In the following, the evaluating remarks are analyzed in terms of the underlying research question:

Which goals of the Intensive Study Programme were achieved and reflected by the students?

The main goal of the Intensive Study Programme was of academic nature, namely that the students get to know outdoor education and feel competent in its use with school students. Figure 4 shows their positive judgment (“1” very bad, “5” very good) of academic outcomes and that these expectations could be realized during the Intensive Study Programme.

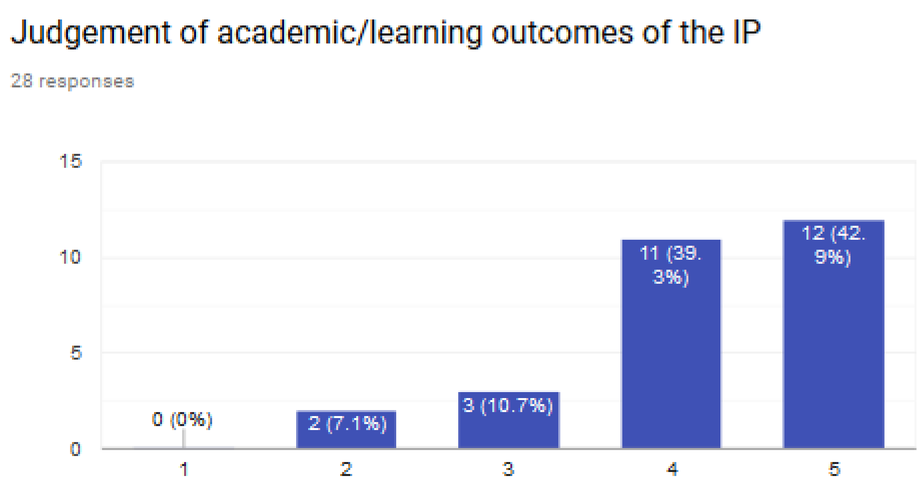

Figure 4: Responses to the Item “Judgement of academic/learning outcomes of the IP”.

A strong learning progress can be observed in the design of (outdoor) task. With the first tasks being tested, reviewed, revised and finally tested with school students, the comparison of the first draft and the final task shows that more task criteria were met.

Taking the example of the task "Fountain of Justice" in Figure 5, one can observe the different task criteria:

- Uniqueness: The object is clearly described through the picture and unique at the task's location
- Attendance: The shape of the fountain is not included in the picture, so it is necessary to be at the task's location
- Activity: The task solver has to recognize the (mathematical) shape of the fountain
- Multiple solutions: The task can be solved in different ways, e.g. by means of a sketch, counting or symmetry
- Reality: The task object is a realistic monument in Frankfurt's inner city. The question focuses on recognizing its shape which is not artificial

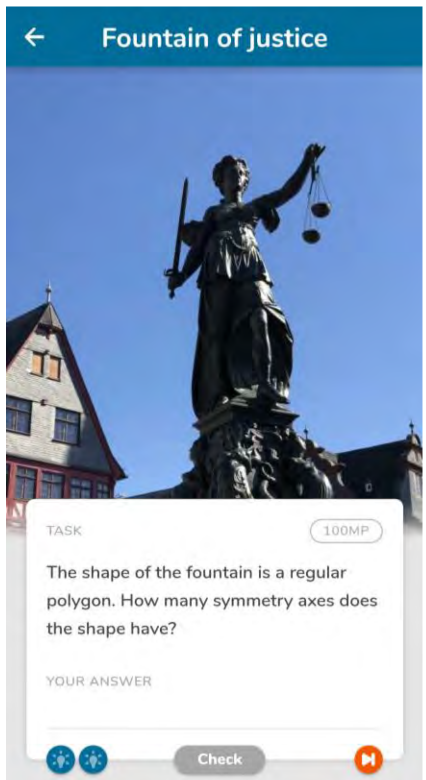

Figure 5: The Task "Fountain of Justice".

With a special focus on the judgement of individual events, the students rated the second visit of the school classes more positively. One can assume that the university students used the feedback and experiences from the first field test, improved their tasks and gained experience in the conduction of a math trail event. This issue can also be observed in some of the final reports. One student reported:

> "When the first class did our final Lower Secondary Trail, they encountered problems and misunderstandings we had not noticed. With the feedback from the students, we were able to solve these problems so that there were fewer difficulties for the class that completed the tasks on the second date. Instead, they were able to concentrate entirely on the mathematical problems to be solved."

The aim of the Intensive Study Programme to educate the university students in the use of outdoor mathematics with MathCityMap seems to be fulfilled. Especially, they had the opportunity to gain practical experiences with the app in two authentic field tests and made realistic observations which are an important basis for future outdoor learning activities. Especially the learning progress within the first and the second activity with school students is a great success as the university students seem to feel more confident in the conduct of outdoor mathematics activities.

Apart from the academic outcomes, the students rated the personal outcomes of the Intensive Study Programme (e.g. practice a foreign language, gain European experiences) very positively (see Figure 6; "1" very bad, "5" very good). Especially the exchange of experiences within four different nations and the chance to visit German school classes was appreciated by the students.

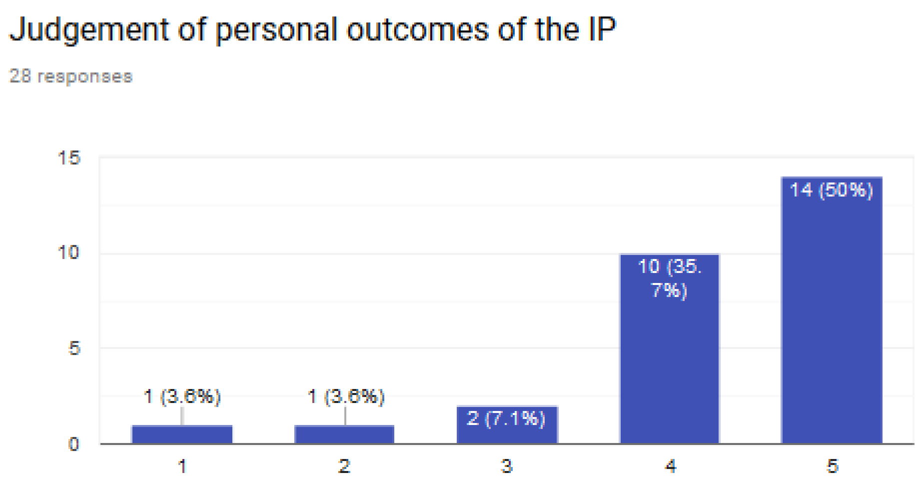

Figure 6: Responses to the Item "Judgement of personal outcomes of the IP"

The questionnaire also asked for opinions on MathCityMap as the system was completely new for all participants. Both, web portal and mobile app were rated as intuitive by most of the students. Solely the interaction between web portal and app was not for every participant easy to understand. The Intensive Study Programme is therefore not only a success for the participating students, but also an important basis for the follow-up of the MoMaTrE project focusing on the continuous development of the usability of the MathCityMap app and web portal.

Summarizing all evaluations and reflections, we can assume that the format of the Intensive Study Programme is appropriate for gaining theoretical and practical experiences in outdoor teaching and learning, for an intercultural exchange on teaching mathematics in Europe, and for getting to know the MathCityMap system with its components web portal and application.

ACKNOWLEDGEMENT

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Erasmus+ Programme, Key Action 2 – Strategic Partnerships under the number: 2017-1-DE01-KA203-003577. We thank all participants (see Figure 7).

Figure 7: Group Picture during one of the Student Activities within the Intensive Study Programme.

References

Borromeo Ferri, R.; Greefrath, G. & Kaiser, G. (2013). Einführung: Mathematisches Modellieren Lehrern und Lernen in Schule und Hochschule. In: *Mathematisches Modellieren für Schule und Hochschule*. Wiesbaden: Springer Spektrum, pp. 1-8.

Cahyono, A. N. (2018). *Learning mathematics in a mobile App-Supported math trail environment*. Cham: Springer.

Gurjanow, I., Jablonski, S., Ludwig, M. & Zender, J. (2019). Modellieren mit MathCityMap. In: I. Grafenhofer & J. Maaß (Eds.): *Neue Materialien für einen realitätsbezogenen Mathematikunterricht 6.* Wiesbaden: Springer Spektrum, pp. 95-105.

Jablonski, S., Ludwig, M., Zender, J. (2018). Task Quality vs. Task Quantity. A dialog-based review system to ensure a certain quality of tasks in the MathCityMap web community. In: H.-G. Weigand, A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, N. Grønbæk & J. Trgalova (Eds.). *Proceedings of the Fifth ERME Topic Conference (ETC 5) on Mathematics Education in the Digital Age, 5-7 September 2018, Copenhagen, Denmark,* pp. 115-122.

Ludwig, M., Baumann-Wehner, M., Gurjanow, I., Jablonski, S. (2019). Mathe draußen: MathCityMap. Mit Aufgaben-Wizard und Digitalem Klassenzimmer zum mobilen MathTrail. *mathematik lehren*, 215, S. 29-32.

Ludwig, M. & Jesberg, J. (2015). Using Mobile technology to provide Outdoor Modelling Tasks – The MathCityMap-Project. In: *Procedia Proceedings of WCES*. Elsevier, pp. 2776–2781.

Shoaf, M.-M., Pollak, H. O. & Schneider, J. (2004). *Math Trails.* COMAP Incorporated.

Vos, P. (2015). Authenticity in Extra-curricular Mathematics Activities: Researching Authenticity as a Social Construct. In: G. A. Stillmann, W. Blum & M. S. Biembengut (Eds.). *Mathematical Modelling in Education Research and Practice*. Heidelberg (u.a.): Springer, pp. 105-113.

A STEM PROJECT IN MOUNTAINS

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Abstract. *We present a STEM project based on Mathematics and GeoGebra software to be carried out in a mountain environment. But this project also includes other key aspects such as an intense use of technology and engineering, with measurements using GPS system and theodolites or optical angular measurement instruments, typical of engineering. At the same time, we must introduce the basic scientific concepts of cartography and geodesy, to which we could add those of satellite geolocalization, in order to know how and why these instruments work. The project actually consists of several independent problems with a common conceptual framework, for whose objectives we can offer precise answers and even study the margin of error in them based on the precision of our instruments and the configuration of our real scenario. The problems described here can be easily adapted to various urban or rural settings to extend their use beyond the mountains alone.*

Key words: Trigonometry, GeoGebra., cartography, app, GPS, theodolite, error measurement.

INTRODUCTION

The mountain environment offers an unbeatable setting to enjoy nature, but it might also become an exceptional learning classroom. The project we present covers all aspects that can be part of STEM education, showing how in some types of problems, all these aspects must be set in action simultaneously for a correct modeling and solution of it.

It is a project designed to be carried out as a team, where the teacher plays a very important role in directing, tutoring and facilitating, as a scenographic producer, the ways to use the varied mix of knowledge, technologies and equipment necessary for accomplishing the goal. Designed to take place in the mountains (or in some cases in the outdoors), taking data, models and views in a real setting. A partial work can be done indoors, in the elaboration of the data and its final visualization through GeoGebra applets. They can be a complement to standard curriculum subjects or, alternatively, become part of a specific STEAM curriculum.

MEASURING THE MOUNTAINS WITH TRIGONOMETRY

Our first problem is to measure the distance from a point where we are, A, to an inaccessible (by distance or physical impossibility) point V.

One of the best real application to our problem is in the mountains, measuring, for example, the distance from the point where we are to a far away peak. But it would also be worth the procedure to measure the distance to a landmark on the other side of a river, without crossing it, or at the top of a high tree in a park. The farther the inaccessible point, the more precise the angular measuring device that we must have to use.

In the activity we will have a goal drawn from a real problem that combines:

1. A model for solving with elementary mathematics (Sine Theorem, Cosine Theorem, Thales' Theorem), easily transferable to secondary school students (See Wikibooks, 2006).

2. Data acquisition techniques with precision instruments used in engineering such as theodolites or optical goniometers. You can find a detailed description of these instruments

Martínez Sevilla, A. A. (2020). A STEM Project in Mountains. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 119-126). Münster: WTM.
https://doi.org/10.37626/GA9783959871440.0.15

in (Nadolinets et al, 2017). Also from smartphone devices such as GPS. These techniques require, by themselves, some mathematical, scientific and technical explanation and allow us to get insight into current technologies such as GPS or topographic measurement. This will be accompanied by a field measurement, i.e. a *field trip for a STEM project.*

3. A data processing and visualization through GeoGebra on a cartographic map image. GeoGebra is well known to secondary school teachers and students in most countries. For a more detailed inspection of the commands and procedures, you can refer to (International GeoGebra Foundation) for a manual on how to proceed. This step will take us along the path of added scientific knowledge of cartography and georeferencing, to present our final solution.

4. Finally, we can add a brief study of how measurement errors due to the instruments used affect the final solution (for a general perspective, see Olusegun Ogundare, 2015).

The result is a project that puts together all the STEM letters. We are going to exemplify, in more detail, a case that takes the measurement from a distant point to one of the peaks of Sierra Nevada (Spain), *Veleta*. We take the measurement from the nearest point to that peak, accessible by road. To do this we must follow some steps, as described in the previous items.

1. The mathematical model and its overlapping to reality

We must first design our model and then identify the actual context on it. For this we will take a new point *B*, accessible by foot from *A*, and in such a way the distance between them could be measured using GPS technology. The farther from A, in the manner of Image 1 (trying to make an isosceles triangle), the better, since higher will be the angle γ, as we will discuss later, the fewer errors the measurement will have. In our case, it must be at least several hundred meters away. What we are doing is transforming the problem of measuring an inaccessible side (at one of its ends) of a triangle, by measuring one side, *AB*, and two accessible angles, *α, β*, of it. Using the Sine Theorem, we will have that the sought distance:

$$(1) \quad \boldsymbol{AV = (AB \cdot \sin(\beta))/\sin(\gamma)\,;\ \gamma = 180° - \ \beta - \alpha}$$

2. Data acquisition techniques

To calculate the distance *AB*, between accessible but distant points we will use a dedicated GPS device or the smartphone sensor with a vectorial cartography app. We will use the app *Oruxmaps* for a small charge (https://www.oruxmaps.com/cs/en/), but there are many for this purpose, for example, *GPX Viewer* and *View Ranger* are two of them free of charge, although they have lower navigation performance. *Google maps* is not suitable, since it is not possible to load topographic vector maps, requires online operation (little available in mountains) or preprocessing, and uses geographic coordinates, which require subsequent conversion to UTM coordinates, more didactic, clear and easy for calculation.

Then we will upload a geo-referenced map of our area (we can get it for free from most of the *National Geographic Services*), and we will create waypoints (marked points in the cartography app) at selected points *A* and *B*, reading their coordinates.

In order to make the calculation easy and understandable, from a mathematical point of view, we must explain the GPS configuration options, such as the *reference ellipsoid* (International or Hayford), the *datum,* that must correspond to that of our map (ED50), and the coordinate system (UTM) that reveal the distance in m. to the Earth's equator and the Greenwich meridian.

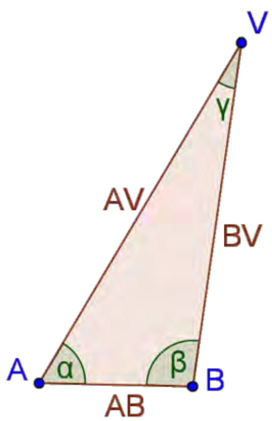

Figure 1: Mathematical model.

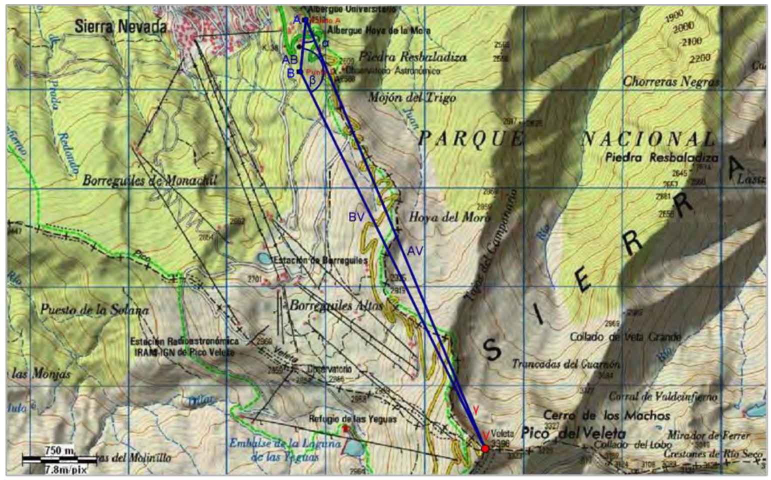

Figure 2: Overlapping the model to our map.

In our case, the coordinates of *A*: 30 S 465821 4105717 and *B*: 30 S 465702 4105148.

To calculate the angles α and β, we will use instruments typical of terrestrial engineering such as the **theodolite** (with precision to the hundredth of a degree) or, if this is not available, from an **optical goniometer** (with usual precision to the tenth of a degree).

In the absence of these resources we can use a **sextant**, an instrument from classical marine navigation, (with less precision) or even a good **alidade compass or digital compass** (for shorter distances and higher γ angle, usual precision up to the degree). For each of them we will need a basic explanation of their opto-mechanical operation and their various leveling, focus and measurement adjustments. We strongly recommend the theodolite by their complete adjustment, superb precision and didactical use.

In our problem, with a theodolite, we can take as values; $\alpha = 32.61°$ and $\beta = 143.19°$.

3. Data processing and visualization with GeoGebra

Although the *A* and *B* coordinate data are on a mountainous surface, is usual to take the distance as the straight line between these two points, and not the distance along the undulating surface of the terrain. In our model we are assuming this convention. Then we have found *AB* then has a value of 581.3 m (using Pythagorean Theorem for a triangle of vertices *A* and *B* at the hypotenuse).

As the value of $\boldsymbol{\gamma}$ is $\boldsymbol{\gamma} = \mathbf{4.20°}$ then, substituing in (1) we have: *AV* = 4755.6 m.

We can also reach this value through the GeoGebra sheet directly. To do this, we must adjust the scale in GeoGebra with the ZoomIn() and ZoomOut() commands and the <Scale Factor> option the grid, so that it coincides, for example in km, with the scale on the image of our loaded map. Then GeoGebra will not only calculate geometrically the final solution (although with less precision than directly, due to higher resolution on the vectorial map than on the raster one) but it will also show a map with all the mathematical objects to scale, much clearer when read.

Figure 3: Theodolite, optical goniometer and Screenshot from our GPS app.

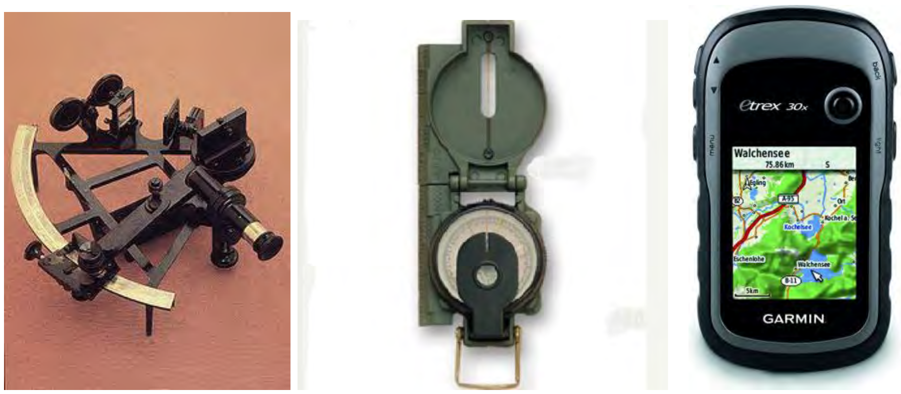

Figure 4: Nautical sextant, alidade compass and GPS device

4. Study of measurement errors

In the measurement process we introduce some error, due to the measurement technique, with which we have to deal. You can inspect. As can be seen, the calculation is very sensitive to the size of γ, since when $\gamma \to 0$, then $\sin(\gamma) \to 0$. So, as this function is in the denominator of expression (1), small variations of this can become in very different values of AV. Hence the importance of taking AB as increasing values when the precision of our measuring instrument decreases. As a complement we can add to the project a small discussion of errors, depending on the precision of the instrument. Assuming an approximation to the tenth of a degree in the measuring instrument, then we can accumulate up to 2 tenths of a degree error in determining the angle γ, and thus:

$\Delta\gamma$ (error)	value of γ	Ratio of $(AB \cdot \sin(\beta))$ in AV
0.2°	3°	13 %
0.2°	2°	29 %
0.2°	1°	117 %

Table 1: Ratio of errors in final value by angle.

Since we have used a theodolite approaching the hundredth of a degree, then each hundredth (+ or -) decreases or increases (respectively) by 11 m. approx. the distance *AV* in our case where $\gamma = 4.20°$. Each tenth of a degree (+ or -) decreases or increases (respectively) by approx. 110 m. the distance *AV*.

We therefore recommend taking a triangle setting that leaves above $3°$ the angle γ, selecting the adequate point *B*, in order to keep the error caused by the angle measurement as low as possible. As a general rule, we should try to arrange a triangle in which points *A*, *B* and *V* are not close to alignment, since this will lead to a larger angle γ, and therefore to a smaller error. But not only the angle γ will determine the error, in addition to the reading error. Also the distance from point *A* to *V* will have an effect: the farthest the point *V* is, the greater the error, following (Ali & Algarni, 2003).

In the measurement of waypoint coordinates with the GPS, and supposing the best acquisition of satellites and location, we could have a minimum error of about 4 *m.* in determining each coordinate. This will give us a total of up to $\sqrt{(x+8)^2+(y+8)^2} - AB \approx 9.5\,m.$ of error in *AB* measurement, being *x* and *y* the difference between the E and N coordinates of *A* and *B*. But it should not be forgotten that the error can be triggered in the event of a lack of precision in the acquisition of satellites due to *selective availability*, a rush in the acquisition of satellites, or a deficit in the quality of the GPS chip or signal reception. Although this error is the least influential, the previous table can be completed taking into account all the errors for our angle γ and measuring instruments.

OTHER RELATED PROBLEMS

We can also consider some problems for which we will need the same measuring instruments and similar approaches. You can get more information about these types of problems in (D'Antonio, 2011) and (Maass, 2001).

- One would be to measure the height of a point *V*, whose vertical *h* is inaccessible from an accessible point, *A*. It is based on the previous problem, we only need to measure the angle φ, which raises the *AV* side with respect to the horizon line (ρ is a right angle). See Image 5, where the vision is the usual topographic: point V (peak) is the highest and all points around it are below V.

$$h = AV \cdot \sin(\varphi)$$

- A second is to measure the distance between two points, *V* and *P*, both inaccessible, from an accessible point *A*. For this task we first proceed as in the exposed problem, taking a point *B*, accessible from *A*, and calculating the *AV* and *AP* distances. Image 6.

$$AV = (AB \cdot \sin(\beta))/\sin(\gamma)\ ;\ AP = (AB \cdot \sin(\delta))/\sin(\epsilon)$$

  Later, we will continue using the *Cosine Theorem*, which allows us to solve any triangle, known two of its sides, *AV* and *AP*, and the angle between them, σ:

$$VP^2 = AV^2 + AP^2 - 2 \cdot AV \cdot AP \cdot \cos(\sigma)$$

- The third is to calculate the distance between two points, one of them being inaccessible, without using angular measurement instruments. This problem supports many variations, and we can do it by applying the *Thales Theorem,* that states: if in a triangle ABC, a segment is drawn parallel to either side, for example $B'C'$, then triangles ABC and $AB'C'$ turn out to be similar triangles. In Image 7 we can calculate the side BC, inaccessible by measurement with tape (a street with traffic, a river, ...) from AC, AC' and $B'C'$ (can be measured with tape or on foot) and the equality:

$$\frac{AB}{AB'} = \frac{AC}{AC'} = \frac{BC}{B'C'}$$

We add here the schema of the mathematical model and the representation of the solution with GeoGebra for the three problems, without going into the details of calculations:

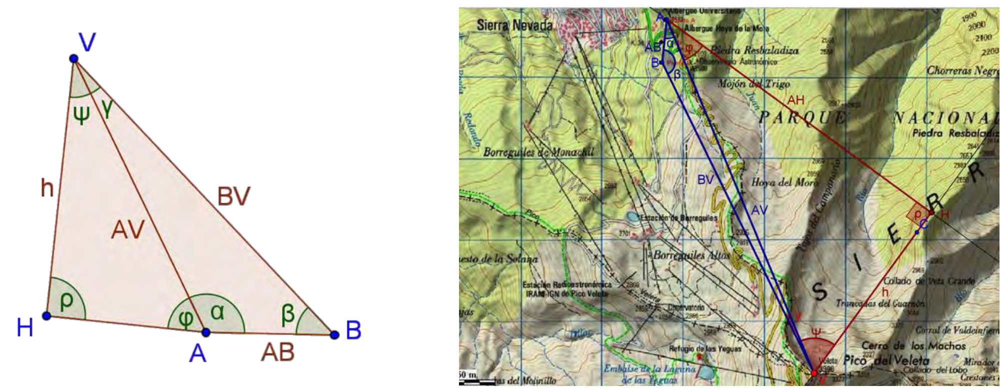

Figure 5: Model and representation for the Height Problem.

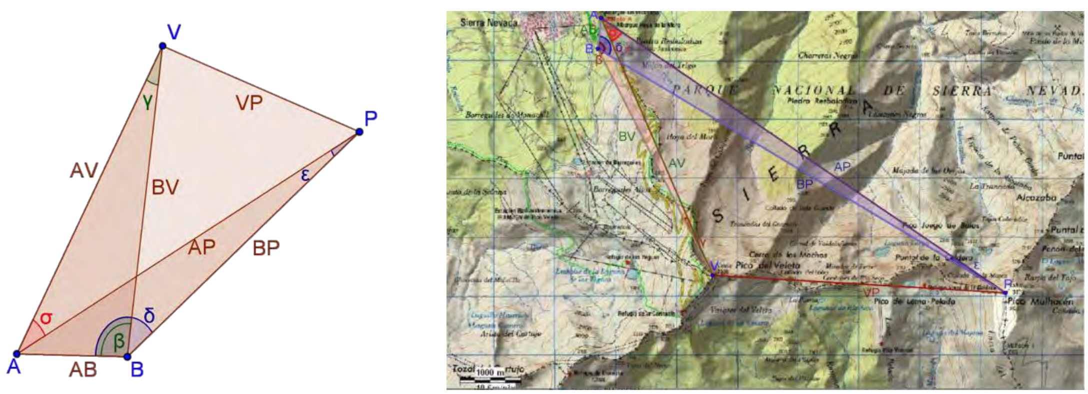

Figure 6: Model and representation for the 2 Peaks Problem.

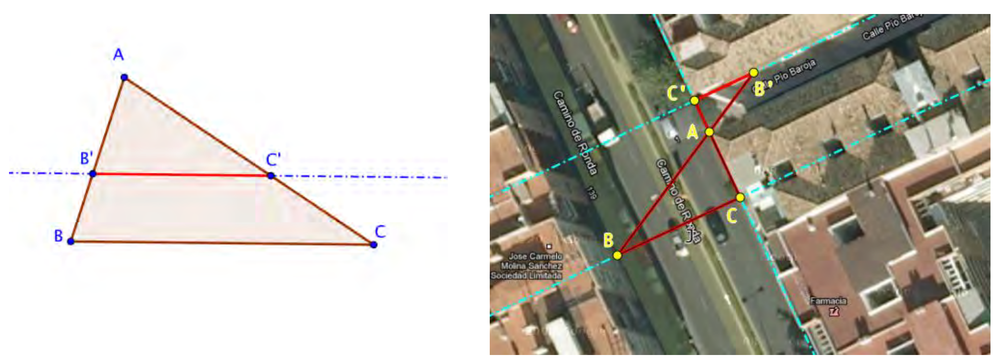

Figure 7: Model and representation for the 3rd problem.

THE EDUCATIONAL PROJECT

This project has been implemented, for three consecutive years, for students in the last years of secondary education. It is part of a broader program to awaken scientific and technological vocations, organized by the Ministry of Science and Innovation in Spain with professors from some universities attached to this project and high school students from all over the country. The project detailed here has been specifically designed by the author in the area of Mathematics and Technology within the scientific program. It has been externally supervised by the FECYT (Spanish Foundation for Science and Technology, a public institution, dependent on the Spanish government). FECYT "works to strengthen the link between science and society through actions that promote open and inclusive science, culture and science education".

The project has also been evaluated, both externally by FECYT, and by the students themselves, achieving a high score in both processes. The students, in evaluation surveys on the content and the methodology taught, have shown their satisfaction with the subjects of the project and its interrelation, as well as with the methodology and, above all, their involvement in it. The project was also finally presented by the students with their own elaboration on the data and materials obtained, before a committee of teachers. Therefore, the entire process has demonstrated high students' participation and great development of STEM learning objectives with very positive evaluation in all phases.

The project has been carried out in 3 scenarios, a classroom for the introduction to section 1, nearby mountain locations previously selected (and an outdoor location in some cases) for section 2 and part of 3 and 4, and finally a computer lab for the final elaboration with GeoGebra in section 3. To annotate, elaborate and study the data acquired in section 2, 3 and 4, students are provided with a form sheet, prepared for this task, in order not to forget any measurements and to organize the computation of the data with scientific calculators or smartphone apps.

Figure 8: Students measuring with goniometer

Figure 9: Students measuring with theodolite.

CONCLUSION

We have exposed a set of related STEM projects in which mathematics, engineering, science and technology interact. They are based on already known trigonometry application problems, but with new instrumentation, data management and scientific representation.

As a result, we have obtained a module in which, apart from the results that are common to expect from many of STEM projects (teamwork, learning to use mathematics in real life, development of critical thinking, ...), students also:

- they must learn to combine different strategies to carry it out in a practical and satisfactory way, combining direct and indirect measures depending on the context.
- they will use tools in which they will learn to assess their precision and performance, and even the crucial influence of errors on the final result.
- they will identify the arrangement and problems of the environment, with a mathematical model and will be able to visualize the problem using GeoGebra and its geometric capabilities, making a representation of it in scientific terms.

Acknowledgments

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References

D'Antonio, L. (2011). How to Measure the Earth. *Mathematical Time Capsules: Historical Modules for the Mathematics Classroom*. D. Jardine & A. Shell-Gellasch, editors. MAA Notes. 7-16.

Elsadig Ali, Abdalla & Dafer Ali Algarni (2003). Modification of the Error Equation in Angular Theodolite Measurements. *Journal of King Saud Univesity-Engineering Sciences*, 15 (2), 157-166.

International GeoGebra Foundation. *The GeoGebra Manual*. https://wiki.geogebra.org/GeoGebra-en-Manual.pdf

Maass, K. (2001). Trigonometry with Reference to Modern Land Surveying Techniques in Math Lessons. *Modelling and Mathematics Education.* Elsevier. 411-422.

Nadolinets, L. & Levin, E. & Akhmedov, D (2017), *Surveying Instruments and Technology.* Routledge Handbooks and CRC Press.

Olusegun Ogundare, John (2015). Accuracy Analysis and Evaluation of Angle Measurement Systems (chapter 4) in *The Precision Surveying: Principles and Geomatics Practice.* John Wiley&Sons.

Wikibooks (2006). Trygonometry: Edition 1.0 https://upload.wikimedia.org/wikipedia/commons/c/cf/Trigonometry.pdf

MAN IS THE MEASURE OF ALL THINGS - MATH TRAILS IN LYON

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Abstract. *We report on our 3 years long experiments in Lyon of Math trails training sessions in the realm of the Erasmus+ MoMaTrE project. We trained students to open a scientific eye on the world around them and envision their own body as the ultimate tool to make sense of it. In particular we present here the results of calibration of body parts by students, their anthropometric identity card, a statistical analysis of the results, comparing it to the Vitruvius antique esthetical cannon, discussions among students and the different strategies proposed by groups of students.*

Key words: Outdoor mathematics, Modelling, Mathematical trails, Embodied Mathematics, Measure

INTRODUCTION TO MATH TRAILS AT UNIVERSITÉ CLAUDE BERNARD LYON 1

Why teacher training with Math Trails?

Opening a scientific eye on the world around us might be the primary benefit of a science faculty education. In this rapidly evolving world, short sighted technical applications don't last long, and educating students as scientists, helping them question the universe around them and finding answers to the questions they deem interesting enough to investigate is a real challenge.

Opening a scientific eye on the world is important for science students and all the more for pre-service and in-service mathematics teachers that have to recognize, in the concepts they teach students, not only scholastic material, but as well tools that were shaped in a certain context in order to solve problems.

Our Institute for Research on Mathematics Education (IREM) in Lyon is involved in training in-service and pre-service teachers as well as university students. We describe in this article lessons learnt during the last three years regarding teaching Mathematical Trails in Lyon, especially training students to measure with their own body. We insisted on that point in order to cause appreciation of measures uncertainty among students. The main result is a confirmation of the resistance of punctual conception of measures despite formal training on measurement interval. Vitruvius ideal human proportions is put to the test with regard to the collected data.

Who we are, what we do

The Institute for Research on Mathematics Education (IREM) in Lyon has been engaged in Realistic Mathematics Education for years, specializing in Long Open Problems (Aldon, 2018; Arsac & Mante, 2007), as a teaching tool in the classroom. In 2000, we celebrated the UNESCO year of mathematics with a large mathematical trail in the city and a mobile exhibition "Why Mathematics" that we are still touring in the region, visiting one school each month. Since 2006 we have been organizing a regional mathematical competition, where around 30 000 secondary students yearly engage in teams to solve challenging problems. We participate in the House of Mathematics and Computer Science (MMI), a peculiar place, mix of a museum of mathematics, an art gallery and a training center, that federates energies around the diffusion of mathematics in the region and we organize

Mercat, C. & Berger, P. (2020). Man is the Measure of all Things – Math Trails in Lyon. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 127-138). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.16

events where mathematical trails are featured in many occasions such as Math.en.Jeans congresses.

Within the Erasmus+ Mobile Math Trails for Europe (MoMaTrE) and now Math Trails in School, Curriculum and Education Environments of Europe (MaSCE[3]) we train students and teachers to create and run mathematical trails. The students we train are undergraduate science students, pre-service mathematics teachers and pre-service primary school teachers. We conducted as well in-service teacher trainings but they are not covered in this article.

Typical training course

During a typical training, students create groups that have to:

- run a math trail, composed by selecting previous years students productions, using the mobile app MathCityMap (often in *Digital Classroom* mode for motivation);
- be trained in mastering the authoring platform (mathcitymap.eu);
- get out in the field in order to come back with pictures, measurements and lots of ideas for designing tasks;
- enter their tasks in the platform, sharing them as a group and setting up a trail;
- run, evaluate and report on the math trails of their fellow students; pairings are done in a way to maximize interactions and feedbacks;
- design and conduct their own math tasks and trail in other locations;
- report on their experience and justify the choices they made, especially diverse ways of tackling the problem, the theoretical tools they used and the associated levels of students.

Designing realistic tasks for students is not easy (Fessakis et al., 2018; Siswono et al., 2018). When *running* a mathematical trail, students are initiated in asking themselves how they can use their knowledge to solve problems; when *designing* math trails, they are *problem posers* and question their environment. In both situations, modelling a phenomenon is the crucial point. Students have to identify the relevant quantities and information, choose what to neglect and what to focus on, name items, make assumptions regarding their relations, predict, approximate, simplify, estimate, sketch diagrams, graphs, tables to collect data, measure, make statistical inferences, compute, interpet, verify, revise... All that is at stake when running or designing a math trail is of great complexity but this broad view on modelling, while guiding our teaching, is not the focus of this article. We refer to the literature for more details (Buchholtz, 2017; Cahyono et al., 2020; Cahyono & Ludwig, 2019, 2018; Druken & Frazin, 2018; English et al., 2010; Fessakis et al., 2018; Gurjanow et al., 2019; Lehrer & Schauble, 2007; Richardson, 2004). Our main point is about relating a specific moment in the course, the introduction of the training, when we teach how and why our body can be used to estimate quantities, and what we learnt from the data we collected.

To briefly give an overview of students' production, trails are located mainly on the UCBL training campuses (La Doua or INSPÉ-Lyon or Saint-Étienne) and near the schools where students teach as trainees. We had 445 tasks designed on the main campus, 38 in INSPÉ-

Lyon, 24 in St Étienne, and 143 in different schools where pre-service teachers conduct their internship.

The location of tasks on the campus is interesting, showing the main locations where students tend to flock around, library, teaching halls, sports halls, parks… and very few pieces of interest are left unchecked after three years on the spot. In order not to submerge the area with visible public tasks, most tasks are kept private.

In a given designer group, we ask students to work together at different levels, finding tasks for their little sister, for their cousins, for their parents, envisioning a scenario, telling a story and addressing different educational levels. This requirement helps students realise what is taught at which age and what it is good for as a problem solving tool. This is somewhat reversed compared to Realistic Mathematics Education techniques where reality should come first: here we require to take into account notions as tools to tackle problems.

In our experience, most questions that students come up with are not realistic in the sense of Realistic Math Education (RME) (Freudenthal, 1968), very few tackle a compelling issue, and most students simply figure out fun questions to quizz their fellow students, but anyway, we believe that having fun with math is a good start to change one's view on what mathematics *is* and what it is to *do* mathematics, even though we have not tried to objectify this belief.

ANTHROPOMETRIC IDENTITY CARD

Before the brief presentation of the mobile app, the actual introduction of the course is done through an enigmatic Identity card. The students are equipped with some measuring devices such as folding rulers and measuring tapes, some string, and are asked to fill in a card that will serve them in the field.

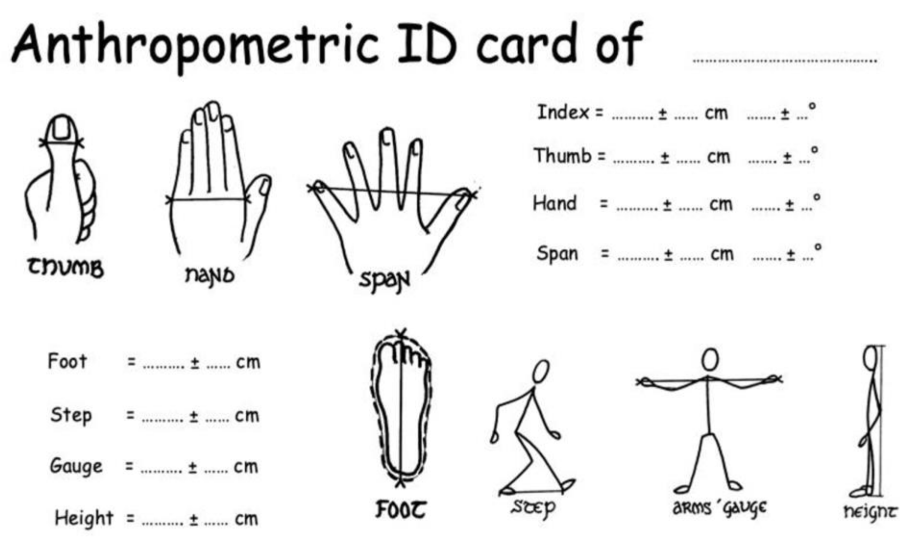

Figure 1: Anthropometric Identity Card.

In a nutshell, students are asked to calibrate their own body in order to perform measurements in the field. Some students are really reluctant at first, saying that such measures are going to be inaccurate, that a ruler gives you THE right measure. It requires some persuasion to make them evolve on the idea of what really is a physical quantity, a measure of it in a given unit and the measurement, the result of actually measuring it with a measuring tool. What follows is the recollection and unfolding of this process, where very complex yet profoundly naturalized ideas about quantities, their measure and incertitudes are discussed with students. We follow the chronological order of the course exposition.

Why do we measure things?

Man has always been counting and measuring, from dividing a stock to planning a trip to the moon, measuring serves to plan, to memorize and communicate about quantities, to validate equitable repartition and transactions, to predict what is likely to happen, in a word to better **understand** the world around us. Science, based on Aristotelian qualitative observation, is now first and foremost a science of measure and its scientificity is based on the fact that measures can be reproduced and verified (Chambris, 2008; Munier & Passelaigue, 2012).

What is a measure?

From the different representations and ideas of the students, we tend to converge after negotiations, towards the following definition, somehow compatible with the International Metrology Vocabulary:

*Measuring is finding the **ratio** between the magnitude of a quantity and the standard magnitude in a system of reference.*

Different cultures established different standards but the use of the human body as a gauge is prevalent, with units such as *foot*, attested since neolithic ages. Students' impression always is that these type of "standards" won't allow for accurate comparisons because it is not universal and not reproducible by another person. Indeed, one needs to calibrate one's own body in order to use it as an accurate tool to evaluate physical quantities, especially lengths. Therefore our first task was to have the students fill in an individual anthropometric card. And this card accounts for an evaluation of the accuracy, with an estimation of uncertainties, the first approach being to round at half a graduation on the ruler.

How to find a measure by direct comparison?

This anthropometric data serves as a calibration for a measurement protocol to be used on the field. For each protocol and each measuring instrument (part of the body) we can compare the result to the value given by a more standard protocol using a ruler, itself vitiated by errors. These errors are of two major kinds: systematic errors (the measure we made, how accurate it could be, is wrong because our protocol or our instrument are faulty) and precision errors (because the right protocol is not well applied and there are some random variations). In a short amount of time, without weather or wear and tear variations, we should consider our instrument to be stable and faithful by looking at the statistics of the results and smoothing out the random dispersion.

This gives a new insight on the choice of body parts: one should be able to observe it (so one can not use one's own nose for example) and it should lead to a robust protocol with a reduced randomness. It should be stable, easily comparable and usually based on an extremal point, where positioning error is of second order. Of course, adequacy is paramount, and the anthropometric card is usable for trails in the city or on the campus, much less in the classroom where a ruler is to be used.

By reproducing the same protocol with the same instrument, students get almost the same result, it is reproducible, but not the same as with another instrument, that is another student. Reducing their own precision errors, students tend to overlook systematic errors and abate the uncertainty of their measures. This point will be seen again during the conception of math trails with the introduction of intervals of validity.

Measuring Errors

At this point of the course students should understand that whatever measure they end up with is not THE correct actual value but they have to choose a bracket of uncertainties for accepting correct answers. But this adoption is very shallow, they still mostly tend to propose narrow options, even when fellow students consistently tell them that they measured something else. As pointed out by (Volkwyn et al., 2008) among others, the *punctual* reasoning, accessible with only one measure prevails and resists over the *statistical* approach. End of course satisfaction questionnaires often report that students confronted to the dismay of their fellows, unable to fall in the expected "good" interval, provided a firsthand experience of their own naive realism. Whether it does evolve towards scientific realism, in a more concrete way than a lecture on epistemology and the Nature Of Science is yet to be confirmed as (Otte et al., 2019) points out, despite teacher's conviction, hard evidence might be deceiving.

Evaluating a bracket

Measuring a length with a ruler for example, it is clear that the accuracy can not exceed half a subdivision, hence all the values computed from it have to take this error into consideration. On simple additions these positive errors never subtract but add up, but are often multiplicative yelding the so called "errors propagation". In more complex formulae, chain rule of derivation is useful. See (Buffler et al., 2008)

Ten Meters are not Ten Thousand Millimeters

Given the misplaced certainty of their accuracy on the part of students, we setup challenges to measure a given physical quantity, such as the length of a corridor, using different tools, whether or not adequate, and experiment the spread of the actual values. Doing so, we validate the fact that steps are as accurate as a folding ruler when measuring lengths of around a decameter. For different orders of magnitudes, different instruments are adequate.

Students are hence set to calibrate their body as a universal measuring tool. Each self-constituted team is invited to come up with their own strategy. To begin with, their first reaction is to measure what is indicated on the ID card: the width of just ONE thumb, the length of ONE step instead of calibrating it as it will be used in the field, that is to say measuring a sufficient quantity of it in order to gain precision by reducing dispersion of each individual unit. We continue to witness the resistance of the punctual conception of a measurement, if not on a single measure, then on the dispersion of measures.

Angles

Whereas estimating lengths is within every student culture, the issue of estimating angles usually opens up gaps in students comprehensions. What can be the angle associated with the span of your hand?

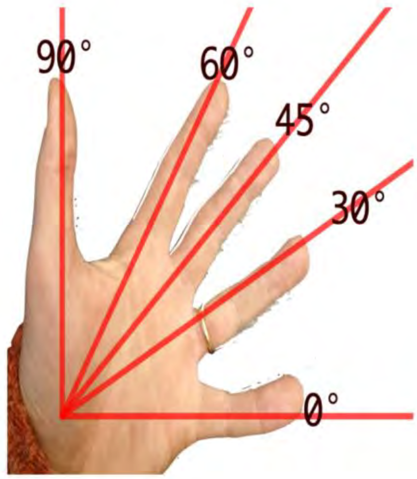

Figure 2: Rough angles.

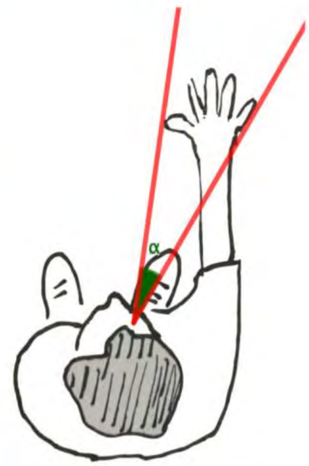

Figure 3: The span angle.

Some have heard about angles made by fingers in an open hand, which are very crude estimates (see Fig. 2). It is not at all what we are asking here. Uneasiness and awkwardness in front of the vague and obscure task, grow in the class, which is typical of the usual didactical contract of math classes where a univocal (albeit not that clear) question is asked, only by the teacher, who already knows the only possible answer to it, and the task is to find how to apply the knowledge at hand in order to please the teacher and get a good mark. Stating that being a scientist is to be able to navigate in the unknown and staying the course is not enough to alleviate students embarrassment in front of the difficulty to model the situation, to give definitions to the terms. Other such embarrassments are yet to come when students will face their first tasks assignments: "How on earth am I supposed to know that?! I have no idea!" And insisting at this point of the course does ease subsequent engagement in the field.

In order to convey what we have in mind, and abate the confusion a little, we usually show the class that, closing one eye and extending your arm, sitting in a corner of a room, you can fit pretty accurately five extended palms from one corner of the room to the next, so that the span of your palm, at arm length, seen from your eye, is approximately 18°. Each one tries and can sees for herself what it means. What we mean is the angle between two vertical planes, meeting in your eye and tangent to whether the left or the right end of the span of your hand.

But to go from this crude estimate to the measure of the angle of your thumb or your index, other strategies have to be devised, you cannot possibly add up around eighty indices in order to make a square angle?! Laying down a drawing is usually the key to understanding.

We are talking here about physical quantities, but making a sketch is usually a good creativity unblocker even in abstract frameworks.

The sketch of the light rays, the position of their intersection point, reveal widespread naive views on vision, leading to discussions on the size of an eyeball and other (yet to be proved irrelevant) topics.

Teasing students using social media, especially those focussing on pictures, such as Instagram, the notion of *Horizontal Field of View* (HFoV) of an image does eventually pop up in the debates. Trying to estimate roughly the 114° of horizontal monocular view of the human eye is a fun exercise. Having students picture what it means usually leads to the right description with the central position of the closed eye and the rays leading to it. But even in this framework, realizing what that implies for the angle of objects *blocking* the view is still another matter.

Trigonometry For The Win

At this point, asking questions such as: *what is the angle made by the door seen from the end of the classroom* does find correct answers. But the *opened* field of view doesn't seem to be as clear for a *blocking* field of view and it requires a lot of conceptual efforts for many students to be able to figure out what it means to measure the angle blocked by an index, a thumb or the span of a hand: it means to measure the angle of the object that is exactly *covered* by this part of the body at outstretched arm length.

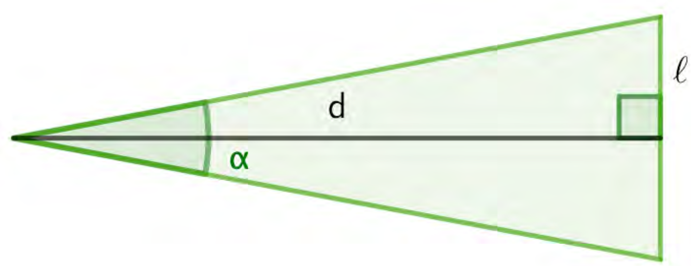

Figure 4: A thin triangle.

In Fig. 4, evaluating the field of view α is based on evaluating the size $2\boldsymbol{\ell}$ of an object, which is hidden by (say) our thumb at a distance d, subject to the formula $\tan \alpha/2= \boldsymbol{\ell}/d$. But for small angles, there is no need for a sophisticated calculator because the measure α in radian is simply the ratio of lengths $2\boldsymbol{\ell}/d$, and it is equal to its tangent and to its sinus up to negligible differences: $\tan(\alpha)=\alpha+o(\alpha^2)$, a formula that all students know but that seems to be largely out of reach in this situation. Hence, whereas the square angle on projection is of a crucial importance, its position right in the middle of the object is not relevant and $\alpha= 2\boldsymbol{\ell}/d \pm 2(\Delta\boldsymbol{\ell}/d+\boldsymbol{\ell}\Delta d/d^2)$ in radian. These very simple observations cause tremendous discussions.

STATISTICAL ANALYSIS

The Vitruvian Man (see Fig. 5) is a drawing by Leonardo da Vinci, representing ideal human body proportions in the Renaissance, based on the work of Vitruvius (Ist century BC), a Roman architect. The described proportions are: the arm's gauge is equal to the height giving the equal sides of the square, from the tip of the finger to the elbow is ⅜ of the height, the foot is 1/7th of the height. Face and shoulder proportions gives a distance from the eye to the hand of about 42% of the height. Since we collected the anonymized anthropological data of the students, we can put these ideal Renaissance figures to the test. We collected 218 entries, evaluating 12 measures: the widths of the index, the thumb, the hand palm, the hand span, in cm, then their associated angles at outstretched arm length, the foot, the step, the arm's gauge and the height. We are in position to question whether these proportions are cultural or still relate to XXIst century young adults.

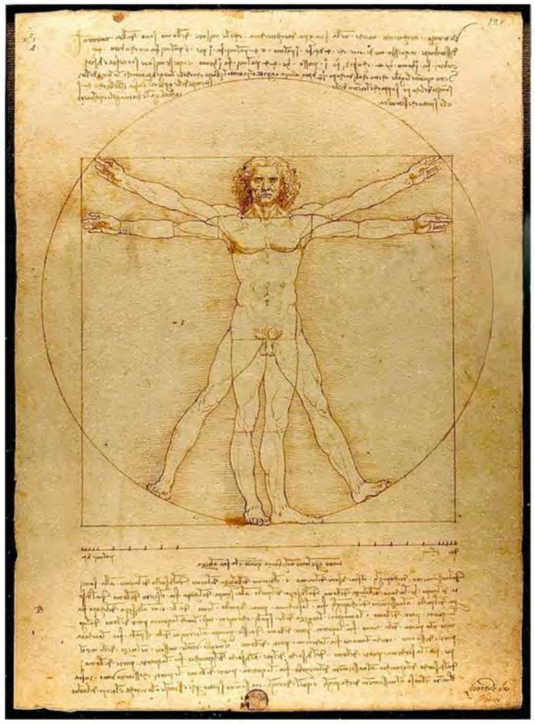

Figure 5: The Vitruvian Man by Leonardo da Vinci, c. 1490.

Another use of statistics is to infer some strategies taken by students. For example, we can compute the distance between the eye and the hand given the ratio of the size of the thumb and the associated angle, likewise for the index, the palm and the span. The deviation between these figures is informative, it is sometimes a constant, meaning that students didn't go into lengthy evaluations of angles in the way we tried to convince them but simply by estimating this arm length and computing (not measuring) the angles given the width of their thumb/index/palm/span. Sixteen students have a standard deviation less than 5% between these four measures and among them four have a zero deviation and many others having very high deviations due to bad protocols (see Fig. 6).

A remarkable result of the analysis is that the distributions of some angles are quite narrow, whether short or tall, most students have approximately 1.6° as an index field of view, 2° for their thumb and 7.2° for their palm. The dispersion of the hand span is much wider (see Fig. 8-9). That means for example that your hand, when stretching your arm and aligning its bottom part with the horizon, corresponds roughly to a half hour of sun's course in the sky, because it takes about 12h from sunrise to sunset, corresponding to 180°, that is 15°/h. More precisely, your index amounts to about 6 minutes, and two thumbs for a quarter of an hour.

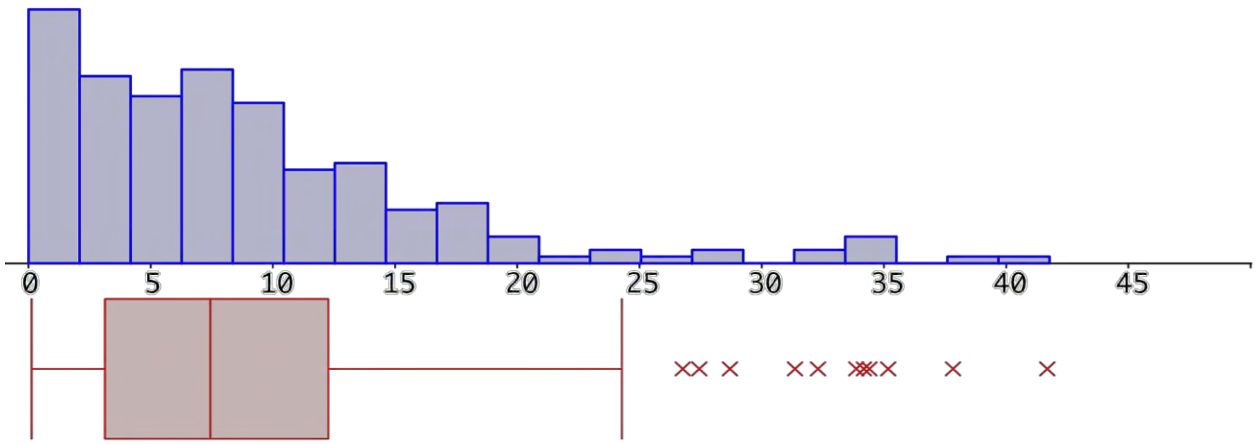

Figure 6: Standard deviation of eye/hand distance (cm).

Of course, these estimates vary with the season, days are longer in summer, shorter in winter and with your latitude (variation between 9h to 15h of daylight in France).

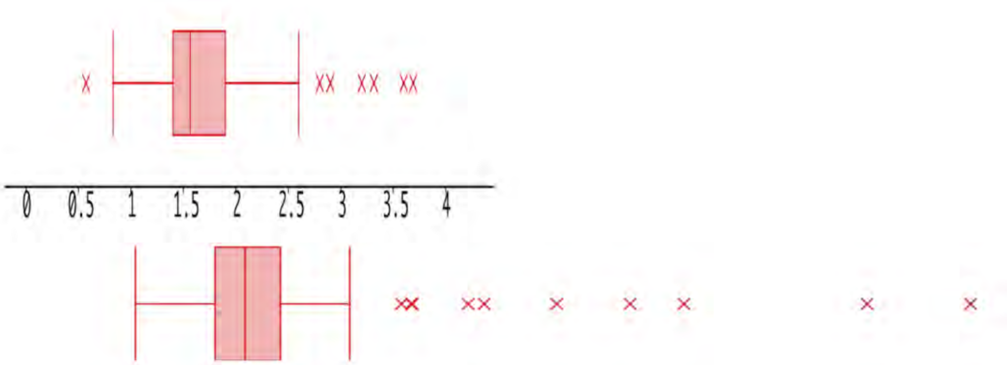

Figure 7: Box plot of index and thumb angles (°).

Dividing all lengths by the height, one can compare the dimensionless data to Vitruvius standard proportions. Some students were already aware of the correspondence between arm's gauge and height and indeed there are 40 cases of exactly same values in both fields. But even without these data, the correspondence is really very good, with a mean and median values of 1 and a standard deviation of only 4%. The foot correspondence is as well very good in terms of dispersion, with less than 10% standard deviation, but since students weren't bare foot, Vitruvius proportion is lower, students' foot, *with their shoes on,* is in a proportion of 7.8 with their height, not 7. The arm's length inferred from the fields of view appears to be more diverse than the one proposed by Vitruvius, with a much higher dispersion, due as well to many bad students protocols (see Fig. 9).

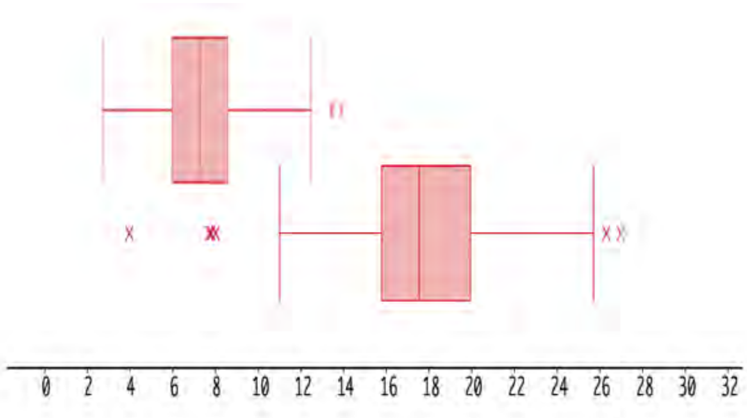

Figure 8: Box plot of the angles, of palm and span of the hand (°).

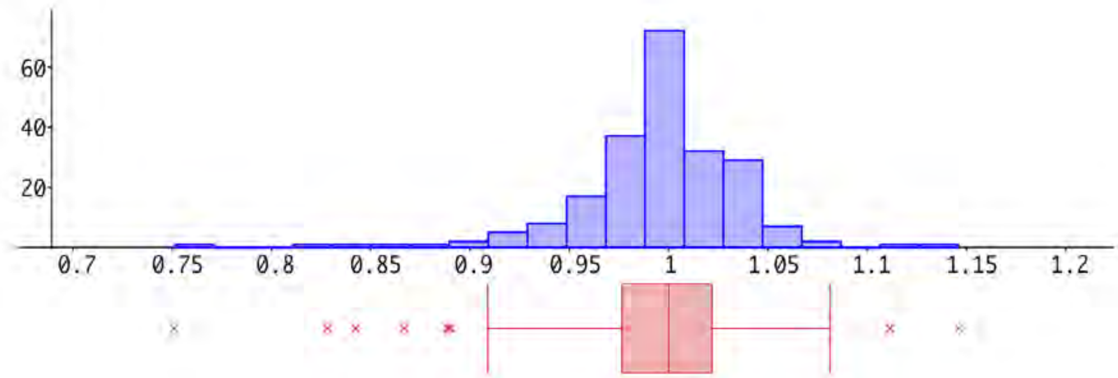

Figure 9: Dispersion of ratio arm's gauge/height

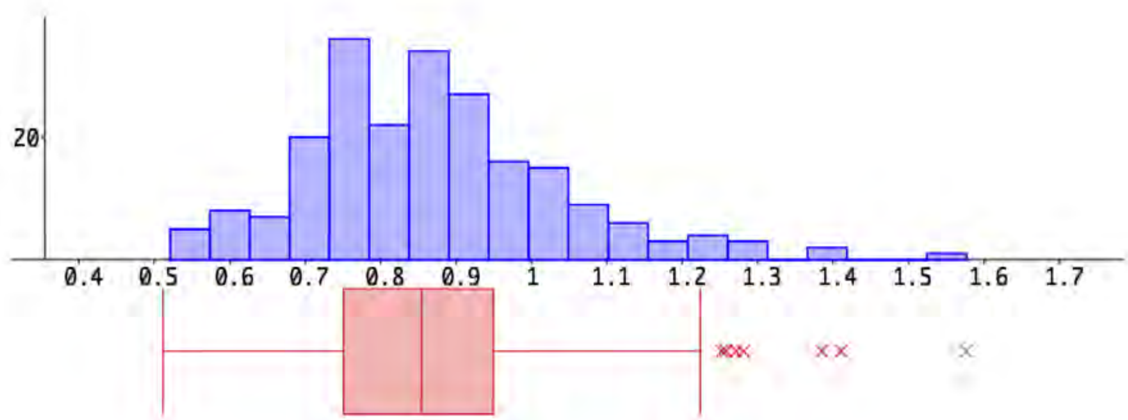

Figure 10: Dispersion of ratio of the distance eye/hand with Vitrivius proportion

CONCLUSION

In this article we have seen that math trails and specifically anthropometry can be a good incentive to open a scientific eye on the world around us, to engage students in a reflexion on exactness of measurements and scientific reasoning. Moreover, we can see that Vitruvius is still right: XXIst century human body continues to follow specific proportions.

Additional information

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References

Aldon, G. (2018). Le Rallye Mathématique dans la classe : Un jeu très sérieux. Canopé - CRDP de Lyon.

Arsac, G., & Mante, M. (2007). Les pratiques du problème ouvert. Canopé - CRDP de Lyon.

Buchholtz, N. (2017). How Teachers Can Promote Mathematising by Means of Mathematical City Walks. In G. A. Stillman, W. Blum, & G. Kaiser (Éds.), *Mathematical Modelling and Applications* (p. 49-58). Springer International Publishing. doi:10.1007/978-3-319-62968-1_4

Buffler, A., Allie, S., & Lubben, F. (2008). Teaching Measurement and Uncertainty the GUM Way. *The Physics Teacher*, 46(9), 539-543. doi:10.1119/1.3023655

Cahyono, A. N., & Ludwig, M. (2019). Teaching and learning mathematics around the city supported by the use of digital technology. *Eurasia Journal of Mathematics, Science and Technology Education*, 15(1). doi:10.29333/ejmste/99514

Cahyono, A. N., & Ludwig, M. (2018). Exploring mathematics outside the classroom with the help of GPS-enabled mobile phone application. *J. Phys. Conf. Ser.*, 983(1). doi:10.1088/1742-6596/983/1/012152

Cahyono, A. N., Sukestiyarno, Y. L., Asikin, M., Miftahudin, Ahsan, M. G. K., & Ludwig, M. (2020). Learning mathematical modelling with augmented reality mobile math trails program : How can it work? *Journal on Mathematics Education*, 11(2), 181-192. doi:10.22342/jme.11.2.10729.181-192

Chambris, C. (2008). Relations entre les grandeurs et les nombres dans les mathématiques de l'école primaire. Évolution de l'enseignement au cours du 20e siècle. Connaissances des élèves actuels. [Université Paris-Diderot - Paris VII]. https://tel.archives-ouvertes.fr/file/index/docid/339086/filename/TheseEnLigne_chambris.pdf

Druken, B., & Frazin, S. (2018). Modeling with Math Trails. *Ohio Journal of School Mathematics*, 79, 43-53.

English, L., Humble, S., & Barnes, V. E. (2010). Trail Blazers. Learning Maths Outside the Classroom. *Teaching Children Mathematics*, 16(7), 402-412.

Fessakis, G., Karta, P., & Kozas, K. (2018). The Math Trail as a Learning Activity Model for M-Learning Enhanced Realistic Mathematics Education : A Case Study in Primary Education. In Guralnick D., Simonics I., & Auer M.E., *Adv. Intell. Sys. Comput.* (Vol. 715, p. 332). Springer Verlag; doi:10.1007/978-3-319-73210-7_39

Freudenthal, H. (1968). Why to teach mathematics so as to be useful. *Educational Studies in Mathematics*, 1(1-2), 3-8. doi:10.1007/BF00426224

Gurjanow, I., Oliveira, M., Zender, J., Santos, P. A., & Ludwig, M. (2019). Shallow and deep gamification in mathematics trails. In Sobke H., Gentile M., & Allegra M., *Lect. Notes Comput. Sci.*: Vol. 11385 LNCS (p. 374). Springer Verlag; doi:10.1007/978-3-030-11548-7_34

Lehrer, R., & Schauble, L. (2007). A Developmental Approach for Supporting the Epistemology of Modeling. *In New ICMI Study Ser*. (Vol. 10, p. 160). Springer; doi:10.1007/978-0-387-29822-1_14

Munier, V., & Passelaigue, D. (2012). Réflexions sur l'articulation entre didactique et épistémologie dans le domaine des grandeurs et mesures dans l'enseignement primaire et secondaire. *Tréma, 38*, 106-147. doi:10.4000/trema.2840

Otte, C. R., Bølling, M., Elsborg, P., Nielsen, G., & Bentsen, P. (2019). Teaching maths outside the classroom : Does it make a difference? *Educational Research*, 61(1), 38-52. doi:10.1080/00131881.2019.1567270

Richardson, K. M. (2004). Designing Math Trails for the Elementary School. *Teaching Children Mathematics*, 11(1), 8.

Siswono, T. Y. E., Kohar, A. W., Rosyidi, A. H., Hartono, S., & Masriyah. (2018). Searching for Authentic Context in Designing PISA-like Mathematics Problem : From Indoor to Outdoor Field Experience. *J. Phys. Conf. Ser*., 953(1). doi:10.1088/1742-6596/953/1/012197

Volkwyn, T. S., Allie, S., Buffler, A., & Lubben, F. (2008). Impact of a conventional introductory laboratory course on the understanding of measurement. *Physical Review Special Topics - Physics Education Research*, 4(1).

CAM CARPETS AS OUTDOOR STEM EDUCATION ACTIVITY

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Abstract. *Concepts can be better understood if teaching is combined with outdoor activities. Cam Carpets offer a unique real-life application of topics in physics and mathematics and can be realised by the students themselves. A Cam Carpets is an advertising carpet, that looks from a specific perspective like an upstanding advertising banner. The idea behind this effect is based on central projection. Thus, Cam Carpets are ideally suited as real-live application for geometrical optics in lower secondary physics education or of geometric analysis in higher secondary mathematics education. Especially instruction in analytic geometry is often very schema-oriented and lacks real application. The paper discusses the potential of a Cam Carpet project from an interdisciplinary and inter-year perspective. Case studies in physics and mathematics education and its project organisational aspects are reported. In addition, possibilities for internal differentiation for different levels of performance are pointed out.*

Key words: outdoor education, STEM education, digital technology, analytic geometry

INTRODUCTION

Cam Carpets are advertising carpets that look from a very specific camera position as if they were upright advertising banners. They are known from sports broadcasts, in particular football matches, or crosswalks. Cam Carpets in football matches are usually placed next to the football goals suggesting a three-dimensional advertising object to the audience at home (Figure 1). If not viewed from the right perspective, it is only a flat lying carpet on the floor with a skewed logo (Figure 1 right). The impression of an upright banner only appears from a single perspective, which is in Figure 1 the grand stand.

The principle of Cam Carpets is based on a (central) projection, in which spatial objects are displayed in two dimensions based on an initial position which is a fixed viewpoint. When advertising with Cam Carpets, this principle is applied to the opposite. A two-dimensional Cam Carpet image (Figure 1 right) creates a three-dimensional (mental) image for the observer (the camera). However, this three-dimensional image can only be seen from a specific viewpoint. From another position is seems to be neither upright nor legible. The three-dimensional letter in Figure 2 is not physically present but due to the representation it is mentally created by our eyes located in the camera position.

Figure 1: Left: Recording of Cam Carpets in the Commerzbank Arena Frankfurt viewd from the grandstand. Right: Cam Carpet identifyable as real carpet. (©Eintracht Frankfurt)

Reit, X.-R. (2020). Cam Carpets as Outdoor STEM Education Activity. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 139-146). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.17

Cam Carpets on the one hand offer an authentic application of mathematics and physics learned in school and on the other hand, they combine those contents with a result that is created by the students themselves outdoors. The latte is not mandatory as Cam Carpets can also be realised indoor but the aspect of creating a large-format Cam Carpet outdoors on the schoolyard which can be admired by the whole school community is an additional motivating factor. Another important feature of realising Cam Carpets outdoors is the implementation of mathematics in the real environment. Everybody knows how to construct a right angle with a triangular or how to draw dots in a coordinate system on quad paper. In the real environment a triangular will not help and the floor is usually not checkered. Authentic, real-world contexts drive the students' questions (Beames & Brown, 2016). Applying mathematics outdoors help students to understand mathematical concepts (Moss, 2009).

A deeper analysis of the Cam Carpet situation leads to geometrical optics or analytic geometry. Especially analytic geometry is a topic in higher secondary mathematics education which is taught mainly schema-driven (Filler, 2007; Borneleit, Danckwerts, Henn, & Weigand, 2001). Cam Carpets can be realised in small- or large-format with help of analytic geometry or geometric optics. Thus, they offer an impressive real-live application of mathematical and physical content. The Cam Carpet project can be realised with or without knowledge in analytic geometry. In the following, the modelling project Cam Carpets is described from a professional and teaching perspective from an inter-year and interdisciplinary perspective.

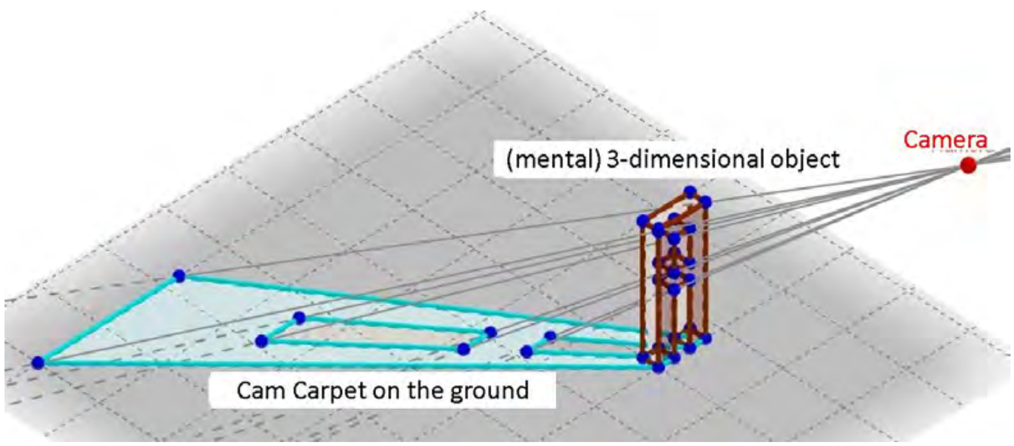

Figure 2: Central projection of a 3-dimensional letter originating from a fixed camera position.

CAM CARPETS IN LOWER SECONDARY PHYSICS EDUCATION

In lower secondary physics education light is approximated by rays. This is a model-based description which can explain related phenomena with sufficient accuracy. Besides conventional (shadow)experiments using common light sources like candles or torches, Cam Carpets offer a different and interesting approach to this light model. If considering the principle behind Cam Carpet as central projection, rays are represented as lines through a fixed point (camera point) conceivable as point-shaped light source (cf. Figure 2). Within the topic "shadows", Cam Carpets allow to penetrate those physical contents with an application used in reality. For high-performing students, considerations of linear functions in three dimensions can be a mathematical challenge and form a link to analytic geometry. The Cam Carpet project thus provides approaches for different performance levels and grades.

Implementation in class

As introduction a qualitative view of the principle of Cam Carpets via LEGO bricks and their shadows is to be considered (Figure 3). Shadows can be traced and attempts can be made to recognise the three-dimensional original LEGO figure by positioning the eyes at the light source. Limits of shadowing become apparent very quickly, which Cam Carpets can overcome: with shadowing only edges bordering translucent surfaces (cf. Figure 3). Other edges, for the three-dimensional impression necessary, can only be added retrospectively but not projected directly. Cam Carpets enable more detailed projections and better three-dimensional impressions.

The goal of a Cam Carpet illustration on the schoolyard should be clear to every group of students. In this phase the following questions must be asked. Which logo should be projected? Which dimensions should the final Cam Carpet have? How to find the projection points? Several possibilities are available according to the students' individual level of performance: GeoGebra (or other DGS), mathematically (taking up linear functions, for high-performing students) or through a handmade model using strings representing the projection lines? Which camera position is reasonable (considering the intended final dimensions)?

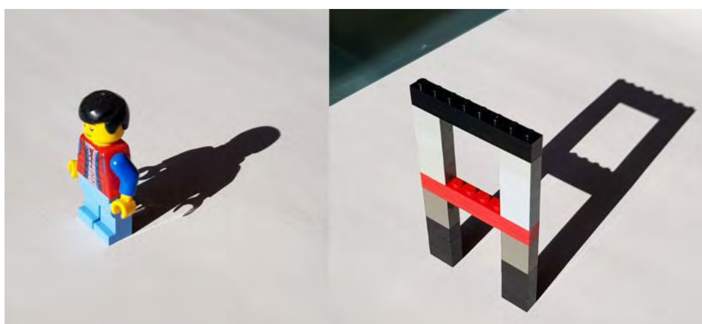

Figure 3: Introducing the Cam Carpet principle via shadowing (photo: D. Sommerbrodt).

Small format (A3/4) realisations can be created with little effort. A (smartphone)camera, tripods, GeoGebra (if applicable) to construct the projection points and a logo idea is needed. If the decision is to use a DGS like GeoGebra, a three-dimensional model has to be created in GeoGebra (Figure 2, left). Subsequently, the projection points can be generated by using the tool "lines" and "intersect" to create lines from camera point to every 3D-logo point and intersect those lines with the xy-plane (Figure 2). The simultaneous display of 3D-model and 2D-Cam Carpet (Figure 4) provides a lucid illustration. If positioned correctly the final Cam Carpet can be observed very impressively (Figure 5).

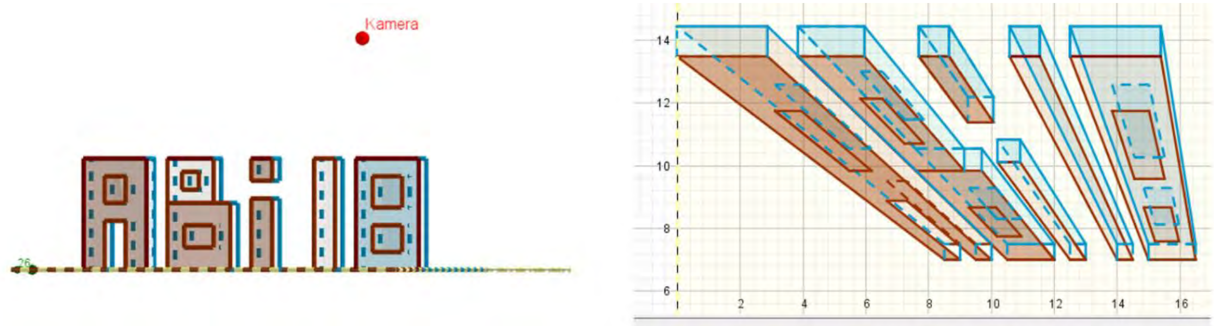

Figure 4: Simultaneous display of 3D-logo model and associated 2D-Cam Carpet.

How the Cam Carpets find their way to the schoolyard is described within the following chapter using a large-format Abi18 Cam Carpet.

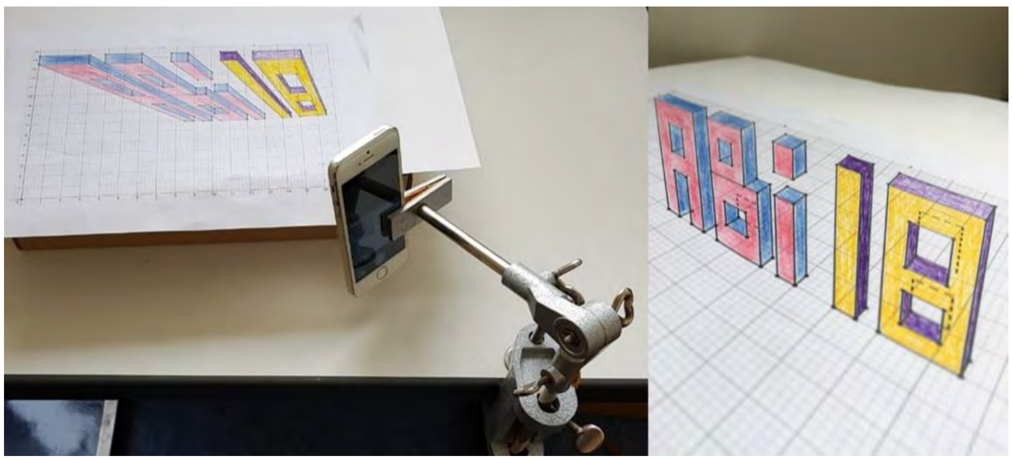

Figure 5: Left: Arrangement of Cam Carpet and tripod (ensuring a correct camera position) Right: 3D-impression of the Cam Carpet viewed from a fixed camera position.

CAM CARPETS IN HIGHER SECONDARY MATHEMATICS EDUCATION

Cam Carpets in higher secondary education provide the possibility for a modelling project worked out from scratch by students themselves with an impressive product result. The students decide on the logo to be developed within their project. Shortly before graduation (in Germany called Abitur), a lettering that can be used for the “Abitur”-newspaper is obvious and an additional motivation factor that makes the modelling project the students' own. When implementing the modelling project, the teacher assumes the role of a moderator, who keeps an eye on the individual project parts and their merging in terms of organization and time.

The aim of the modelling project for the higher secondary class was to create a large format Cam Carpet logo on the schoolyard (Figure 6). The 22 students of the mathematics course, who are about to graduate, opt for "Abi18". A total of four 90-minute courses and the homework time are available to develop and realise the Cam Carpet. A lot of preparation has to be done before the Carpet can be drawn on the schoolyard. A possible Cam Carpet position on the schoolyard, including a camera position, must be found in order to then determine the coordinates of the letter points projected on the xy-plane. Those coordinates depend on the camera position. Following those organisational aspects, the student groups started to create a 3D-model with GeoGebra3D.

The mathematics behind the Cam Carpets can be located in analytical geometry. Whether with track points, projection vectors or, usually at the level of the advanced course, with projection matrices – Cam Carpets allow performance-differentiated work. The following alternatives are described as examples for a point that is to be projected from a camera position onto the xy-plane (Figure 2).

Figure 6: Large-scale "Abi18" Cam Carpet on the playground (letter height ca. 7m).

Approach 1: Track points

The camera is in $\mathrm{K}(5|-5|6)$ and a point of the 3D model is $\mathrm{A}(0|0|4)$ (Figure 2). From this, a line $\mathrm{g_{AK}}$ can be determined that contains both points. $\overrightarrow{\mathrm{OA}}$ forms the support vector and the vector $\overrightarrow{\mathrm{AK}}$ represents the direction vector of the line $\mathrm{g_{AK}}$.

(Eq 1) $$\mathrm{g_{AK}}: \vec{x} = \overrightarrow{\mathrm{OA}} + r \cdot \overrightarrow{\mathrm{AK}} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + r \cdot \begin{pmatrix} 5 \\ -5 \\ 2 \end{pmatrix}, r \in \mathbb{R}$$

The projection point of A in the xy-plane is obtained by determining the track point S_{xy} of the line $\mathrm{g_{AK}}$. The track point has the z coordinate $z = 0$. The z-coordinate of the general line point is $z = 4 + 2r$. In the xy-plane, $0 = 4 + 2r$. This gives $r = -2$. The track point $\mathrm{S_{xy}}$ can now be obtained by inserting $r = -2$ into the line equation:

(Eq 2) $$\vec{x}_{xy} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + (-2) \cdot \begin{pmatrix} 5 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \\ 0 \end{pmatrix}$$

This leads to the track point $\mathrm{A_{xy}}(-10|10|0)$, which is also the projection point of the original point A. Analog calculations with all points of the 3D logo lead to the coordinates of the 3D model projected onto the xy-plane (cf. Figure 2).

Approach 2: Camera point dependent projection vector

Since many projection points have to be calculated, the pupils quickly discovered the general pattern behind approach 1 and developed an efficient method from it: After deriving a general projection vector that depends on the camera position, it can be used for the calculation of the projection points by inserting the points to be projected and the camera point. The implementation of this projection vector in a spreadsheet program, similar to the way implemented by a group of students with variant 3, enables an efficient determination of the projection points.

In general, a letter point $\mathrm{B(x|y|z)}$ with a camera at point $\mathrm{K(k_1|k_2|k_3)}$ in camera direction $\vec{v} = \begin{pmatrix} k_1 - x \\ k_2 - y \\ k_3 - z \end{pmatrix}$ on the xy-plane are projected, which creates the point $\mathrm{B_{xy}}$. The approach results from the general intersection problem (see approach 1)

(Eq 3) $$\vec{b} + r \cdot \vec{v} = \overrightarrow{b_{xy}}, r \in \mathbb{R}$$

and thus,

(Eq 4) $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} + r \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} x_{xy} \\ y_{xy} \\ 0 \end{pmatrix} = \overrightarrow{b_{xy}}$$

The third coordinate of $\overrightarrow{b_{xy}}$, solved for r, returns

(Eq 5) $$r = -\frac{z}{v_3}$$

Insertion of Eq 5 in Eq 4 yields

(Eq 6) $$\overrightarrow{b_{xy}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \left(-\frac{z}{v_3}\right) \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} -z \cdot \frac{v_1}{v_3} \\ -z \cdot \frac{v_2}{v_3} \\ -z \end{pmatrix} = \begin{pmatrix} x - z \cdot \frac{v_1}{v_3} \\ y - z \cdot \frac{v_2}{v_3} \\ 0 \end{pmatrix} = \begin{pmatrix} x - z \cdot \frac{k_1 - x}{k_3 - z} \\ y - z \cdot \frac{k_2 - y}{k_3 - z} \\ 0 \end{pmatrix}$$

The track point searched has the general coordinates $(x - z \cdot \frac{k_1 - x}{k_3 - z} | y - z \cdot \frac{k_2 - y}{k_3 - z} | 0)$, where $(x|y|z)$ is the coordinate of the point to be projected and $(k_1|k_2|k_3)$ represents the coordinates of the camera point.

If here the point $A(0|0|4)$ to be projected and the camera point $K(5|-5|6)$ from approach 1 are used, the projection point A_{xy} of A is obtained

(Eq 7) $$\overrightarrow{a_{xy}} = \begin{pmatrix} 0 - 4 \cdot \frac{5}{2} \\ 0 - 4 \cdot \frac{-5}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \\ 0 \end{pmatrix}$$

and thus, the same result as with approach 1 (Eq 2).

Approach 3: Camera dependent projection matrix

On a higher level, a camera point dependent projection matrix can be derived from Eq 6 for calculating the projection points. This projection matrix, equal to approach 2, can be used for the calculation of the track points (projection points). Using the matrix notation, Eq 6 is rewritten as follows

(Eq 8) $$\overrightarrow{b_{xy}} = \begin{pmatrix} x - z \cdot \frac{k_1 - x}{k_3 - z} \\ y - z \cdot \frac{k_2 - y}{k_3 - z} \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\frac{k_1 - x}{k_3 - z} \\ 0 & 1 & -\frac{k_2 - y}{k_3 - z} \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = P \cdot \vec{b}$$

where P is the projection matrix sought, which maps an arbitrary point B depending on the camera position on the xy-plane.

For the initially considered point $A(0|0|4)$ and the camera position in $K(5|-5|6)$, the following projection matrix results, which is dependent on the camera position

(Eq 9) $$P_A = \begin{pmatrix} 1 & 0 & -\frac{5}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

Inserting in Eq 8 results

(Eq 10) $$\overrightarrow{a_{xy}} = P_A \cdot \vec{a} = \begin{pmatrix} 1 & 0 & -\frac{5}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \\ 0 \end{pmatrix}$$

thus $A_{xy}(-10|10|0)$ which is the same track point as with the projection vector in Eq 7 and previously calculated with approach 1.

Since the projection vector and the projection matrix both depend on the camera point, a calculation must be made for each projection point using these approaches. In order to make the work easier, a group of students used a spreadsheet program in variant 3, in which the projection matrix was implemented to automatically calculate projection points.

Implementation in class

For a large-scale implementation of a Cam Carpet on the schoolyard four 90-minutes courses and a full school day was necessary. A large-scale implementation on the schoolyard requires preliminary organizational considerations, but rewards with an impressive result for the whole school community.

It is important to take into account the space available on the school premises. The larger the individual letters of the logo, the more impressive the result, but large letters also mean more drawing and material. The projection letters of the Cam Carpet in Figure 6 are $3m\ x\ 7m$ in size, with the camera positioned at a height of $5.6m$. The camera position from which the Cam Carpet can be viewed should be chosen so that it is accessible to a wide audience in order to give the whole school community access to the artwork.

Once all the letter coordinates have been determined, they can be realized on the schoolyard. Before the coordinates can be drawn in, a coordinate system must be placed on the schoolyard. The perpendicularity of the x- and y-axis of a coordinate system on a "non-checkered" surface that is sufficiently large for the Cam Carpet is a real challenge. After some consideration, the students decide to use a twelve-knotted cord (3-4-5 triangle) and apply their knowledge of the Pythagorean theorem (or its inversion) and Pythagorean triples from lower secondary mathematics: If you stretch a cord with the edge lengths

$3:4:5$, there is a right angle between the two shorter edges. This allows a right angle to be constructed with sufficient accuracy.

Another problem is the sufficiently precise drawing of the letter coordinates on the 18mx15m area. The students need a check pattern that on the one hand allow the coordinates to be drawn in with sufficient accuracy, but on the other hand is not visible in the later image so as not to disturb the 3D impression. The use of a chalk line (marking line) creates the desired and at the same time only slightly visible check pattern. Available in a conventional hardware store, a chalk line consists of a housing filled with colored chalk in which the line is wound up. Creating $1m\ x\ 1m$ checks is a sufficiently precise scaling.

A good contrast between the front and side surfaces of the letters is important for the 3D effect in order to be able to clearly differentiate between the respective surfaces. To estimate the amount of street chalk to be bought, the students used GeoGebra to have the area of the 2D projection displayed. A test with a common street painting circle cylinder shows that this is sufficient for an area of approximately $1.5\ m^2$.

After having finished the work, the 3D impression cannot be seen from the schoolyard. The surprise is great when looking at the Cam Carpet from the camera position. Many photos are taken with students sitting “in” the letters or “leaning on” them. But the most impressive is the jump video/photo from Figure 6.

SUMMARY AND OUTLOOK

The modelling project “Abi18” Cam Carpet allows not only performance-differentiated, but also inter-year and interdisciplinary project work due to the variety of solution and implementation options. The implementation in lower secondary physics education produces impressive results without mathematical penetration of the underlying principle. As part of analytical geometry, Cam Carpets offer a motivating application for the otherwise all too often schema-driven subject area and additionally reward with an impressive result for the whole school community if implemented in large format on the school yard.

Human vision and the effect of optical illusions allow an exciting expansion of the topic in the direction of biology, physiology and psychology. Further considerations regarding the quality of the 3D impression depending on the deviation from the calculated camera point as well as the effect of binocular vision as the basis of 3D vision give reasons for an interdisciplinary expansion of the subject.

References

Beames, S., & Brown, M. (2016). *Adventurous Learning: A pedagogy for a changing world.* New York: Routledge.

Borneleit, P., Danckwerts, R., Henn, H.-W., & Weigand, H.-G. (2001). Expertise zum Mathematikunterricht in der gymnasialen Oberstufe. *JMD 22*(1), pp. 73-90.

Filler, A. (2007). *Einbeziehung von Elementen der 3D-Computergrafik in den Mathematikunterricht der Sekundarstufe II im Stoffgebiet Analytische Geometrie.* habilitation thesis, Humboldt-University Berlin.

Moss, M. (2009). Outdoor Mathematical Experiences: Constructivism, Connections, and Health. In B. Clarke, B. Grevholm, & M. R, *Tasks in Primary Mathematics Teacher Education* (pp. 263-). New York: Springer.

THE PAST, PRESENT AND FUTURE OF SERIOUS GAMES AND GAMIFICATION IN STEM LEARNING

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Abstract. *I present here a short personal position paper on the role of serious games and gamification in school children STEM learning. I argue that the technology is ripe and there is a strong theoretical foundation on Game Design, but still there is a lack of good games or gamified activities that can show the potential of this novel learning tool. I have been designing or collaborating in the design of games for many years, both boardgames and videogames, both serious and non-serious, and have implemented gamified classroom approaches.*

Key words: Education, Learning, Serious Games, Gamification.

INTRODUCTION

Serious games have a long history. Mammals are biologically primed to use play as a learning and training tool. One has just to look to a dog or cat playing with an inanimate object like a ball or small rock to understand this. The potential of directing children's playfulness to serious purposes was recognized already in ancient Greece, as attests the following passage by Plato (D'Angour, 2013):

"*For example, if a boy is to be a good farmer or a good builder, he should play at building toy houses or at farming and be provided by his tutor with miniature tools modeled on real ones. ... One should see games as a means of directing children's tastes and inclinations to the role they will fulfill as adults*"

Clark Abt coined the term "serious game" with its current meaning in 1970 (Abt, 1970). A century earlier, the first highly visible serious games were simulations of land battles for the training of military officers. But while the Prussian Officer Corps were playing *Kriegspiel*, at the same time other young adults were learning how to do fireworks on their own and in the process being inspired by Chemistry.

STEM AND PLAY: LOOKING INTO THE PAST AND PRESENT

While the connection between play and learning was made since antiquity, one would have to wait until the industrial revolution for the society to give importance to STEM education, in the Darwinist nation on nation competition that characterized the later part of the nineteen century.

STEM Toys

Industrial production of STEM toys started with the interest in Chemistry and Mechanics in the 1800s (Nicholls, 2007). The first Chemistry sets for children appeared in the 1830s. In 1901 Frank Hornby patented a toy based on the principles of mechanical engineering called "Mechanics Made Easy" that was to become the *Meccano* line of construction toys. Whole generations of future engineers were inspired by playing with these toys. But to my

Santos, P. A. (2020). The Past, Present and Future of Serious Games and Gamification in STEM Learning. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 147-154). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.18

dismay, more than one century later when I tried to buy a Chemistry Set for my daughters, I could not.

My first memories of the power of STEM learning via play come from my childhood, when a small Electricity playset with lamps, switches and buzzer made me understand how electricity worked. As a pre-teenager a few years later, a Chemistry set (see Figure 1) made me understand the basics of chemistry. At ten years old, it was a revelation to me the table sea salt was just a simple chemical compound with the esoteric name sodium chloride that you could actually manufacture. I started to connect STEM knowledge with everyday life. Science could explain the whole world!

Figure 1: The Chemistry Set from my youth; you cannot find them in stores anymore...

Another couple of years later I understood the principle of a gearbox and how a driving wheel turns the front wheels of a car by assembling a complex LEGO set. Cogwheels were really interesting. If you connected a cogwheel with another one with double the cogs, one would rotate at double speed of the other. Building a chain of interlocking cogwheels, this process could be repeated. So I imagined I could build a bicycle that would run at incredible speeds. Trying to build such a mechanism using LEGO pieces, showed me first-hand the trade-offs involved.

What those sets gave me was much more then I could get from a book. By physically manipulating, working and experimenting with chemical compounds, electrical components and cogwheels I was getting insights that no other medium could give me at the time. And Mathematics was the language that could describe many of the things that I was experiencing[1].

[1] The math I was learning at school did not make that connection, though.

Unfortunately it seems that safety concerns and consumer regulation got the better out of children STEM education, by removing the most interesting experiments and components from chemistry and science sets (it might be argued that sets like the 1950's Gilbert U-238 Atomic Energy Laboratory were in fact a bit over-the-top)[2]. These types of toys also appeal to more self-motivated children, who are comfortable with setting their own goals.

While there was a decline in the attractiveness of traditional chemistry and electricity/electronics sets, a new generation of STEM toys appeared in the 1990's, mixing the mechanic parts of the build set toys like LEGO's Technic line and Meccano with programming. These are of course the Robotics sets such as the Mindstorms line from LEGO and the Meccanoid Robot-Building Kits from Meccano, which for twenty years now have brought many kids and teenagers to interact with very good STEM toys.

STEM Games

One moves from playing with toys to playing games by the introduction of arbitrary (i.e. not prescribed by the environment or physical properties of the artifact) rules and the acceptance of specific goals in an activity (Santos, 2008). Compared to STEM toys, both physical and video STEM games are a relatively recent phenomenon (with the exception of John Spinello's *Operation* published by Milton Bradley in 1965). In terms of physical (board) games, most STEM games on the market are of the puzzle genre, for instance marble or domino runs: *Gravitax* (2018) from Ravensburger, *Gravity Maze* (2014) and *Domino Maze* (2019) from Thinkfun.

Taking into account that one of the most famous boardgames in history – Monopoly – started its existence as a serious game[3], it is maybe disheartening to verify that very few STEM boardgames exist. A notable exception is John Coveyou's *Cytosis* (2017) published by Genius Games, where the players use a worker placement mechanic to simulate a human cell's metabolism.

STEM videogames

The potential application of videogames for serious purposes was perceived almost immediately (Wilkinson, 2016). But videogames were understood by the generation that grew up in the 1970s and 1980s, playing in the arcades and programming their own games. For their educators and political decisors, videogames were violent and addictive, a menace to society. It was necessary to wait for the 1990s for both a generational shift and the ubiquity of videogames to clear the way for the widespread acceptance of videogames as a learning tool.

Djaouti et al (2011) counted 2218 Serious Games launched until 2009. Of the 1265 games launched from 2002 to 2009, about a quarter were classified in the "Education" category. But how much good STEM games were produced? Not many, unfortunately. As several authors mention at the time most earlier educational games were *"Shavian reversals":*

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[2] I did continue devising my own experiments, for instance playing with fire or trying to build small solid propellant (home-made powder) rockets at age thirteen (would that be considered an outdoor STEM activity nowadays?).

[3] The Landlord's game, patented by Elizabeth J. Magie in 1904, see the interesting account in Adams, C.J. (1978)

offspring that inherit the worst characteristics of both parents (in this case, boring games and drill-and-kill learning) (Papert, 1988 and Van Eck, 2006).

The situation seems to have improved somewhat in the last decade (Chen, 2016). Some Serious Game designers, while still not having the resources lavished on AAA entertainment titles, started to develop games applying the concept of "Stealth Learning", where learning is happening implicitly while the player is having fun with the game. Also, they use game design techniques and ideas common with entertainment games. For instance, in the NASA-published *Moon Base Alpha* (2010)[4], players play as astronauts and are put in a typical videogame situation: a meteor strike damages the outpost, gamers are tasked with repairing vital systems using a variety of tools, from the lunar rover to remote-controlled robotic units. But the game uses actual NASA Constellation program design details developed by NASA for mankind's return to the Moon, with immersive 3D graphics.

The mobile game *Math Evolve* (2011) from InterAction Education is also an interesting example of good educational game design. It teaches the basic math operations, but contrarily to the (in)famous *Math Blaster* Series (1983-2013), where the math content interrupts the normal shooter gameplay of the game, in Math Evolve the math content is tightly integrated with the gameplay.

A third example of this new generation of games is *Kerbal Space Program* (2015)[5] from Squad. The basic gameplay is assembling rockets from parts and attempting to explore a solar system. The point is that the laws of physics are realistic (at least equivalent to the models actually used by scientists and engineers in the 1960s). The player will fail a lot, but failures are fun and make the player learn and try other approach. The game includes intelligently designed interfaces that make it easier to understand what will happen with the player decisions and space maneuvers (that must be performed manually). It is surprising how even 8 years old children can learn very complex aeronautics and space exploration concepts like thrust to weight ratios or orbital mechanics with this game. And they do learn all of these while having a lot of fun!

Lastly, I will mention a project I was personally involved, the game Treme-Treme (2015)[6]. It is a game designed to teach 9 to 12-year-old children what to do in case there is an earthquake. The game had three chapters (before, during and after the earthquake, each with its own mechanics) For the design of the gameplay during the earthquake, the idea was for the child to know where it was safe and not safe to stay, and what to do in the moments immediately after an earthquake (if possible switch of electricity and other utilities, pick the emergency kit and leave the house). I took inspiration from two entertainment games, not usually associated with that age bracket: *Dark Souls* (2011) and *Limbo* (2010). From *Dark Souls* I took the concept that you learn by failing and dying, and quickly trying again. From *Limbo* I took the idea that if you are going to die a lot, then one should make it interesting.

[4] https://sservi.nasa.gov/articles/moonbase-alpha/

[5] https://www.kerbalspaceprogram.com/

[6] https://www.treme-treme.pt/

Figure 2: Gameplay screenshot from the game Treme-Treme (2015).

The gameplay in that part of the game is very simple (see Figure 2): The player has side view of the interior of the house and moves by clicking where he wants to go. We do not say where the safe places are. He starts in a random division and has 10 seconds to choose a place and get there. After those 10 seconds, if he is not on a safe place (e.g. under a table or bed), something will happen (a lamp falls on his head, the balcony falls, and so on) and he dies. A small angel gets out of his body and flies up, while a short text explains why that place is dangerous. Then that level starts again. We have observed that the children try several times different places and, in the end, learn **by experience** the places where it is safe to be. And they have fun!

STEM learning with Gamification

The most common accepted definition of Gamification is that gamification is the act of using game design elements in non-game activities to make them more enjoyable (Deterding et al, 2011). While gamification is often equated with the use of badges and rankings or the enveloping of activities within a narrative, it is possible to re-design activities and the learning process to incorporate game design principles and gameplay mechanics, thus attaining a deeper level of gamification (Santos, 2015).

Gamification works, at least in my experience. If implemented properly, it changes student attitudes, increases motivation and creates an environment where failure is seen as a step towards success. Unfortunately, there is still a lack of academic studies to formally analyze its effects in controlled tests. In the MoMaTrE[7] project we have implemented and tested a gamified version of the system versus a non-gamified version, with positive results (Gurjanow et al, 2019). We have continued to develop gamified aspects of the application,

[7] http://momatre.eu/

like introducing the possibility of adding a fictional narrative to envelop the student activities. Unfortunately, the Covid 19 situation let to the cancellation or postponement of studies that were planned to test its effects.

In fact, in what regards the effects of STEM games and Gamification, there is still a dearth of information and studies. While some studies show in fact that there is no effect in the use of interactive material compared with alternative methods (workbooks) in student performance (e.g. Reinhold, 2019), other studies show statistically significant improvements with the use of STEM games in schools (e.g. Freina et al, 2018).

More studies to shed light on this and related subjects are sorely needed. A large-scale study involving the MoMaTrE project results and the MathCityMap App is planned to start in the next school year (Barlovits and Ludwig, 2020). Hopefully, it will bring interesting results.

STEM AND GAMES: A BRIGHT FUTURE?

As we have seen by the examples above, in the last decade there has been a trend where new STEM serious games were developed considering modern game design principles, where the learning component is tightly integrated with the gameplay. So, in a certain way, for the player, he is playing the game just for fun and learning occurs naturally. Of course, designing serious games using this design philosophy is harder than designing pure entertainment games or the more traditional Edutaiment type games. The designer must design the gameplay to be both fun and to teach the player the knowledge that was the purpose of the game, thus provoking stealth learning.

It does not help that funding for the development of STEM games is usually very restricted. Entertainment games publishers avoid serious games, because they think it is harder to have success, and so the development of Serious Games is usually done by small companies with little resources, doing many times work-for-hire. But while there are companies that have interest in financing advergames , who is interested in financing STEM games? I think that here state education systems and other stakeholders should have an active role in this process.

Technological evolution is also presenting new opportunities and risks. New XR – technologies, that is, virtual reality and augmented reality technologies, are becoming common and open a new frontier for STEM applications. What is too dangerous or expensive for children and teenagers to do in real life (manipulating dangerous chemicals, working with lab equipment) can now be simulated in immersive environments. But not only that, VR opens a whole new world where we can experience things and make activities which are impossible in “real life”, all in a safe environment for experimentation. There are already experimental games and interactive experiences that allow one to grab atoms and play with them (Rodrigues and Prada, 2018). One can also replicate what I experienced as a teenager manipulating LEGOs, but now on a real-life scale: it is possible to show how any machine works inside, while letting the student manipulate and observe everything from different angles. It is possible to virtually disassemble any machine, to be inside a working engine, or the human body.

The imagination is the limit. I can only advise all educators to try XR technology to understand its potential for education in general, and STEM education in particular. Any description or video does not make it justice, one needs to experience it.

CONCLUSIONS

STEM games are fundamental if we want the current generation of computational native children to learn and have interest in STEM, together with developing problem solving and critical thinking skills. Of all the new technologies that appeared in the last years, I think XR technology is the most promising to bring a revolution in STEM serious games.

I will finish with a few pieces of advice for the design of new STEM games that enhance student learning:

- Design for Fun, and integrate learning in the core gameplay loops;
- Heed the teachings of the wise (game design) men;
- Consider both shallow and deep gamification techniques;
- Harness the potential of new technology.

I believe that if one follows these simple four lines of advice, together with some financial support for this important aspect of education, there will be indeed a bright future for STEM Game-based learning.

References

Abt, C.C. (1987). *Serious Games*. University Press of America

Adams, C.J. (1978). MONOPOLY: From Berks to Boardwalk. Historical Review of Berks County, Vol XLIV, Number 1 Winter, 1978-1979

Barlovits, S. & Ludwig, M. (2020). Mobile-supported Outdoor Learning in Math Class: Draft of an Efficacy Study about the MathCityMap App. Proceedings of Roseta Conference 2020 (in press).

Chen, S. (2016). Facing Edutainment's Dark Legacy, http://www.gamesandlearning.org/2016/01/25/facing-edutainments-dark-legacy/.

D'Angour, A. (2013). Plato and Play: Taking Education Seriously in Ancient Greece. Am. J. Play. 5, 293–307.

Deterding, S., Dixon, D., Khaled, R., & Nacke, L. (2011). From game design elements to gamefulness: defining "gamification". Proceedings of the MindTrek '11. doi:10.1145/2181037.2181040

Djaouti, D., Alvarez, J., Jessel, J., Rampnoux, O. (2011). Origins of Serious Games. Serious Games Edutainment Appl. 25–43.

Freina, L., & Bottino, R. (2018). Visuospatial Abilities Training with Digital Games in a Primary School. International Journal of Serious Games, 5(3), 23 - 35.

Gurjanow, I., Olivera, M., Zender, J., Santos, P., & Ludwig, M. (2019). Mathematics Trails: Shallow and Deep Gamification. International Journal of Serious Games, pp. 65-79.

Nicholls, H. (2007). The chemistry set generation. Royal Society of Chemistry. Retrieved 28 March 2020, https://www.chemistryworld.com/features/the-chemistry-set-generation/3004820.article

Papert, S. (1988). Does easy do it? Children, games, and learning. Game Dev. Mag.

Reinhold, F. (2019). Wirksamkeit von Tablet-PCs bei der Entwicklung des Bruchzahlbegriffs aus mathematikdidaktischer und psychologischer Perspektive [Effectiveness of tablets in the development of the fractional number concept]. Wiesbaden: Springer Spektrum.

Rodrigues, I., Prada, R. (2018). Virtual Reality Game to teach Organic Chemistry, In Proceedings of Videojogos 2018.

Santos, P.A. (2008) Games - Refining the model. In Proceedings of Digital Games 2008, 6-7 Nov 2008, Porto, Portugal

Santos, P.A. (2015) Deep Gamification of a University Course, in Proceedings of SciTecIN'15 Sciences and Technologies of Interaction, Coimbra, Nov. 2015

Van Eck, R. (2006). Digital Game-Based Learning: It's Not Just the Digital Natives Who Are Restless. Educ. Rev. 41, 16–30.

Wilkinson, P. (2016). A Brief History of Serious Games. In *Entertain Comput Serious Games LNCS 9970,* 17–41.

MODELING QUADRATIC FUNCTIONS IN THE SCHOOLYARD

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Abstract. *Teaching in the schoolyard and using digital technologies is one of a lot of components of the structure of mathematics lessons. One example of mathematical analysis lesson, which has been tested in class, will be presented in this paper: a basketball throw in correlation with quadratic functions. 15-year old pupils experience haptic mathematics, the functional graph arises using transparencies and pins, the functional equation can be developed and tasks can be solved in the fields of sports' application. Pupils use their own smartphones (bring your own mobile device) to this lesson: They film their activities and collect and evaluate the measured data of their video. Furthermore, this paper describes different ways of execution of this lesson and one possibility of interdisciplinary teaching approach in physics. The pupils compare the trajectory as a ballistic curve with a parabola of the quadratic function and they calculate the initial velocity.*

Key words: schoolyard, quadratic function, basketball throw, initial velocity

THE BASKETBALL THROW – ONE EXAMPLE OF MODELING MATHEMATIC

To create a math lesson which is creative, varied, exciting, close to reality, based on competences, differentiated and motivating for pupils is very difficult. A lot of pieces of the puzzle have to interlock. One of these puzzle pieces is learning in the schoolyard. Outdoor education may promote learning by improving pupils' attention, interest and enjoyment on acquisition of knowledge, and physical activity (Kuo & Jordan, 2019). If you have experienced contents, you don't have to learn them (Kramer, 2016).

Another puzzle piece is mathematics learning using mobile phones. The influence of the mobile phones on pupils' motivation is acknowledged in the literation (for example the study "Students' perceptions of Mathematics learning using mobile phones" of Baya'a & Daher). The benefits are learning mathematics in authentic real-life situations, visualizing mathematics and investigating it dynamically, performing diversified mathematical actions using new and advanced technologies and learning mathematics easily and efficiently (Baya'a & Daher, 2009).

In this described lesson, the mathematical modeling competence can be increased, a process of formulating real world situation in mathematical terms (Gablonsky & Lang, 2005). Mathematical modeling is a valuable and meaningful activity and it is important to introduce to mathematics learners in school (Ang, 2019). The pupils can understand that mathematics is useful and the basis for many professions, see the interdisciplinary nature of mathematics and understand what mathematical modeling means (Borromeo Ferri, 2018).

This lesson can be done if the pupils learned to graph quadratic equations in vertex form and to determine equation parameters. The materials for each group of three are one basketball, one smartphone, one pin, a part of transparency and one transparency pen.

Schubert, M. & Ludwig, M. (2020). Modeling Quadratic Functions in the Schoolyard. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 155-162). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.19

Stating the question

Does the trajectory of a basketball correspond to a parabola?

Figure 1: Analysis of a basketball throw as a parabolic trajectory.

This question was explored with a class of 15-year old pupils in Germany. The following elements were included in this lesson:

The first part of the lesson takes place in the schoolyard. Two pupils (first pupil and second pupil, see Figure 2) throw the ball to each other and a third pupil films this with his or her own phone. The pupil who films the action (third pupil, see Figure 2) should stand at the same distance from the other group members in order to film an undistorted trajectory. This pupil should also stand at a certain distance from the thrower and the catcher so that the central perspective approximately conforms to the parallel-perspective.

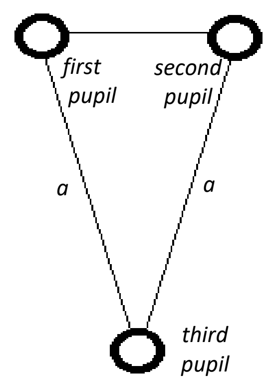

Figure 2: Position of the three pupils.

It should be noted that the ball is tossed from left to right if the footage is used for further researches. The pupils ought to throw a big ball (for example a basketball or a medicine ball), so that the trajectory can be seen clearly in the video afterwards. The ball should also be heavy so that the air friction has a low impact on the movement. It is possible that the pupils bring their own balls to school.

Curve fitting with quadratic function

After the basketball throws in the schoolyard, the class returns to the classroom and the pupils meet in their groups. It is important to discuss the perfect position of the coordinate system. Pupils can express their ideas and explain them mathematically. Effective communication is critical for more precise instruction and deeper mathematic learning and creating classroom environments in which pupils practice multiple forms of communication is imperative (Sammons, 2018).

For teaching more mathematical or physical issues, the teacher and the pupils agree on the origin of coordinates, which conforms the feet of the thrower. The body size (head and feet) is marked on the transparency to illustrate the scale.

Now the question arises how to draw the trajectory parabola as a graph of a function onto paper. The pupils take a transparency and place it on the screen of their phone. The learners draw the flight curve onto the transparency by playing the video in slow motion. Multiple loop and repeatedly stopping are another possibility. The pupils mark the points with little circles because the coordinates are approximate values.

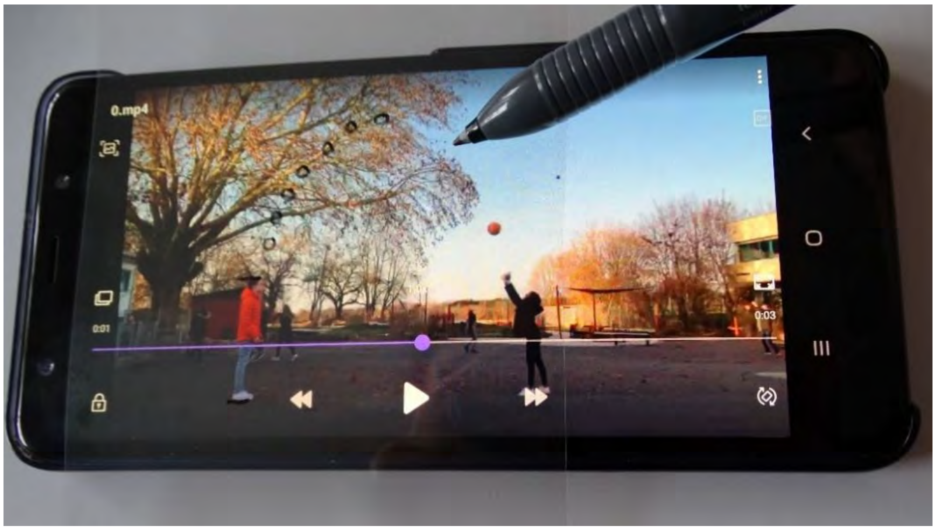

Figure 3: The points are marked on a transparency put onto the screen.

In a next step, the circles and the body size are transferred to millimeter paper, either with pins or by projection.

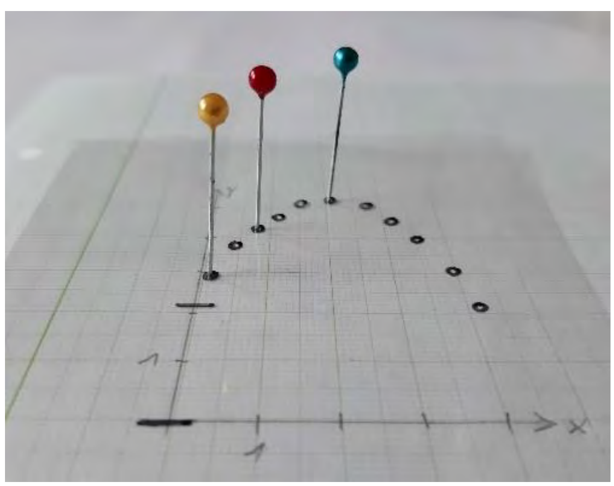

Figure 4: Pins are pierced through the transparency.

Alternatively, the pupils work with a document camera. They put their transparency onto the document camera, hold their mathematics exercise books against the wall, draw the projection of the trajectory in their exercise books and enlarge so their drawing to a corresponding size.

The pupils estimate the coordinates of the point of throw-out and the highest point as vertex. They substitute the points into the vertex form and reconstruct the quadratic function. Now, the learners calculate points of the function and draw the parabola in the same coordinate system.

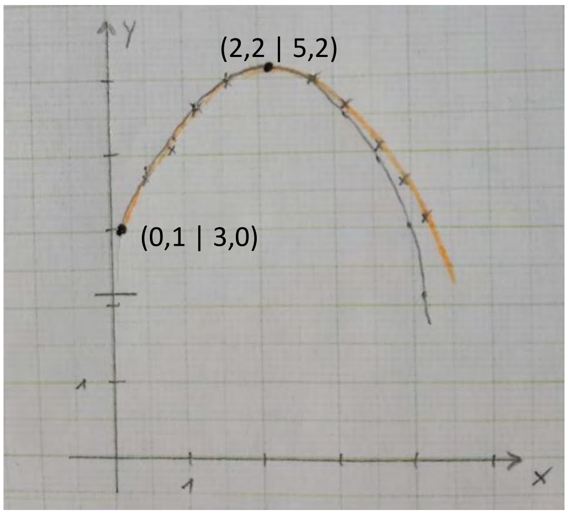

Figure 5: Trajectory (black) and parabola (orange)

Coordinate axes in unit of length.

A pupil's solution; see Figure 5:

$$f(x) = a \cdot (x - d)^2 + e$$

$$3{,}0 = a \cdot (0{,}1 - d)^2 + e$$

$$3{,}0 = a \cdot (0{,}1 - 2{,}2)^2 + 5{,}2$$

$$f(x) = -\frac{220}{441}(x - 2{,}2)^2 + 5{,}2$$

Interpret solution

The pupils detect the deviations from these two graphs. If the ball soars, the trajectory (the ballistic curve) is almost identical in the parabola (reconstructed with only two points). If the ball drops, the trajectory is below the parabola and the difference is greater, the longer the ball flies. The deviations may be pronounced in environmental influences, such as the friction between the air and the ball.

The learners can calculate the maximum height of the ball. The thrower tells his body size (as marked onto the transparency) and the proportion is determined between the body size and one unit of length of the squared paper or millimeter paper. Then they use the value of the y-coordinate of the vertex and, together with the proportion, calculate the maximum height.

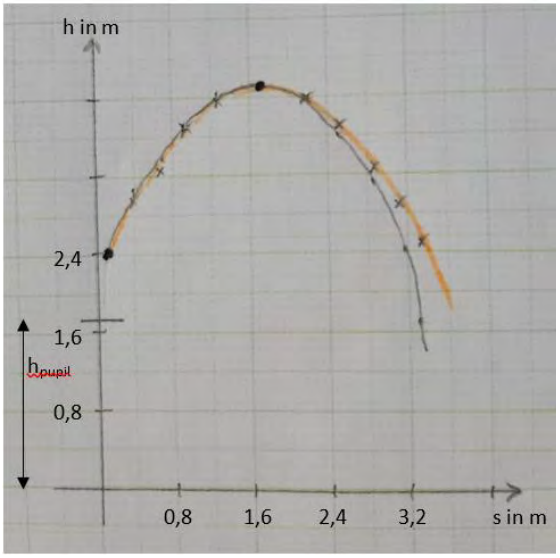

Figure 6: Drawing of the calculation of the maximum height of the ball (h_{pupil} is the body size of the thrower).

A pupil's solution; see Figure 6:

The thrower said that his body size is 1,76 m.

$h_{pupil} = 1{,}76\ m \triangleq 2{,}2\ unit\ of\ length$

$one\ unit\ of\ length \triangleq 0{,}8\ m$

$vertex's\ y - coordinate:$

$5{,}2\ unit\ of\ length \triangleq 4{,}16\ m$

The maximum height of the ball is approximately 4,2 m.

Another possibility to investigation the proportion is the distance between the thrower and the catcher. Both pupils mark their location with chalk in the schoolyard, measure the length and calculate the proportion between the distance and one unit of length of the squared paper or millimeter paper.

Another calculation is the study of the initial velocity v_0. The time of the throw is noticeable in the film. The pupils draw a tangent at the point of throw-out.

A pupil's solution; see Figure 7*:*

$s_x = 3{,}2\ m\ \ (determined\ by\ the\ drawing)$

$t = 0{,}922\ s\ (determined\ by\ the\ video)$

The velocity in x-direction v_x is constant:

$$v_x = \frac{s_x}{t} = \frac{3{,}2m}{0.922s} = 3{,}47\frac{m}{s}$$

Investigation v_0: The pupils draw with the tangent a rectangular triangle and measure the lengths of the arrows for v_0 and v_x.

$$\frac{v_0}{v_x} = \frac{5{,}5\ cm}{2{,}5\ cm} \qquad v_0 = \frac{5{,}5\ cm}{2{,}5\ cm} \cdot v_x$$

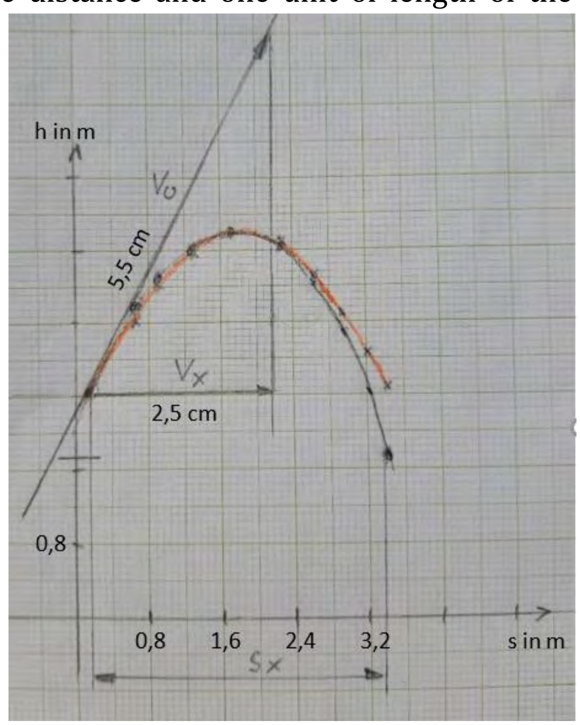

Figure 7: Drawing of the calculation initial velocity.

$$v_0 = \frac{5{,}5\ cm}{2{,}5\ cm} \cdot 3{,}47\frac{m}{s} = 7{,}63\frac{m}{s} \quad The\ initial\ velocity\ of\ the\ ball\ is\ approximately\ 7{,}63\frac{m}{s} \approx\ 27{,}5\ \frac{km}{h}$$

Second possibility to calculation the study of the initial velocity v_0: The pupils measure the angle between the arrows for v_0 and v_x. This part requires knowledge of trigonometric knowledge.

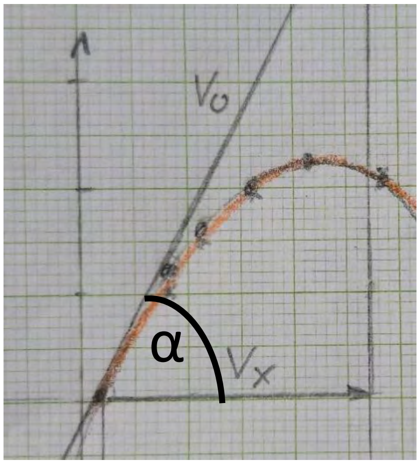

Figure 8: A part of Figure 7.

A pupil's solution; see Figure 8:

$$\alpha = 63°$$

$$\cos(\alpha) = \frac{v_x}{v_0}$$

$$v_0 = \frac{3{,}47\frac{m}{s}}{\cos(63°)}$$

$$= 7{,}64\frac{m}{s}$$

Third possibility for older pupils in use the first derivative:

A pupil's solution:

$$f(x) = -\frac{220}{441}(x-2{,}2)^2 + 5{,}2 \qquad f'(x) = -\frac{440}{441}(x-2{,}2) \qquad \tan(\beta) = f'(x_0)$$

See; Figure 5: $x_0 = 0{,}1$ $f'(0{,}1) \approx 2{,}095$ $\beta \approx 64{,}48°$ $v_0 = \dfrac{3{,}47\frac{m}{s}}{\cos(64{,}48°)} = 8{,}05\dfrac{m}{s}$

The quality of the results can be checked with video players for sports analysis. The program determines a position of the ball every one or two milliseconds.

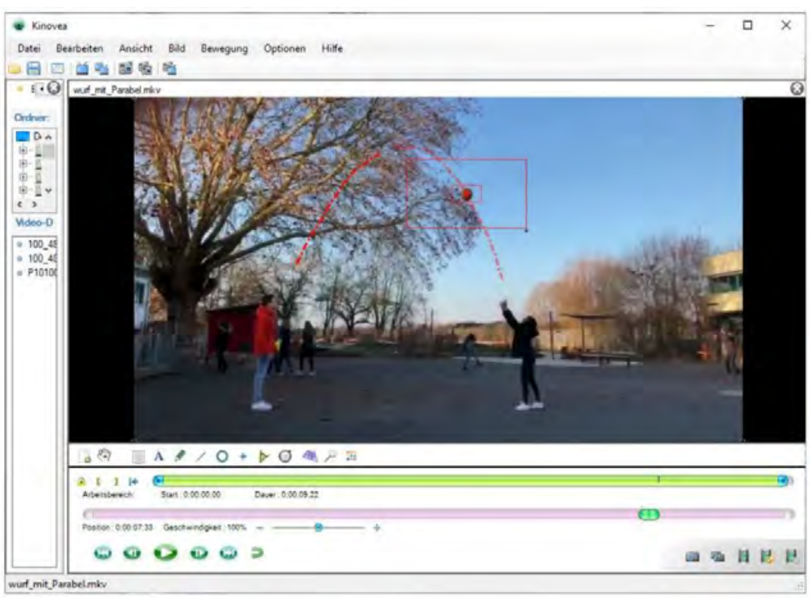

Figure 9: An example of a video player.

This data can be transferred to a spreadsheet program.

	A	B	C	D	E	F	G	H	I	J
1	video evaluation			convertion		approximate function		trajectory		
2	Track					f(x)=-220/441*(x-2,2)²+5,2		g(x)=tan β * x -g/2 * x² /(v² * cos² β) + h		
3	Label :	Label						β	v	h
4	Coords (x,y:px; t:time)							64	7,63	2,785
5	x	y	t	x	y	x	y	x	y	
6	0	0	0:00:00:00	0	2,785	0	2,785487528	0	2,785	
7	4	9	0:00:00:11	0,0589474	2,9175305	0,058947368	2,913144214	0,0589474	2,9043365	
8	8	18	0:00:00:22	0,1178947	3,050061	0,117894737	3,037333987	0,1178947	3,0206261	
9	12	26	0:00:00:33	0,1768421	3,1678659	0,176842105	3,158056846	0,1768421	3,1338688	
10	17	35	0:00:00:44	0,2505263	3,3003963	0,250526316	3,304085075	0,2505263	3,2711373	
11	23	44	0:00:00:55	0,3389474	3,4329268	0,338947368	3,472168441	0,3389474	3,4295753	
12	27	51	0:00:00:66	0,3978947	3,5360061	0,397894737	3,579890378	0,3978947	3,5313919	
13	32	59	0:00:00:77	0,4715789	3,653811	0,471578947	3,709667452	0,4715789	3,6543778	
14	38	67	0:00:00:88	0,56	3,7716159	0,56	3,858249433	0,56	3,7956767	
15	42	75	0:00:00:99	0,6189474	3,8894207	0,618947368	3,952970446	0,6189474	3,8860672	
16	45	82	0:00:01:11	0,6631579	3,9925	0,663157895	4,021736044	0,6631579	3,9518606	
17	49	87	0:00:01:22	0,7221053	4,066128	0,722105263	4,11038996	0,7221053	4,036919	
18	54	94	0:00:01:33	0,7957895	4,1692073	0,795789474	4,216332008	0,7957895	4,1389572	
19	58	100	0:00:01:44	0,8547368	4,257561	0,854736842	4,297185369	0,8547368	4,21716	
20	62	106	0:00:01:55	0,9136842	4,3459146	0,913684211	4,374571818	0,9136842	4,2923158	
21	64	110	0:00:01:66	0,9431579	4,4048171	0,943157895	4,41196495	0,9431579	4,3287511	
22	67	114	0:00:01:77	0,9873684	4,4637195	0,987368421	4,466429532	0,9873684	4,3819758	
23	70	118	0:00:01:88	1,0315789	4,522622	1,031578947	4,518943976	1,0315789	4,4334866	
24	72	120	0:00:01:99	1,0610526	4,5520732	1,061052632	4,552870196	1,0610526	4,4668749	
25	76	125	0:00:02:10	1,12	4,6257012	1,12	4,618122449	1,12	4,5313664	
26	81	130	0:00:02:22	1,1936842	4,6993293	1,193684211	4,69481242	1,1936842	4,607696	
27	85	134	0:00:02:33	1,2526316	4,7582317	1,252631579	4,752264119	1,2526316	4,6653319	
28	90	137	0:00:02:44	1,3263158	4,8024085	1,326315789	4,819203397	1,3263158	4,733092	
29	96	140	0:00:02:55	1,4147368	4,8465854	1,414736842	4,892380023	1,4147368	4,8081197	
30	98	143	0:00:02:66	1,4442105	4,8907622	1,444210526	4,915038775	1,4442105	4,8316055	
31	102	145	0:00:02:77	1,5031579	4,9202134	1,503157895	4,957756094	1,5031579	4,8762919	
32	107	148	0:00:02:88	1,5768421	4,9643902	1,576842105	5,006277398	1,5768421	4,927865	

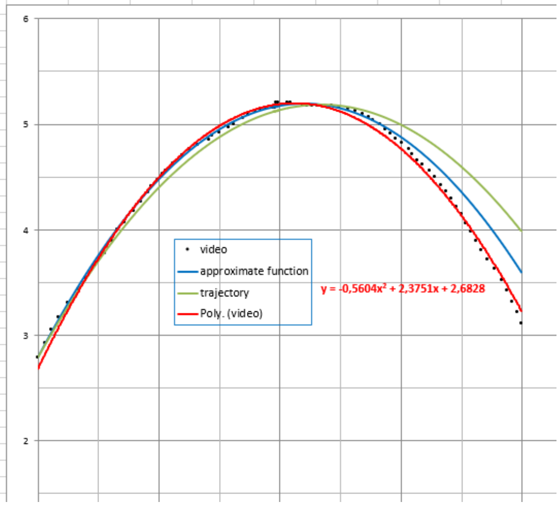

Figure 10: Comparison of video data and calculated data.

Video data are in columns A, B and C. The x and y values of the video data are transformed using the maximum y value and the x value of the time of catching in columns D and E (see Figure 10 black points). The approximation of the values can be shown with a trend line. (see Figure 10 red curve). The x and y values of the approximate function f (see Figure 5) are in the columns F and G (see Figure 10 blue curve).

The explicit equation of the trajectory in the space area considering the height h (h = f(0)) is $g(x) = -\frac{g}{2} \cdot \frac{x^2}{v_0^2 \cdot cos^2\beta} + x \cdot tan\beta + h$. The x and y values of g are in columns H and I (Figure 10 green curve).

Results

The lesson works well if the pupils divide themselves into groups of three. All pupils enjoyed the work with their own smartphone and most pupils achieved good flight curves and good quadratic function approximations.

One group used a handball because a girl from this group plays handball in her free time and she brought her own ball to school. The handball was difficult to see in the video.

In another group, the trajectory was flat and this group had problems to reconstruct the quadratic function. It is important for the teacher to announce that the pass must be thrown up high into the air.

The evaluation with the computer shows that the different types of calculation, approximation of measured values every one or two milliseconds, reconstruct the quadratic function in use the point of throw-out plus the vertex and equation of the trajectory in use the angle and the initial velocity, lead to a similarly good result.

References

Ang, K. C. (2019). *Mathematical Modelling for Teachers: Resources, Pedagogy and Practice.* London: Routledge, Taylor & Francis.

Baya'a, N. & Daher, W. (2009). Students' perceptions of Mathematics learning using mobile phones. *In Proceedings of the International Conference on Mobile and Computer Aided Learning,* 4, 1–9.

Borromeo Ferri, R. (2018). *Learning How To Teach Mathematical Modeling – in School and Teacher Education*. New York: Springer.

Gablonsky, J., & Lang, A. (2005). Modeling Basketball Free Throws. *Society Industrial and Applied Mathematics, 47*(4), 775-798.

Kramer, M. (2016). *Mathematik als Abenteuer. Band III: Analysis und Wahrscheinlichkeits-rechnung. Erleben wird zur Grundlage des Unterrichtens*. Seelze: FriedrichVerlag GmbH Klett Kallmeyer.

Kuo, M., & Jordan, C. (2019). Editorial: The Natural World as a Resource for Learning and Development: From Schoolyards to Wilderness. *Frontiers in Psychology and Frontiers in Education, 10,* 4-5.

Sammons, L. (2018). *Teaching Students to Communicate Mathematically.* Alexandria, Virginia USA: ASCD.

All images used in this study are the author's own.

TEACHING PRIVACY OUTDOORS – FIRST APPROACHES IN THE FIELD IN CONNECTION WITH STEM EDUCATION

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Abstract. *This paper presents an approach to teach privacy in secondary schools. The research field is highly topical and there is a lack of educational concepts. In this approach the students analyze pictures taken with smartphones. They are taught about hidden information stored in the pictures and how to use them to find the owner of the pictures. Smartphone pictures as learning content are related to their daily life, what makes it relevant and interesting for the students. Two versions of the approach were tested in the last years in classrooms and are presented in this paper. The evaluation shows that the students were very engaged and gained privacy awareness. A third version of this approach takes this activity outdoors and connects it with STEM subjects.*

Key words: privacy, teaching, computer science education, STEM education, meta data, problem-based learning

INTRODUCTION

In the digital age the smartphones are our constant companions, what makes the life at first glance easier and smarter. However, topics like digital addiction and the effects on social life are being researched. It was found that the smartphone usage, especially WhatsApp, dominates our daily life (Montag, Błaszkiewicz, Sariyska et al. 2015). This applies equally for youngsters and adults. People are sending tones of data without reflecting what kind of data is sent. That shows on the one hand, that privacy awareness is not high enough. On the other hand, most people do not know what information is hidden in their data, for instance in pictures. In the K-12 Computer Science Framework "privacy and security" is described as a crosscutting concept (K-12 Computer Science Framework Steering Committee 2016). Thus, privacy is an important area in the digital age that needs to be brought into school. The barriers are high, because the topic is closely related to law and it seems to be tough to teach it in an interesting way. "I have nothing to hide" and other rejections are additional hurdles when topics such as data minimization are brought up.

In this paper an educational approach for computer science lessons in secondary school is presented. The analysis of meta data out of smartphone pictures is clearly settled in the area of data protection and privacy in computer science. Two approaches were realized until now. A third version connects the approach with partial outdoor activities and interdisciplinary entry points.

First, related literature regarding teaching data protection and problem-based learning is presented. Afterwards, the educational approach is described in more detail, including an overview of the evaluation of the first and second version and data out of the studies. In the third version arguments are given on why it is planned to take this approach outdoors. Then, all three versions of the approach will be compared. Finally, the implementation in school and the empirical evaluation of the three versions are discussed and an outlook is given.

Schulz, S. (2020). Teaching Privacy Outdoors – First Approaches in the Field in Connection with STEM Education. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 163-170). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.20

RELATED LITERATURE

Since 2018 the General Data Protection Regulation (GDPR) is a legal requirement for all European citizen. It addresses companies on how to handle specific personal data. However, it effects students as well. On the one hand, everyday personal data is uploaded by students in social networks and the students are responsible for personal data of themselves and others, for instance when they are part of an uploaded picture. On the other hand, the students need to know which data can be collected by companies and how to protect themselves, for instance through the right of restriction of processing.

Privacy education is highly topical and claimed in education standards. For instance, the K-12 Computer Science Standards (2016) embeds privacy at the end of grade 2, going up to grade 12. For grade 12 it is explained, what should be understood by the students in the area of data collection:

> "Data can be collected and aggregated across millions of people, even when they are not actively engaging with or physically near the data collection devices. This automated and nonevident collection can raise privacy concerns, such as social media sites mining an account even when the user is not online […]" (p. 117).

These contents are highly related to privacy teaching. The declaration as a crosscutting concept also shows the relevance. After the GDPR became effective in 2018, the relevance of privacy teaching has been increasing. However, there are not many concepts on how to teach privacy in school. "If you are not paying for it, you are the product" is one approach presented by Berendt and Dettmar (2018). It is a lesson series, in which the effects of internet tracking and data analysis are uncovered. The tracking of data is visualized and the students learn about biased data by using an example case and how algorithms work in the background.

"Email for you (only?)" is another approach, focussing on how to send e-mails securely. The encryption of e-mails and the importance of encrypted communication are part of the lesson series (Gramm, Hornung & Witten 2012).

Most approaches lack an empirical evaluation or are not relevant for students. The context of encrypted e-mails might work much better for adults than for students, which rarely send e-mails. Information about the underlying methods are often missing. For this approach the problem-based learning seems to be the best fit. In that, the students get a real-world problem and need to figure out how to solve it appropriately. This is quite common in computer science education, for instance to foster critical thinking and problem-solving skills (Kay et al. 2000). For an interdisciplinary approach it seems to be appropriate, because there exist a variety of working techniques to improve problem-solving skills in STEM education as well. Problem-solving, particularly scientific inquiry, is addressed since decades in science curricula (American Association for the Advancement of Science et al. 1993).

EVOLUTION OF AN EDUCATIONAL APPROACH

The kind of data, that is used in this approach is called EXIF data. It is an acronym for "Exchangeable Image File". This data is stored in every digital picture (hidden) and can contain information about camera type, operating system, smartphone type, date and time of the image, GPS information and much more. The GPS information is stored, depending on the settings of the smartphone, for instance if the GPS location is activated. With only a couple of pictures a profile of the owner can be constructed. Thus, the GPS coordinates contain the most important meta data.

In this paper a couple of pictures is called a “scenario”. Having approximately 5 pictures, the included EXIF data can be enough to find the owner of the pictures. If that is not enough, more data can be collected from other sources. For instance, it is possible to search on the internet to find companies or sports clubs around the found GPS coordinates. Afterwards, employees or a training plan on the related websites can be detected. If there are no GPS coordinates included, it is also helpful to have a closer look at the picture to find out about the place it was taken and perhaps an occurred event, using the internet. Street signs, tourist attractions and registration plates can be helpful to figure out a location.

Figure 1 shows a screenshot of the program GIMP 2.10 (GNU Image Manipulator Program), which is a freely-available software to analyze and edit pictures. On the picture (left) a Tic Tac Toe game is shown. The content of the picture is visible for whomever it is sent to. On the right side the EXIF data is shown, which is accessible with programs like GIMP. Into this case, the GPS coordinates, which are pointing to a secondary school in Berlin, can be found. It seems plausible, that the author of the picture is a teacher or a student from this school. Playing games like Tic Tac Toe is more likely done by students than by teachers. The timestamp of the picture is included as well. Thus, it is possible to draw conclusions about when this student is in school and about the timetable (for example a free hour).

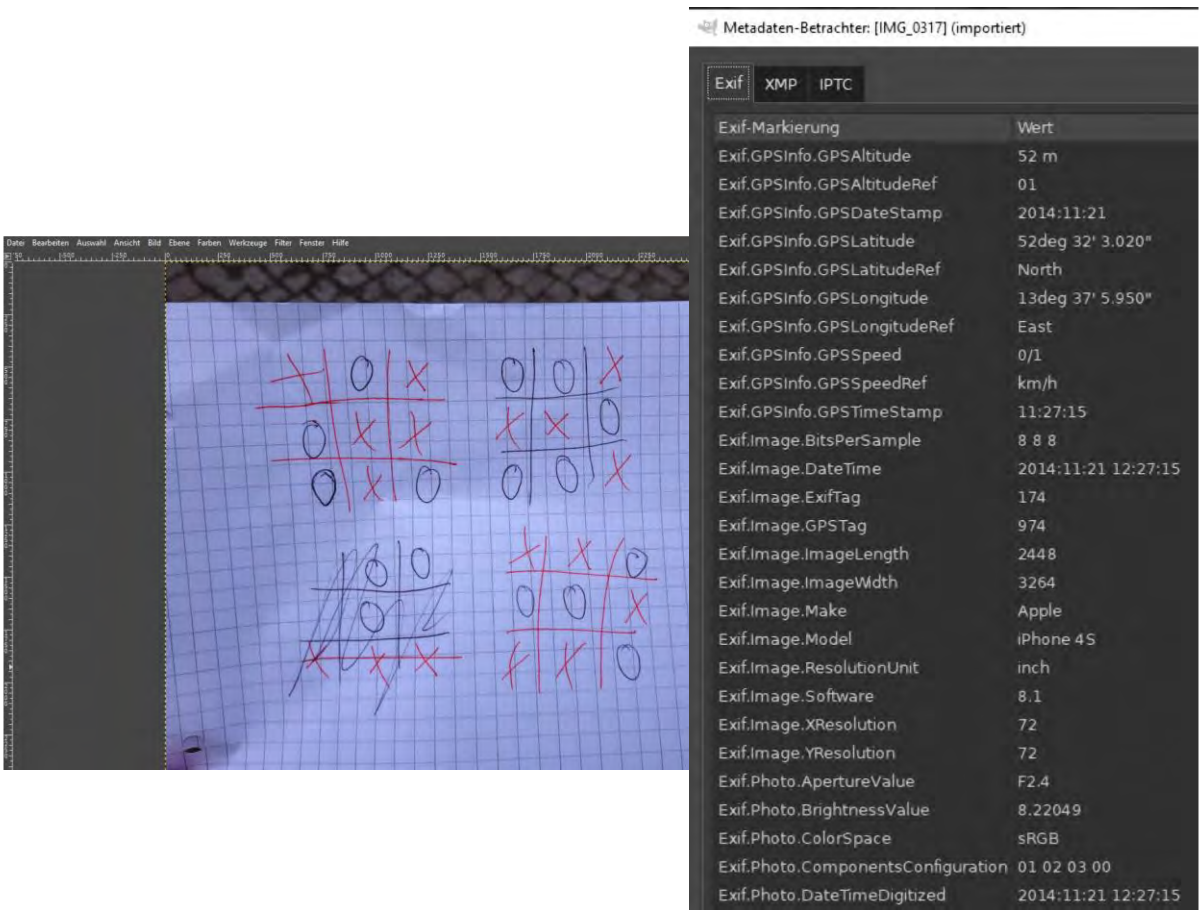

Figure 1: Picture of Tic Tac Toe game (left) and its EXIF data (right) in GIMP software.

First Version (tested in 2014)

In a project to enhance the awareness of data protection and privacy a study with students in secondary school was conducted in 2014 (Schulz and Strickroth 2019). It was tested in a computer science course in grade 11 with 15-17 aged students. In total 9 students participated in this course and the study. The whole project consists of 6 lessons, 45 minutes each, held in 3 blocks. The students got information about privacy laws in Germany and newspaper articles showing risks and chances of the analysis of meta data. The analysis of the meta data was the main part of the project. The students received information on how to insert GPS coordinates into Google Maps. This is necessary, because there exists different metrics about GPS coordinates, which are similar, but will lead to different locations (for instance the sexagesimal system or decimal system). Understanding the differences is important for the success in further lessons. Then, they got some pictures from the teacher to practice the analysis of EXIF data. The students received 5 pictures (generating a scenario), which were taken by the same person and had to build a profile of that person. The students needed to act like detectives and figure out who took the pictures. To analyze the pictures' EXIF data, the students needed the program GIMP, which is a standard freely-available software to view pictures. In the menu bar the students had to choose "picture" and afterwards "meta data" to uncover the EXIF data.

For the evaluation of the project, students' data was collected using questionnaires (pre- and post-test) and the teacher was interviewed. Both showed that the students gained privacy awareness and became more sensitive towards their personal data. The part of meta data analysis was described as "impressive for the students" by the teacher. In the questionnaires the students said that they were surprised by the hidden data and that they would also tell their friends about it. In general, it was an eye-opener, even though the project was not using the students' personal data. This could have been even more impressive.

Second Version (tested in 2019)

In 2019 a revised approach was realized in a 10th grade computer science course consisting of 16 students, aged between 15 and 16. Before this data protection topic was introduced, the students were asked to send 5 pictures each (without people on them) to the teacher. They were given instructions, for instance they had to pick pictures everybody is allowed to see, but not the purpose of the task. The students were introduced to privacy law and meta data. They received the same scenario of 5 pictures to analyze, like the group in 2014. In addition, the students were introduced in the general functionality of geolocation and its different notations. The detailed information about it turned out to be valuable for the successful analysis of EXIF data. As next exercise, the students were given pictures from other students in the course to analyze. They worked in groups of 4 to 5 students and analyzed the pictures of one other student in the course. All groups figured out the owners of the pictures by connecting data and their knowledge about the classmates. Some gave arguments like: "I know Lara was in the zoo on the 20th of November. We had a field trip with the biology course we are sharing. The picture must be from her […]" (having a picture with an animal in the zoo). Other students were using Google Street View to find out about the surroundings of a specific place on the picture to get more information.

After every lesson three random volunteers were asked with the flashlight method to name positive and negative aspects of the lesson. Directly after the data analysis lessons all students mentioned the problem-based approach as a positive aspect. They said: “It was really fun to act like a detective and to find out from whom the pictures are” and “I like to analyze the meta data of the pictures and to search for further data to connect them.” The problem-based approach seems to be very motivating for students following the students’ answers and the observations of the teacher. Particularly the data analysis from the students’ social surroundings raises the awareness and credibility of privacy and uncovers potential problems.

Third Version (planned)

This phase is not yet tested but will combine the actions of version 1 and 2 with outdoor learning and interdisciplinary content. Therefore, the students’ tasks will be the same as in the second version. But instead of the analysis of data from the classmates, the students will have to construct own scenarios for their classmates, before the different scenarios will be distributed to all students. It is necessary for the construction of scenarios that the pictures are gathered outside and for the students to travel through the city to make it more interesting, with different places included. The scenario is more interesting when different kinds of data are provided or need to be found. It is also possible to deactivate GPS during the process of taking pictures, when the students are supposed to look at the picture in more detail. The students are allowed to manipulate EXIF data to create contradictions as long as they give clear hints of the manipulation. The EXIF data of a pictures might for instance show that it was taken by night, even though it shows bright daylight. Consequently, the complexity of the scenario may be adjusted to fit different levels of difficulty. Students that solve their tasks quicker than others might be given additional tasks as developing scaffolds for their scenarios. The main task for the analysis is always to find the author of the pictures (if possible) and to gather a lot of information about that person.

Bringing the third version outdoors has the following advantages for the students and the complexity of the project:

- **Variety of GPS coordinates**: If data is just gathered inside, for instance the school or the students’ homes, the complexity of the scenarios is reduced and the variety of GPS coordinates is more limited. If the students want to point at something known by everyone, classrooms would be their only option.
- **Protection of students’ data**: In this project it is necessary to protect the students’ data and the students should feel secure. Information like their place of residence should not be figured out by other students if the owner of the pictures does not want them to.
- **Authentic data**: Taking pictures from the internet and change the EXIF data could be an alternative, but it is hard to find pictures on the internet, in which EXIF data is still included. Furthermore, during the analysis with software like GIMP, the changes of the EXIF data would be traceable. To increase the authenticity of data and of the fact, that many information is hidden, it seems to be logic to provide “real” data of their daily life and not do manipulate meta data of the pictures.

- **Differentiation in complexity**: Making pictures from tourist attractions or street signs make it easier to find a location, even if the GPS coordinates are not provided.

EDUCATIONAL PERSPECTIVES FOR COMPUTER SCIENCE AND STEM

All three versions contain the EXIF data analysis as the main part (see table 1). The first version is the shortest and can be integrated in usual school lessons in the classroom. The second version encompasses more lessons, because the students are analyzing two scenarios and are more engaged in understanding the function of GPS location. This leads to deeper learning because the students get more practice in the analysis of data and they get a scenario out of the real-world. Having pictures from the classmates can show, that the teacher's scenario is from the real-world and their pictures contain the same data. Finally, the third version needs at least 12 lessons, because the students are going outdoors to construct their own scenario. Therefore, it is necessary to get guidelines from the teacher on how to construct interesting scenarios with an adequate level of difficulty. The construction of own scenarios should be prepared in the classroom to plan which places to visit and what data to collect. Gathering the data will need 2–4 lessons, depending on restrictions by the teacher, for instance how far to travel via train. Afterwards the students need to process their scenarios in classroom. At that stage, interdisciplinary problems may be indicated (see below). Depending on the implementation of this version in school, it can be valuable to give the interdisciplinary tasks before the outdoor activities.

	First Version	Second Version	Third Version
Number of lessons (45 minutes)	6	10 – 12	> 12
Further subjects to connect with	-	Geography	Geography, Mathematics, Physics
Environment	Classroom	Classroom	Classroom, outdoor
Students' activities	Analysis of foreign scenarios (teacher)	Analysis of foreign scenarios (teacher and students)	Constructing an own scenario; analysis of foreign scenarios (teacher)

Table 1: Comparison of the three versions.

Especially the third version of this approach seems to be perfectly suitable for problem-based STEM learning. GPS navigation is a topic addressing multiple school subjects and is connectable to real-world problems. For instance:

How is GPS navigation working? It is necessary to explain the functionality and purpose of satellites and the geocentric coordinate system (Geography). By using GPS it is also possible to measure the speed of an object. This is applicable because of the Doppler-effect and differential calculation (Physics and Mathematics).

What factors make GPS navigation imprecise? Students might find differences between their actual location and the location that is shown in an app like Google

Maps. This can happen when they are surrounded by high buildings, because of reflection (Physics) or different weather conditions (Geography). General discussions on how signals can be sent and disturbed could be connected, too.

What is the difference between GPS location and Wi-Fi location? This question adds an ethical perspective to this topic. Is it ethically correct to use private Wi-Fi data to locate foreign devices? When the Wi-Fi location is used to improve an outdoor game, is the information stored by the manufacturers of the game? What are they doing with this information?

These questions are just a few examples of a multitude of real-world problems. Constructing meaningful and interdisciplinary problems for students can improve their motivation. Furthermore, the subject of EXIF data can be the hook to address other disciplines in computer science as ethics, for instance, which is oftentimes neglected. Nowadays many data analyzing algorithms can be implemented, but the question needs to be addressed, if the implementation is ethically correct. Using EXIF data is one example, that needs to be discussed in the field of ethics. On the one hand, EXIF data is an effective way to organize data and to provide more information about data. On the other hand, EXIF data cause damage, if the user is not aware of what personal information is being revealed. Questions to discuss in class can be: Is it ethically right to use EXIF data information without the knowledge of the owner? Can I upload pictures of others on social media platforms without asking them? What about strangers being in the picture, when taking pictures outdoors? Building the connections to different disciplines seems to be challenging for many teachers. Teachers should not insist in tackling the project by themselves. Privacy officers and teachers from related subjects can easily get involved to make this project even more interesting and diverse.

OUTLOOK

Bringing these approaches into school seems to be valuable. The first and second version showed, that the students were engaged with privacy and gave very positive feedback concerning the project. For an interdisciplinary implementation it is suitable to use project weeks to bring the related subjects together and to have enough time to address problem-based questions. Working together with STEM teachers seems to be appropriate to accurately explain interdisciplinary phenomena and to make visible, that problems are not limited by the boundaries of single subjects. After the implementation in STEM, other connections should be made, for instance to Art or History. This can be achieved by focusing on specific themes for the pictures, like street art or statues. This can improve media competences in different subjects, which are fundamental today.

For an empirical evaluation a pre- and post-test of privacy awareness might be a good choice. Additionally, an interview with the teacher should be conducted to validate the students' answers. A qualitative analysis of a portfolio is also an appropriate way to implement a phase of reflection for the students and to find out about the students' view, if they are planning to change their data handling or to delete the EXIF data before sending pictures. It is necessary to repeat questionnaires and interviews after a couple of weeks to validate, if the project has brought a sustainable change to the students' privacy awareness.

References

American Association for the Advancement of Science et al. Benchmarks for science literacy. Oxford University Press, 1993.

Berendt, B., & Dettmar, G. (2018). If you're not paying for it you are the product: A lesson series on data, profiles, and democracy. In Paderborn Symposium on Data Science Education 2017: The Collected Extended Abstracts. (pp. 55-58). University of Paderborn.

Gramm, A., Hornung, M., & Witten, H. (2012). Email for you (only?) design and implementation of a context-based learning process on internetworking and cryptography. In Proceedings of the 7th Workshop in Primary and Secondary Computing Education (pp. 116-124).

Intersoft consulting services AG (2018). General Data Protection Regulation (GDPR) https://gdpr-info.eu (last checked 04.03.2020).

Kay, J., Barg, M., Fekete, A., Greening, T., Hollands, O., Kingston, J. H., & Crawford, K. (2000). Problem-based learning for foundation computer science courses. Computer Science Education, 10(2), 109-128.

K-12 Computer Science Framework Steering Committee. (2016). K-12 Computer Science Framework (2016). Retrieved August, 28, 2017. https://k12cs.org/wp-content/uploads/2016/09/K–12-Computer-Science-Framework.pdf

Montag, C., Błaszkiewicz, K., Sariyska, R., Lachmann, B., Andone, I., Trendafilov, B., Eibes, M. & Markowetz, A. (2015). Smartphone usage in the 21st century: who is active on WhatsApp? *BMC Res Notes* 8, 331. https://doi.org/10.1186/s13104-015-1280-z

Schulz, S. & Strickroth, S. (2019). Versteckte Meta-Daten und Datenschutz - Die Förderung des Datenschutzbewusstseins durch das praxisorientierte Auslesen von Meta-Daten. *LOGIN Verlag*, 191/192, 67-73.

SOME REMARKS ON 'GOOD' TASKS IN MATHEMATICAL OUTDOOR ACTIVITIES

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Abstract. *Starting with a brief theoretical discussion of academic tasks, I shall point out two desiderata of task design in mathematical outdoor activities and subsequently propose a four-point scheme for assessing such tasks. I shall conclude with* three illustrative examples.

Key words: task design, outdoor mathematics, real world contexts

SETTING THE SCENE

Mathematical knowledge in an educational context is often organised along sets of tasks[1]. According to Lenné (1969, p. 50-54) this concept has its origin in the institutionalisation of schooling which has become part of the bureaucratic apparatus of modern societies. What is more, this very idea can be traced back to the training of bureaucratic elites in ancient societies. Yet – as I will argue – it is still reverberating in today's mathematics classrooms as well as in current research on mathematics education. Thus, it might be worthwhile to recall Lenné's description of 'task didactics':

> Each subdomain is determined by a specific type of task, which is to be dealt with systematically proceeding from simple to sophisticated patterns. Sophisticated tasks can be construed as combinations of simple tasks. Intrinsically, individual domains, therefore, appear to be rigorously systemised. Without being linked to each other, though, they are dealt with in a relatively isolated manner. 'Application tasks' are *separately* allocated to each domain. To make sure that one domain provides the required pre-conditions for the next, only the sequence of domains is determined. Domains which have been covered are considered as done, the pertinent subject-matter is taken for granted; interconnections along comprehensive ideas or structures are hardly worked out clearly – at least not systematically. In any case it applies: 'we have dealt with it' or 'we have not yet dealt with it'. Therefore, students do not meet mathematics as a whole intrinsically ideal entity, but as a mere repository of different types of tasks. I shall call this concept of how to organise subject-matter in traditional mathematics 'task didactics'. (Lenné 1969, p. 34-35)

It goes without saying that, from an educational point of view, this attitude towards mathematics leaves us wanting. Fortunately, however, there is no lack of alternative concepts, the *locus classicus* concerning tasks in the context of primary and especially secondary education being Doyle (1983).[2]

Doyle distinguishes four general categories of tasks: (i) memory tasks, (ii) procedural or routine tasks, (iii) comprehension or understanding tasks, and (iv) opinion tasks. This scheme has been widely accepted, and the differentiation of procedural and understanding tasks in particular has been stimulating research in education ever since. Its educational implications, however, are not yet fully realised. In theory, schools are thought to aim at comprehension and understanding, i.e. higher-level cognitive processes, in order to foster cognitive (and linguistic) development. In practice, though, schools all too often fail to achieve this objective. According to Doyle this could be explained by students' efforts to avoid both ambiguity and risk. In other words, in a school setting of constant monitoring and continuous accountability, students are not only striving for the security of tasks with

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low ambiguity and risk, but are actively avoiding or 'mis-performing' the didactically desirable, but highly ambiguous and risky understanding tasks (Doyle & Carter 1984, p. 144-147).[3]

MATHEMATICAL OUTDOOR ACTIVITIES

As a consequence, a central target of mathematics education is to design didactically desirable tasks,[4] withal empowering students to overcome the related structural hindrances of performance. In this case, an outdoor-setting[5] may provide the necessary and favourable environment to meet these challenges, especially when digital devices can be employed in addition. My arguments will be organised along the theoretical lines of Borba et al. (2013).

Collaboration

Working in small groups without direct monitoring by the teacher in an out-of-school environment can prompt self-determined and self-responsible action – but this requires a careful didactical framing. There has been extensive research on this topic, its very essence being summarised in three general, but powerful rules: Outdoor activities have to be (i) integrated into math classes with regard to content, (ii) prepared thoroughly, and (iii) individually revised afterwards (cf. Klaes, 2008).

Multimodality

To encounter real, i.e. physical, objects in an out-of-school environment and to regard them in the perspective of school-learning necessitates the active construction of interconnections between different domains of knowledge with the aim of fostering higher-level cognitive processes. This is especially true if the task on hand requires a thorough examination of the object. With access to the internet, information in the form of text and pictures can be easily retrieved to complement the outdoor learning environment.

Performance

Working on tasks in an out-of-school environment may contribute to deepen the very notion of mathematical knowledge. Mathematics would then be "no longer just the property of teachers and textbooks, nor [… be] constrained by the communication forms of traditional textbooks." (Borba et al. 2013, p. 692) Instead, it can be 'performed' outside the formal classroom setting, encouraging thereby an awareness of self-efficacy which may help in counteracting the well-known marginalising force of mathematical knowledge (cf. Sriraman et al., 2010).

REAL WORLD TASKS

As pointed out above, the didactically desirable aspects of mathematical tasks are intimately linked with real-world settings. To encounter real world objects and perceive them in a mathematically informed way, i.e. (mathematical) modelling, however, seems to be fraught with several difficulties. Two of these seem to me of particular importance.

First, mathematical modelling entails the competent handling of data, i.e. statistical literacy.[6] According to Gal (2002, p. 4) this encompasses knowledge elements (literacy skills, statistical knowledge, mathematical knowledge, context knowledge and critical questions) as well as dispositional elements (beliefs & attitudes and a critical stance).

Notably, focussing too much on mathematical (and at best statistical)[7] knowledge, the crucial role of context knowledge in accomplishing outdoor tasks is all too often underrated. A case in point of this *déformation professionnelle* is the bad habit of supplying the (allegedly) required context knowledge within the task itself – in striking contrast to the didactical considerations above.

Second, the very aim of mathematical modelling is insight and control, or in the famous words of Hertz:

> The most direct, and in a sense the most important, problem which our conscious knowledge of nature should enable us to solve is the anticipation of future events, so that we may arrange our present affairs in accordance with such anticipation. […] In endeavouring thus to draw inferences as to the future from the past, we always adopt the following process. We form for ourselves images or symbols of external objects; and the form which we give them is such that the necessary consequents of the images in thought are always the images of the necessary consequents in nature of the things pictured. […] When from our accumulated previous experience we have once succeeded in deducing images of the desired nature, we can then in a short time develop by means of them, as by means of models, the consequences which in the external world only arise in a comparatively long time, or as the result of our own interposition. We are thus enabled to be in advance of the facts, and to decide as to present affairs in accordance with the insight so obtained. (Hertz, 1899, p. 1)

Therefore, an outdoor task should allow for substantial insights into the object of concern to afford an adequate notion of mathematical modelling.[8] Expressed somewhat pointedly: Pay attention to the real world!

'GOOD' TASKS IN MATHEMATICAL OUTDOOR ACTIVITIES

As recapitulation of my above line of argumentation, I shall suggest a four-point scheme for assessing out-of-school tasks:

A 'good' task in mathematical outdoor activities has to meet four conditions: (i) students can gain substantial knowledge about real-world objects; (ii) students resort to substantial (school) mathematical knowledge; (iii) mathematics contributes to the understanding of the real-world context; (iv) a thorough exploration of the object of concern forms the core of activities.

To illustrate the above criteria, I shall discuss a generic example from the research literature and suggest two examples from my own teaching project.

Example 1: Art Gallery

I shall begin with a task by Buchholtz (2017), re-reading it as a generic illustration of a blind spot widespread among mathematics educators, acknowledging that it has been devised in a diagnostic setting, albeit – in the words of Buchholtz – 'based on meaningful reality-based tasks'.

The example, chosen from a mathematical city walk, is the problem called 'Art Gallery', which deals with the ramps (and stairs) leading up to the forecourt of *Kunsthalle Hamburg (Hamburg Art Gallery)*. It reads as follows:

(i) The platform [i.e. the forecourt] can be accessed via four long ramps, which are divided by the stairs into a left and a right section. Compare the two sections of the ramp with each other. What do you notice?

(ii) At the stairs one can measure what height one section of the ramp overcomes [sic]. Record the measured height in the worksheet.

(iii) Also wheelchair users would like to reach the platform. Wheelchair ramps may however, for safety reasons, only have a slope of no more than 6%. Find out how long a section of the ramp is. How can you determine the slope in percent (%)? Can wheelchair users navigate safely on the ramp?

In his book chapter, Buchholtz discusses the task mainly in regard to its diagnostic potential concerning the calculation of percentage and of slope. The real-world context in the form of wheelchair users is mentioned only once: “If the sizes were determined correctly, the result is a slope of about 6.5% […]. With regards [sic] to the problem for wheelchairs, it can be stated that driving onto the ramps would not be safe […], although […] certainly still possible.” (cf. Buchholtz, 2017, p. 53 f.)

With regard to the real-world context, this passage falls way too short. To begin with, there is no discussion of the accuracy of measurement (except for measuring a false quantity). Furthermore, there is neither a discussion of the accuracy required in the construction of ramps (which coincidentally amounts to 0.5%) nor a discussion of further specifications of wheelchair ramps. A short glance in the relevant (German) regulation, DIN 18040-1, reveals numerous shortcomings: The mandatory rest platform for every 6 m of length is missing (the ramp at *Kunsthalle Hamburg* runs for over 11 m without a rest platform) just as the prescribed hand and wheel rails. Going beyond national regulations and comparing them, for example, with the corresponding regulations in the USA, the *Americans with Disabilities Act (ADA)*, one finds a maximum permitted slope of 1 : 12 (one foot ramp for one inch of rise), thus exceeding 6% by far.

To sum up: the reference to wheelchairs in the third subtask seems dispensable in terms of diagnostics, while in terms of mathematising real-world contexts, it is a missed opportunity of learning something substantial about wheelchair ramps. Of course, it may be granted that the task is primarily about slope and not about wheelchair ramps – but this is exactly what I mean by saying that ‘task didactics’ is still reverberating in current research.

Example 2: Humboldt Penguins

The following task originates from my current teaching project *Out-of-School Learning in Frankfurt*. For two years now, every semester I have chosen an out-of-school place in Frankfurt, visiting it weekly with ten students to find situations containing mathematics and trying to design ‘good’ math tasks. In *Frankfurt Zoo*, a student proposed the following problem for pupils of age 10-12 (due to place constraints, I shall shorten the text):

(i) How many Humboldt penguins are living in *Frankfurt Zoo*? First, count on your own, then compare your results among each other. Did you all get the same result? (ii) Count the penguins methodically, ensuring no penguin is overlooked or counted twice. Describe how you went about the task. (iii) How many penguins are male, how many female? (Hint: Take a look at the name badges.) (iv) Look at the penguins more closely. Can you differentiate males and females without looking at the name badges? If so, how? (v) Observe one penguin of your choice for ten minutes while s/he is diving. How often does s/he submerge and for how long does s/he stay under water each time? Record your data in a table. (vi) Present your data in the form of a box plot.

In preparation for the excursion to the zoo, the pupils are asked to catch up on Humboldt penguins on the internet, and afterwards are assigned to find out about the biological characteristics allowing penguins – a species of a bird (!) – to dive.

This task allows pupils to gain knowledge about Humboldt penguins, by the way a vulnerable species (*inter alia* due to climate change): Their sexual dimorphism, for example, is very subtle, and only very thorough examination by experts allows for differentiating between males and females. Information (suitable for various age classes) about their adaption to aquatic life is readily and abundantly found online. Mathematically, not only the intricacies of counting and measuring are addressed, but also the value of repeated and systematic observations to gain insight in the diving behaviour of Humboldt penguins beyond a trite 'can stay under water up to … minutes' – even with the prospects of discussing behaviour in captivity.

Example 3: Macrotermes spp.

The following problem originates from the same project, this time visiting *Senckenberg Museum*, a museum of natural history in Frankfurt. The (shortened) task of my own design reads as follows:

Figure 1: A showcase at *Senckenberg Museum* displaying a termite mound of *macrotermes spp.* on a scale of 1 : 1 (picture taken by the author).

(i) Find the model of the termite mound of *macrotermes spp.* [cf. Fig. 1] and read the information panel (*spp.* standing for: varying species of the genus *macrotermes*). (ii) Higher termites live in highly organized colonies often exceeding over a million individuals. Usually there are three castes: the reproductive caste (as a rule one queen and one king), the soldier caste and the worker caste. Inform yourself about eusociality. (iii) How many times larger is the queen in comparison to a worker? Refer to the models of *macrotermes spp.* in the display next to the mount [cf. Fig. 2]. (iv) If proportions in the world of humans and termites would be the same, what would be the size of a human 'queen'? (v) Determine the height of the termite mound as accurately as possible. Again, if proportions in the world of humans and termites would be the same, how high would be a human 'mound'? Compare your result with the height of the Frankfurt Trade Fair Tower.

Further subtasks cover topics such as biological systematics, hemimetabolism and physiogastrism.

As an addendum, there is an excerpt from Smeathman (1781), a letter written to the *Royal Society of London* concerning observations on termites, containing a footnote outlining a line of thought similar to that in subtask (v):

> The labourers are not quite a quarter of an inch in length; however, for the sake of avoiding fractions, and of comparing them and their buildings with those of mankind more easily, I estimate their length or height so much, and the human standard of length or height, also to avoid fractions, at six feet, which is likewise above the height of men. If then one labourer is = to one-fourth of an inch = to six feet, four labourers are = to one inch in height = 24 feet, which multiplied by 12 inches, gives the comparative height of a foot of their building = 288 feet of the building of men, which multiplied by 10 feet, the supposed average height of one of their nests is = 2880 of our feet, which is 240 feet more than half a mile, or near five times the height of the great pyramid; and, as it is proportionably wide at the base, a great many times its solid contents. If to this comparison we join that of the time in which the different buildings are erected, and consider the Termites as raising theirs in the course of three or four years, the immensity of their works sets the boasted magnitude of the ancient wonders of the world in a most diminutive point of view, and gives a specimen of industry and enterprize as much beyond the pride and ambition of men as St. Paul's Cathedral exceeds an Indian hut. (Smeathman, 1781, p. 148 f.)

This task tries to stimulate an 'in-depth' exploration of (higher) termites along the exhibits of *Senckenberg Museum*. As social insects, termites provide a wealth of features that have fascinated mankind for centuries, notably their highly developed construction activity (parental care would be another feasible topic). The measuring and reckoning approach taken – mathematically a mere exercise in elementary proportions – was perceived favourably by my students as a way of coming to terms with the wondrous 'industry and enterprize' Smeathman so aptly describes.

Figure 2: A showcase at *Senckenberg Museum* displaying ideal-typical individuals of different castes of *macrotermes spp.* on a scale of 10:1 (picture taken by the author).

EPILOGUE

In both of my own examples, the real-world context comes first. In fact, at the beginning several students of mine felt the mathematics involved would not be sufficient. While it is true that counting, measuring, and proportions are quite elementary, it is equally true that they form a substantial part of the core of mathematical reasoning – a point neglected on occasion. Many of the reservations, it seems to me, are voiced mainly due to unfamiliarity. My students, as a case in point, came forward with beautiful ideas rather quickly, and I can see no reason, why teachers with all their experience and expertise should not be able to do the same, provided they dare to transgress the line of 'proper school mathematics' without the sense of guilt passed on by tradition and fueled – as I have tried to argue – by the tradition of 'task didactics'.

Admittedly, teachers looking for a set of 'application tasks' (to take up Lenné's term) in a specific domain of school mathematics would perhaps not find anything worthwhile in the kind of examples I propose. I would assert, nevertheless, that there are already too many examples of 'application tasks' of high quality to add but another one; and I would add for consideration, that there is a price to pay for trading school mathematical concerns above value: a quite twisted image of what (school) mathematics is all about.

If you are bent on doing (school) mathematics, the classroom seems the place to be. But when you are out for a genuine engagement in outdoor activities, let your maxim be: real world first, mathematics second.[9]

Notes

1 In specialised contexts such as problem-solving, it is common practice to differentiate between tasks and problems; in the following, I shall use these words synonymously.

2 For the adaption of Doyle's work to mathematics education cf. Stein et al. (1996).

3 In mathematics education, instead of 'understanding tasks' a variety of terms are in use: 'cognitively demanding tasks', 'rich tasks', 'powerful tasks' or 'substantial tasks', to name but a few.

4 Task design has never been at the core of research in mathematics education. Nevertheless, the last decade has provided some relevant literature, notably Sullivan et al. (2013) and Watson et al. (2015). For the German discourse, the pertinent reference is Leuders (2015).

5 In the following, I shall extend the notion of outdoor activities to include any out-of-school activity.

6 Astonishingly, this obvious fact is not worthy of note in nearly all educational concepts of mathematical modelling, a pertinent example being the well-known modelling circle of Blum et al. (Blum & Borromeo Ferri 2009, p. 46).

7 To name but the most salient aspect of statistical knowledge: Quantitative results are only meaningful as far as one has an idea of its bounds of error. In stark contrast to this commonplace, there is virtually never a trace of a rough guess about the accuracy of the results obtained in mathematical modelling tasks in educational settings.

8 Stillman et al. (2013, p. 2) suggest four dimensions of 'authentic' modelling: content authenticity, process authenticity, situation authenticity, and product authenticity. Arguably, not any kind of modelling is obliged to authenticity (cf. Kaiser & Sriraman 2006), especially in an educational context; but the respective arguments are not easily transferred to an out-of-school context.

9 It is my pleasure to cordially thank Gerhard Bierwirth and Lutz Führer for their invaluable advice once again.

References

Blum, W., & Borromeo Ferri, R. (2009). Mathematical Modelling. *Journal of Mathematical Modelling and Application*, 1(1), 45-58.

Borba, M., Clarkson, P., & Gadanidis, G. (2013). Learning With the Use of the Internet. In M. A. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third International Handbook of Mathematics Education*. New York, NY: Springer New York, 691-720.

Buchholtz, N. (2017). How Teachers Can Promote Mathematising by Means of Mathematical City Walks. In G. Stillman, W. Blum, & G. Kaiser (Eds.), *Mathematical Modelling and Applications*. Cham: Springer International Publishing, 49-58.

Doyle, W. (1983). Academic Work. *Review of Educational Research*, 53(2), 159-199.

Doyle, W., & Carter, K. (1984). Academic Tasks in Classrooms. *Curriculum Inquiry*, 14(2), 129-149.

Gal, I. (2002). Adult's Statistical Literacy. *International Statistical Review*, 70(1), 1-51.

Hertz, H. (1899). *The Principles of Mechanics Presented in a New Form*. London: Macmillan.

Kaiser, G., & Sriraman, B. (2006). A Global Survey of International Perspectives on Modelling in Mathematics Education. *ZDM*, 38(3), 302-310.

Klaes, E. (2008). *Out-of-School Learning in Science Classes* (orig.: Außerschulische Lernorte im naturwissenschaftlichen Unterricht). Berlin: Logos.

Lenné, H. (1969). *An Analysis of Mathematics Education in Germany* (orig.: Analyse der Mathematikdidaktik in Deutschland). Stuttgart: Klett.

Leuders, T. (2015). Tasks in Research and Practice (orig.: Aufgaben in Forschung und Praxis). In R. Bruder, L. Hefendehl-Hebeker, B. Schmidt-Thieme, & H.-G. Weigand (Eds.), *Handbook of Mathematics Education* (orig.: Handbuch der Mathematikdidaktik). Berlin & Wiesbaden: Springer Spektrum.

Smeathman, H. (1781). Some Account of the Termites. *Philosophical Transactions of the Royal Society Series B*, 71, 139-192.

Sriraman, B., Roscoe, M., & English, L. (2010). Politicizing Mathematics Education. In B. Sriraman, & L. English (Eds.), *Theories of Mathematics Education*. Berlin & Heidelberg: Springer, 621-643.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building Student Capacity for Mathematical Thinking and Reasoning. *American Educational Research Journal*, 33(2), 455-488.

Stillman, G. A., Kaiser, G., Blum, W., & Brown, J. P. (2013). Mathematical Modelling. In G. A. Stillman, G. Kaiser, W. Blum, & J. P. Brown (Eds.), *Teaching Mathematical Modelling*. Dordrecht: Springer Netherlands, 1-24.

Sullivan, P., Clarke, D., & Clarke, B. (2013). *Teaching with Tasks for Effective Mathematics Learning*. New York, NY: Springer New York.

Watson, A., & Ohtani, M. (Eds.) (2015). *Task Design in Mathematics Education*. Cham: Springer International Publishing.

PHOTOGRAPHY: A RESOURCE TO CAPTURE OUTDOOR MATH

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Abstract. Several researchers refer that photography outside the classroom motivates students, for the understanding of math contents. *This paper, is part of a broader project centred on active learning of mathematics outside the classroom, describes a study where elementary preservice teachers capture photos in the environment that allow task design. In particular, we want to identify what aspects of the environment were privileged by the mathematical eye of the future teachers and to understand what are their main difficulties* when designing mathematical tasks. Results suggest that the participants chose photos privileging elements related with buildings, which mobilized Geometry and Measurement subjects. They expressed that the design of high-level cognitive tasks was not an easy process. The use of photography had a positive impact on them, providing a "closer look" at everyday objects, searching for the underlying mathematics more consciously, developing their "math eye".

Key words: Photography; Task design; Connections; Visualization; Problem solving; Teacher training.

INTRODUCTION

School mathematics requires effective teaching that engages students in meaningful learning through individual and collaborative experiences, giving them opportunities to communicate, reason, be creative, think critically, solve problems, make decisions, and make sense of mathematical ideas (NCTM, 2014). However, very often students don't develop such abilities, and they aren't able to make connections among different topics and use diversified tools to approach the same problem, which can be attributed to curriculum features and its extension that lead teachers to avoid this type of exploration. In this context, we must stress the importance of complementing formal learning with other environments, like the outdoors. The classroom is just one of the "homes" where education takes place (Kenderov et al., 2009). The process of acquiring information and the development of knowledge by students can occur in many ways and in many places. Whereas the stimulus for an affective environment can influence the initial expectations and motivations of students, the use of the surroundings as an educational context can promote positive attitudes and additional motivation for the study of mathematics, allowing them to understand its applicability (Vale, Barbosa & Pimentel, 2015). Students need understanding and, in mathematics, connections are fundamental to achieve this goal. In this scope, we consider that seeing through photos or images the connection between the mathematics discovered in and outside the classroom, and not view them as separate entities, can be a good learning strategy.

So, this work intends to promote contact with a contextualized mathematics, focusing on everyday life features, walking through and analyzing the place where we live, connecting some of its details through mathematical problem solving tasks, designed by preservice teachers. The main purpose is to promote a new attitude towards mathematics, through the observation and exploration of the urban environment. It is important that future teachers are aware of the mathematics around them, with all the complexity but also beauty and challenge that it encloses. It is also an opportunity for students to formulate problems, which implies making decisions about what to consider and what to ignore in the situation under analysis, applying and mobilizing personal mathematical knowledge in

Vale, I. & Barbosa, A. (2020). Photography: A Resource to Capture Outdoor Math. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 179-186). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.22

face of that particular situation, specifically a realistic one, and promotes creativity in its different dimensions. This motivates students' engagement in an active learning of mathematics assuming that learning emerges from experiences and interactions between the intellectual, social and physical dimensions (Vale & Barbosa, 2019b). On the other hand, images play an important cognitive role in the teaching and learning of mathematics, as an aid to thinking, as a means of communicating mathematical ideas and as a useful tool in problem solving (Arcavi, 2003). An image of a real object, situation or phenomenon, through photography or a drawing, has a fundamental importance in solving and formulating problems. In this particular case, we are interested in the use of photography.

The aim of this study was to capture photographs in the environment that allow the formulation of mathematical tasks, in order to establish connections between mathematics and reality, and its purpose was to answer the following questions: 1) What features of the environment were privileged by the preservice teacher's mathematical eye?; 2) What difficulties did they show in the tasks design; and 3) What reactions did they evidence during this experience?

PHOTOGRAPY AND VISUALIZATION

Several researchers (e.g. Meier, Hannula & Toivanen, 2018; Munakata & Vaidya, 2012; Rizzo, Rio, Mancenido, Lavicza & Houghton, 2019) have been working on photography outside the classroom as a way to motivate students, increasing interest and understanding of content, through the connection between mathematics and everyday situations. In addition, this type of approach gives students the opportunity to conduct their own transformative and aesthetic experience. This type of photograph, that we call mathematical photograph (or problem picture), is a photograph of an object, phenomenon or situation that is accompanied by one or more questions or a problem based on the context of the photograph (adapted from Bragg & Nicol, 2011). According to these authors, an image-based question can stimulate students' curiosity in answering the question, as well as their engagement during the process of creating immediate questions or a problem. Gutstein (2006) argues that good tasks do not necessarily reside in the task itself but rather in the relationship between the task and the solver (student or teacher), related to students' interests and lives, aspect that reinforces the use of photos (digital images), because they are chosen by the user. Taking a photo creates an affective connection between everyday situations and mathematical concepts, which engages students with the tasks (Barbosa & Vale, 2018; Vale & Barbosa, 2019a).

The role of visualization in mathematics learning has been a subject of undeniable attention because there is a great need to think and reason visually in problem solving, and it can be a very important cognitive tool in the development of mathematical concepts and processes, which can be applied to problem solving not only in mathematics education but in a variety of areas (e.g. Arcavi, 2003; Gutiérrez, 1996; Rivera, 2011; Zimmermann & Cunningham, 1991). We can find in the literature several definitions of visualization, but the one proposed by Arcavi (2003) is more in line with our work:

> Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings (p. 217).

This definition states that visualization can be considered as a tool of thinking in the sense that is fundamental in the process of mathematical discovery, involving components of creative thinking. Within the scope of visualization appears the concept of mathematical eye that it is an actual competence that students must acquire, because we live in a world where the visual features are a crucial component either in society or in many professions. Having a mathematical eye encourages the use of the real world as a starting point for a relevant exploration of mathematics and for recognizing the usefulness of mathematics. We use the common term "mathematical eye" when we want to refer to the use of mathematics as a lens to see and interpret the things/elements that surrounds us. It means to see the unseen, interpret things in the world as a boundless opportunity, and discover the mathematics involved by seeing the world around us with new eyes, *eyes that are open to the beauty of mathematics and its relation to the beauty of nature* (Stewart, 1997). This is important because, for most people the mathematics that surrounds them often remains "invisible" to the untrained or inattentive eye, and we have to educate their look, i.e. their mathematical eye, so that they can identify contexts and elements that can support rich mathematical tasks (Vale & Barbosa, 2019a). Connected with the term "mathematical eye" we may find another, which is also widely used in specific cases, and that is "geometrical eye", coined by Charles Godfrey in 1910 as the power of seeing geometrical properties detach themselves from a figure.

So, when we say that students must develop their mathematical or geometrical eye it means that we have to discover new ways of looking and consider familiar things either in daily life, work or inside/outside the classroom. It means seeing common objects from a new perspective, whose level of detail varies with each individual's knowledge and experience. Barnbaum (2010) uses the metaphor of a detective when observing a crime scene. The detective will certainly see more details than an ordinary person. He also refers that the art of re-seeing must be taught. According to Arcavi (2003) visualization must become more visible in the teaching of mathematics. He discusses mathematical visualization in a more figurative and deeper sense, as *seeing the unseen*, not only what comes *within sight* but also what we are unable to see, and become a tool for students to learn mathematics (Vale, Pimentel & Barbosa, 2018).

Using photos (images) provides opportunities to use the real world as the starting point and context to develop the mathematical eye and build mathematical problems and for further mathematical development, providing teachers with knowledge about students' visual attention. Furthermore, according with Meier, Hannula and Toivanen (2018) the use of photography motivates students, increases creativity and provides that "everyday life outdoors and science/mathematics can be connected in a meaningful way through the experience of photography" (p.147). So, the ability to see, contributes to the identification of mathematics in everyday life, reinforcing this type of connections, which leads to the use of new approaches to teach and learn mathematics (e.g. Barbosa & Vale, 2018).

DESIGNING TASK THROUGH PHOTOS

We defend that students must have mathematical experiences outside the classroom getting to know the natural, architectonic heritage surrounding their schools, and discover the connections of school mathematics with buildings, gardens, streets, solving tasks in real contexts. Hence, it is important that preservice teachers apply their knowledge about problem posing and creating tasks outside the classroom, so they can design tasks and trails for their own students.

For the tasks design, we followed the ideas of Sullivan and Liburn (2002) and also ideas of authors that have been working with problem solving/problem posing (Brown & Walter, 2005; Silver, 1997; Stoyanova, 1998), that embrace all kinds of tasks from simple exercises to challenging problems. Silver (1997) considers problem posing either being the generation (creation) of a new problem or the reformulation of a given problem. Stoyanova (1998) considers problem posing as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems. Brown and Walter (2005) propose two strategies to formulate problems. The first strategy, *Accepting the given*, starts from a static situation, which can be an expression, a table, a condition, an image, a diagram, a phrase, a calculation or simply a set of data, from which we formulate questions in order to have a problem, without changing the starting situation. The second strategy, *What-if-not*, extends a given task by changing what is given. From the information contained in a problem, we identify what is known, what is asked for and what limitations the response to the problem involves. By modifying one or more of these aspects or questions, new and more questions can be generated (Barbosa & Vale, 2018; Vale, Barbosa & Pimentel, 2015).

For Sullivan and Liburn (2002) there are three main features for posing good questions, that require more than remembering a fact or reproducing a skill. Students can learn by answering the questions, and the teacher learns about each student from their attempts. These authors propose a practical and accessible method for posing open-ended questions using a three-step process. Method 1: *Working Backward*, includes - Identifying a topic; - Thinking of a closed question and writing down the answer; - Making up a question that includes (or addresses) the answer. Method 2*: Adapting a Standard Question*, includes: - Identifying a topic; - Thinking of a standard question; - Adapting it to make a good question. These methods can provide also information about the way we choose a mathematical photography. If we look for the math potential of an object (or phenomena) in a photo or if we go looking for an object that matches a predefined subject.

We believe, in the sense of Bragg and Nicol (2011), that through creating open-ended problem photos, teachers and students will see mathematics through a new lens.

METHODOLOGY AND SOME RESULTS

An exploratory qualitative methodology was adopted with a group of 13 elementary preservice teachers of a course in primary education (6-12 years old) conducted in a school of education in a Didactics of Mathematics curricular unit. Throughout the classes, that acted as the context for the development of the study, these students were provided with diversified experiences, distributed in curricular modules, focusing on problem posing and solving, learning outside the classroom, creativity, and the establishment of connections, particularly those involving mathematics and daily life; and other mathematical processes (e.g. communication, reasoning, representations). For this paper we chose to present a particular structure to propose the tasks, which was a new experience for these future teachers – the creation of a *Photography task poster*. These preservice teachers were asked to propose tasks that would cover the elementary school levels (6-12 years).

The preservice teachers were asked, in pairs: 1) to explore the surroundings, taking a city tour analyzing the urban area where they had to capture a set of life shots, with potential to formulate mathematical tasks. They captured those photos with their mobile phones; 2) to choose some of the photos. The choices resulted from the analysis of the mathematics underlying each photo and the group discussion; 3) to formulate the tasks and solved them. In order to create a task based on photos we used the respective picture, applying the problem posing strategy *accepting the given* (Brown & Walter, 2005) and then the future teachers used Method 1 or Method 2 to pose questions (Sullivan & Liburn, 2002); 4) each pair created a poster that should include photographs, the created tasks and solved them; 5) the posters were presented, discussed and evaluated by all the students who participated in this experience, using an assessment grid that focused on the assessment of the tasks and the poster in global terms At the end, future teachers also made a written report. Figure 1 shows one example illustrating the different moments of this experience, some objects chosen along the city tour, a photography task poster, and two moments during the class when they were observing the different tasks in the posters and assessing the posters.

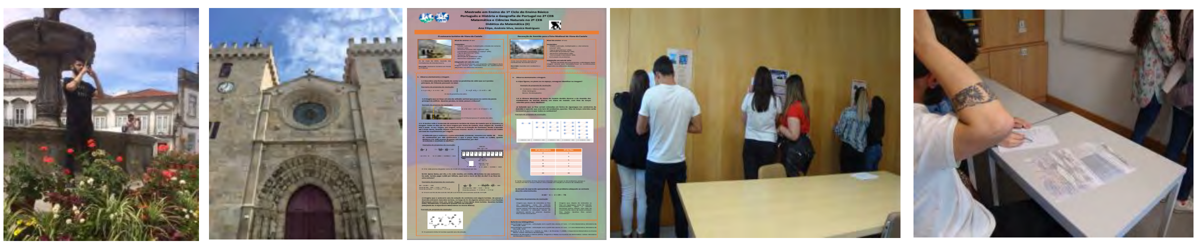

Figure 1: Examples of the different moments of the activity.

Data was collected in a holistic, descriptive and interpretive way and included observations along the whole experience (except the poster printing), the set of photos chosen, the written report (describing their reaction throughout the different phases of the experience, including how they chose and created the tasks) and the assessment grid. All the data was analyzed in an inductive way, according to the nature of that data and the research questions.

Analyzing the photographs picked by the preservice teachers in the presented posters, it appears that their gaze focused mainly on elements such as buildings/facades, flower boxes, and prices disposed on stores with information that could be explored mathematically. The participants supported this selection by referring to the mathematical content suggested by the captured images. They mentioned that these photographs were the ones that most inspired them to identify the contents that allowed to formulate the tasks. Through this dynamic, the objects of reality were transformed into mathematical objects, having aroused, for the most part, the mobilization of contents within the scope of Geometry and Measurement, followed by Numbers and Operations. In this sense, the establishment of connections between mathematics and real life was facilitated, being explored in a significant way through photography.

Figure 2 shows four of the tasks created by the participants. We consider that the first two tasks have a low level of demand and also can be solved without the solver being present on the site. In our opinion, the other two tasks have a high level of demand and the solver needs to be on the site to collect the necessary data to solve the task.

 The flower box shows a pattern where you can see symmetry of reflection and of rotation. Characterize all the symmetries you see.	 Margarida received a money for her birthday totalizing €100,40. With this money, she decided to go shopping at a fashion store, that had the price table shown in the image. Help Margarida decide what she can buy in the store with the money she received.
 In the flower bed you can see some flowers. Find out the part of the flower bed occupied by each group of flowers and then find out which area of the flower bed has no flowers. Explain your reasoning.	 Watch the Avenue closely. For the Medieval Fair, the Avenue will be decorated with ribbons of colored handkerchiefs placed in a zigzag pattern supported on the lamps along the Avenue (1 ribbon for every two lamps). How many ribbons will it take to decorate all the lamps?

Figure 2: Examples of tasks proposed by the students.

The formulation of tasks was not an easy process for these future teachers, a situation that can be explained by the fact that they did not have much experience at this level. Since the formulation of problems is a higher order capacity, it implies regular work so that there is a positive impact on the quality of the proposals. For this reason, they expressed difficulties in going beyond the traditional tasks, showing lack of creativity and little flexibility in the mobilization of different contents. In agreement with the ideas advocated by Barnbaum (2010), the more knowledge, training and experience we have, the more detailed and deeper the look will be. Despite having had the opportunity to contact with the objects in a real context, some of the pairs were unable to adapt the data used in the tasks to reality, having mobilized information that was not real (e.g. associating 3 meters in length to the radius of a flower pot). We can, however, point out that some of the participants were able to formulate challenging tasks with multiple approaches.

CONCLUDING REMARKS

To conclude we will synthesize the main ideas of this study, taking into consideration the research questions that guided the investigation, and also the data that emerged from the empirical work.

First of all, it was clear that the main features of the environment privileged by the preservice teacher's mathematical eye were objects, in particular buildings. The architectural details caught their attention in terms of possibility for mathematical exploration (Rizzo et al., 2019; Vale, Barbosa & Pimentel, 2015). The choice of photos was based on "possibilities for good questions". Only one group chose photos that "fit" what they had already thought of proposing (Bragg & Nicol, 2011). In general terms, it was a successful experiment that allowed these future teachers to identify the potential of photography in the mathematical exploration of everyday objects (Vale & Barbosa, 2019a). The use of photography as a means for promoting learning in Mathematics had a positive impact on students, providing a "closer look" for everyday objects, looking for the underlying mathematics in a more conscious and intentional way (Meier et al, 2018, Vale & Barbosa, 2019a). They admitted that formulating high-level cognitive tasks was a difficult endeavor. One group was not able to adapt the data used in the tasks to reality, mobilizing information that was not real or accurate. The greatest difficulties resulted from the little experience they had in terms of task design, beyond tasks that only involved formulas or calculations.

We may conclude that the participants responded with interest and motivation to the proposed challenge, committing themselves to the formulation of tasks. Based on the photos, they managed to build proposals with suitable contents for elementary school students, being able to naturally highlight connections between mathematics and the environment (Gutstein, 2006). It was an opportunity for these future teachers to contact with other (teaching) learning contexts and also an opportunity to work together, create their own tasks, grounded on their personal perceptions of reality, posing questions and problems, thinking creatively (Vale & Barbosa, 2019a, 2019b; Rizzo et al., 2019). It is important to include an instruction for (preservice) teachers to develop their mathematical eye as well as to create rich tasks so that they can propose to their pupils in the scope of outdoors mathematics education and of mathematical connections. This type of experience can be used with elementary school students, motivating them to learn mathematics outside the classroom, to strengthen mathematical knowledge and provide an excellent opportunity for connecting mathematics with real life on their own surroundings.

References

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215-241.

Barbosa, A., & Vale, I. (2018). Math trails: a resource for teaching and learning. In V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne, & L., Trouche (Eds.), *Proceedings of the Re(s)sources 2018 International Conference* (pp. 183-186). Lyon: ENS de Lyon.

Barnbaum, B. (2010). *The art of photography: An approach to personal expression*. San Rafael, CA: Rockynook.

Bragg, L., & Nicol, C. (2011). Seeing mathematics through a new lens: using photos in the mathematics classroom, *Australian Mathematics Teacher*, *67*(3), 3-9.

Brown, S., & Walter, M. (2005). *The art of problem posing*. Mahwah, NJ: Erlbaum.

Godfrey, C. (1910). The Board of Education Circular on the Teaching of Geometry, *Mathematical Gazette*, *5*, 195-200.

Gutiérrez, A. (1996). Visualization in 3-dimensional geometry: In search of a framework. *Proceedings of the 20th PME Conference, 1*, 3–19.

Gutstein, E. (2006). *Reading and Writing the World with Mathematics: Toward a Pedagogy for Social Justice*. New York: Routledge.

Kenderov, P., Rejali, A., Bartolini Bussi, M., Pandelieva, V., Richter, K., Maschietto, M., Kadijevich, D., & Taylor, P. (2009). Challenges Beyond the Classroom—Sources and Organizational Issues. In E. Barbeau & P. Taylor (Eds.), *Challenging Mathematics In and Beyond the Classroom – New ICMI Study Series 12* (pp. 53-96). Springer.

Meier, A., Hannula, M., & Toivanen, M. (2018). Mathematics and outdoor photography experience – exploration of an approach to mathematical education, based on the theory of Dewey's aesthetics, LUMAT. *International Journal on Math, Science and Technology Education, 8*(2), 146-166.

Munakata, M., & Vaidya, A. (2012). Encouraging creativity in mathematics and science through photography. *Teaching Mathematics and its Applications, 31*, 121-132.

National Council of Teachers of Mathematics (2014). *Principles To Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Rivera, F. (2011). *Toward a visually-oriented school mathematics curriculum: Research, theory,practice, and issues*. Dordrecht, Netherlands: Springer.

Rizzo, K. , Rio, L., Mancenido, M., Lavicza, Z. & Houghton, T. (2019). Linking Photography and mathematics with the use of technology. *Open Education Studies*, *1*(1), 262- 266. https://doi.org/10.1515/edu-2019-0020

Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM, 3*, 75-80.

Stewart, I. (1997). *The Magical Maze - seeing the world through mathematical eyes.* NY: John Wiley & Sons.

Stoyanova, E. (1998). Problem posing in mathematics classrooms. In A. McIntosh & N. Ellerton (Eds.), *Research in Mathematics Education: a contemporary perspective* (pp. 164-185). Edith Cowan University: MASTEC.

Sullivan, P. & Liburn, P. (2002). *Good questions for math teaching*. Sausalito, CA: Math Solutions publications.

Vale, I., & Barbosa, A. (2019a). A fotografia na aula de matemática: uma experiência promotora de conexões [Photography in math class: an experience that promotes connections]. In N. Amado, A.P. Canavarro, S. Carreira, R. Tomás Ferreira, I. Vale (Eds), *Livro de Atas do EIEM 2019* (pp. 183-186). Lisboa: SPIEM.

Vale, I. & Barbosa, A. (2019b). Gallery walk a collaborative strategy to discuss problem solving. *Quaderni di Ricerca in Didattica (Mathematics), 2(3), 151-156*.

Vale, I., Barbosa, A. & Pimentel, T. (2015). Math trails a rich context for problem posing - an experience with pre-service teachers. *Quaderni di Ricerca in Didattica (Mathematics)*, *25*(2), 221-227.

Vale, I., Pimentel, T. & Barbosa, A. (2018). The power of seeing in problem solving and creativity: an issue under discussion. In N. Amado, S. Carreira, K. Jones (Eds.), *Broadening the scope of research on mathematical problem solving: A focus on technology, creativity and affect* (pp. 243-272). Cham, CH: Springer.

Zimmermann, W., & Cunningham, S. (1991). *Visualization in teaching and learning mathematics*. Washington, DC: Mathematical Association of America.

EXPERIENCING COMPUTATIONAL THINKING AND THE CONCEPT OF LOOPS IN AN OUTDOOR CS UNPLUGGED APPROACH

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Abstract. *Computational Thinking (CT) is an important skill in the 21st century which should be fostered not only in computer science classes but also in K-12 STEM education in general. Taking a step back ignoring computers and other smart devices, an unplugged approach to computational problems can help students develop competencies related to CT without the need for advanced prior knowledge such as programming skills. Taking unplugged activities outside the classroom can ignite higher motivation within students to engage in these activities and elements of embodiment can lead to a deeper understanding. This paper presents a possible approach to experience CT and the concept of loops outside where students simulate a programmable Turtle on an outdoors area such as the school yard.*

Key words: Computational Thinking, Algorithmic Thinking, Computer Science Unplugged, Outdoor Learning, Turtle Graphics, Loops

THEORETICAL BACKGROUND

Teaching children how to program is not a new endeavor with first efforts to foster school students' programming skills already emerging in the 1970s and 1980s (see e.g. Pappert, 1980). However, in recent years, a different approach can be observed as well in which the focus has shifted from teaching the mere skill of programming to its underlying fundamental strategies and thinking skills. These skills were subsumed by Wing (2006) under the term of Computational Thinking (CT). Although Wing was not the first one to use this term, her paper can be seen as the starting point of the current discussion of CT (Hoppe & Werneburg, 2019, p. 14). Since then, many researchers have tried to capture the essence of CT with most definitions agreeing that it is a specific way of thinking about and solving problems (Bocconi et al., 2016). Core skills of CT include – but are not necessarily limited to – abstraction, algorithmic thinking, automation, decomposition, debugging and generalization (ibid.).

To thrive in the technology-driven world of today, becoming skilled at CT is of uttermost importance (ISTE, 2016). However, there is no consistent embedding of CT in most countries' school curricula with a great variation among the countries (Bocconi et al., 2016). Even though CT is considered a key competency in the digital age, engaging with CT does not necessarily mean working with a computer or other digital devices. The so-called Computer Science (CS) Unplugged approach, which was developed at the University of Canterbury in New-Zealand, aims at engaging people with key ideas of computer science without actually using computers (see Bell, Alexander, Freeman and Grimley, 2009). In mainly kinesthetic activities, participants are confronted with problems which they should solve. While some activities specifically aim at educating the participants in computer science, some also foster the more general skill of CT (ibid.). Since the threshold to participate in CS Unplugged activities is usually quite low as they are independent from any kind of programming knowledge and there is no necessity for any kind of hardware or software, they seem promising to foster CT especially in younger children.

Wetzel, S., Milicic, G. & Ludwig, M. (2020). Experiencing Computational Thinking and the Concept of Loops in an Outdoor CS Unplugged Approach. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 187-194). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.23

With CS Unplugged not relying on the use of technology, appropriate activities can also be transported outside the classroom. As such, they could present an opportunity to experience elements of CT outdoors. Participating in such activities outside the classroom, could increase students' motivation and lead to a higher level of engagement. Depending on the activity, working outside also offers positive side-effects like having more space and not being limited by constraints created through the properties of a classroom. In the following, we present a lesson design and describe an experimental test run of it. Apart from testing the quality and meaningfulness of the lesson design, we exemplarily take this lesson as incentive to explore if this and similar activities can engage younger children with CT outside the classroom.

DESIGN OF AN OUTDOOR CT LESSON

Bell et al. (2009) argue that a single lesson where students experience a CS Unplugged approach can already have a significant impact on the students' concepts of computer science. This clearly distinguishes CS Unplugged approaches from other, more complex approaches that cannot exist as stand-alone lessons but have to be embedded in a profound curriculum. We aim at transferring this positive effect to an outdoor lesson targeting young children in grade 5 or 6 where they can intuitively experience some of the core skills of CT. The lesson we present revolves around the idea of a Turtle (see e.g. Pappert, 1980) which is a programmable object with a position and a direction sitting on a canvas on which it can draw. Pappert (1980) argues that by identifying with the moving Turtle and imagining what they would do if they were the Turtle, children can combine the knowledge about their own body and its movement with a problem they aim to solve (p. 56). As such, a Turtle can be an intuitive approach to programming for younger children. However, solely imagining the Turtle's movement might not always suffice as Benton et al. (2018) argue that students often confuse directions in which the Turtle should move. They thus propose to engage in related unplugged activities to help diminish these errors. In our approach, instead of only controlling the Turtle, students also act as the Turtle which strongly relates to the idea of embodiment as proposed by Tall (2003). Tall defines embodiment as "thought built fundamentally on sensory perception" (p. 4) and argues that an embodied approach to a topic can help students build meaning.

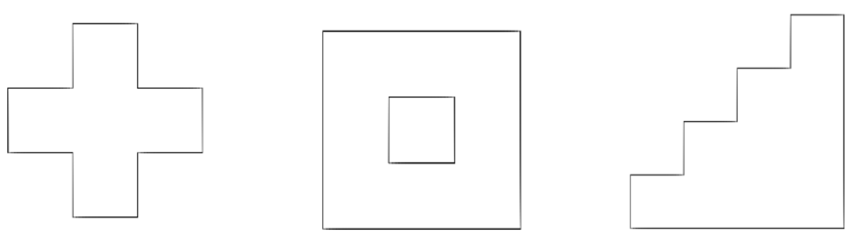

Figure 1: Target figures to draw.

In the lesson, which is designed for ninety minutes, students develop algorithms to draw a designated object which is made up of squares since squares are easy to draw and can also be found outside, for example on paved yards. Students work together in groups of three and each team member gets assigned his or her own target figure (see Figure 1). The students shall use a predefined set of instructions for their algorithms (see Figure 2). Instead of only providing the students with textual commands, we decided to illustrate them in the form of blocks, similar to Scratch (https://scratch.mit.edu/) or other block languages. Available blocks include forwards movement, ninety degree turns to the right and left, instructions if the chalk should be used or not, a command to go to the starting point and a bigger block for a loop, as it is represented in scratch, with the possibility to insert the commands that get repeated inside. The provided code blocks can be printed and cut out so they can be used as a kind of puzzle, however, working with blocks made of paper can prove quite fiddly and it takes a long time to cut out enough blocks. Another possibility is to just provide these blocks as a guideline to the students and let them draw the blocks on paper. As such, the students can flexibly adjust the size of the loop block, which is also possible in Scratch (Maloney et al., 2010).

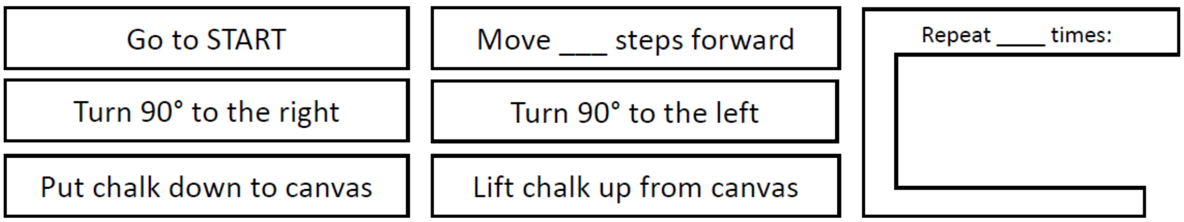

Figure 2: Set of commands.

One key element of the lesson is the division of the students into groups of three. Each student gets assigned one target figure which no one else is allowed to see. All three students individually conceptualize an algorithm with which the image can be drawn. Thus, all of them get the opportunity to equally engage in algorithmic thinking. During the testing phase of their program outside, each student of a group has a different role. Students take turns so that every student adopts each role once. The student who wrote the code is the developer who, in a real programming situation, would have no influence on what the program does after hitting the run button. He or she can only wait and see if something goes wrong and, if necessary, ponder possible errors in the code. Thus, in our lesson, the student who wrote the code is a quiet and passive observer while the other two students actively work with the code. One of the remaining two students reads out the code line by line, not knowing what the result is supposed to look like and thus not able to correct the code while reading it out. This student therefore serves as the machine or compiler translating the code into something that can be executed. The third student is the Turtle and executes the commands exactly like they are read out. This strict separation of responsibilities helps the students to recognize the difference between the developer, the machine or compiler and the executing unit, in our case the turtle, with the latter two both knowing nothing of the developer's intentions.

Learning goals of the lesson include a first introduction to sequential coding and an intuitive understanding of the advantages of loops. Concerning the core skills of CT following the notion of Bocconi et al. (2016), firstly, students engage in algorithmic thinking when conceptualizing and writing their code. During the lesson, it should be highlighted that debugging is a normal part of every development process and that students should not fear errors but instead analyze them closely, ideally learn from them and then change their code accordingly. They thus practice debugging, which, according to Bocconi et al. (2016) is "… the systematic application of analysis and evaluation". Since all three target figures contain repetitive elements and parts of squares, students can simplify their code considerably if they recognize these patterns and use loops which is a part of generalization (Bocconi et al., 2016). However, recognizing the squares and solving the problem for a single square first can also be considered a part of decomposition: instead of solving the whole problem at once, a smaller and easier problem namely the one of a single square gets addressed first. The lesson could hence foster the four core CT skills algorithmic thinking, debugging, generalization and decomposition.

METHODS

According to Bell et al. (2009), a CS Unplugged approach should be developed in three steps: designing, testing and adjusting. It is thus essential to do a test run of a proposed lesson. We tested the lesson with students of grade 5 in January 2020. As this trial run was meant to pilot the lesson so that the concept could, if necessary, be modified and improved before conducting it with a complete class of 30 students, we decided to invite six students to participate in the test run so that we would have two groups of three students each.

Apart from observing the students during such a test run, it can provide additional insights if students fill in pre- and post-questionnaires. The purpose of the questionnaires is threefold: we want to raise data concerning the students' pre-knowledge and knowledge gain through the lesson, ask them for their opinion on outdoor learning before and after the lessen as well as generally find out how they like the lesson. As one goal of the lesson is to introduce the concept of loops, we ask students before and after the lesson whether they know what a loop is and, if they do, whether they can describe its purpose and advantages. All other data, i.e. their attitudes towards computer science and outdoor learning as well as whether they like the lesson, is raised using a four-point Likert scale. We decided to use a Likert scale to better describe whether there are differences before and after the lesson.

EXPERIENCES DURING THE TEST RUN

The lesson started with a short input phase inside in which we explained the goal of the lesson and the commands, with special focus on how to use the loop block, as well as the structure of and responsibilities in the groups. Exemplarily, one of us read out some lines of code while someone else embodied the Turtle to show the students how the role division works when executing someone's code. We also gave them two examples how a loop command could be read out by either repeating the code inside the loop as many times as needed and leaving out the loop completely or by making clear which commands needed to be repeated how many times.

Afterwards, the students each received a target figure (one of those in Figure 1) which they were not allowed to show to anyone else. Before they started to develop their programs, we informed them that there would be a debugging phase after the first test execution of their programs. Each student received a clipboard with a sheet in foil on which they should write their code with a non-permanent marker so that they could easily change their code. Students designed their programs inside before we went outside for the first test run. Even though everyone was occupied at first, some students were significantly faster than others. For example, the student who wrote the code depicted in Figure 3 on the left finished first. By recognizing that a pattern was underlying his target figure and consequently using a loop, his code was considerably shorter than that of another student in Figure 3 on the right who took longest and did not use any loops. However, the code of the student on the left was erroneous and did not produce the target figure since he used only left turns but never a right turn (the translated version of his code is displayed in the bottom right corner of the picture). When asked to recheck his code while he was waiting for the others, the student assured us that his solution was correct and that the task was far too easy. Hence, he could not recognize his mistake in the conception phase of the lesson.

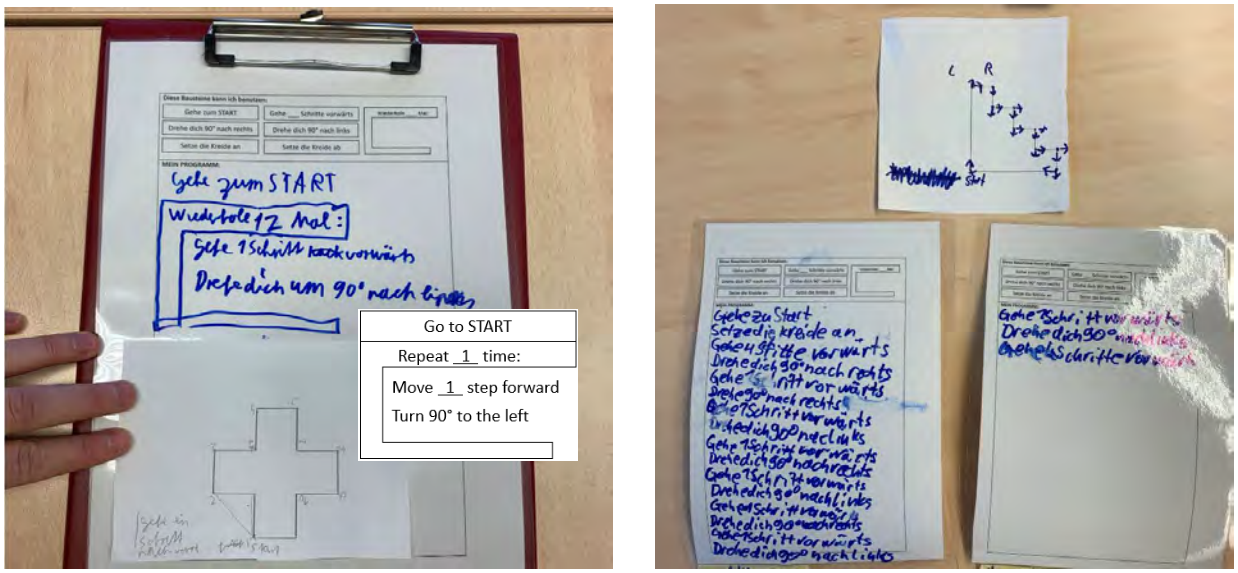

Figure 3: Two student solutions, one with a loop and one without one.

For the drawings, we used the yard in front of the university which is paved with square tiles (see Figure 4) to facilitate the drawing process. Even though the weather conditions were not perfect, the students were eager at work. The code of one member of each team already worked correctly in the first test run while the others' codes contained errors. The errors were mostly related to the direction in which the Turtle should turn. When there was an error and the code did not produce the intended result, the student in the role of the supposedly passive developer often showed signs of discontent and attempted to interact with his team mates to correct the errors. We therefore needed to remind the students to simply observe and think about possible corrections which could be implemented in the following phase but not to intervene during the execution of the code. Some of the students also had difficulties in the role of the compiler when reading the code out loud, especially when dealing with a loop. Keeping count of the number of iterations and giving the corresponding commands in the right order proved challenging for some of them. After a while, though, the students got more proficient and gave the commands in the right order.

In a subsequent debugging phase, students should adapt their code collaborating with their team mates. We observed that the students actively used their bodies during this phase to decide whether the Turtle should turn left or right. Already on the second attempt, all students' programs worked. Students were also asked to simplify their code if possible, which prompted some students to use a loop who had not used a loop before. In the end, five of the six students had used loops. At the end of the lesson, the concept of loops and its usefulness in programming was discussed briefly.

Figure 4: Students in action.

RESULTS AND DISCUSSION

When we asked the students what a loop was in the pre-questionnaire, most drew a ribbon since both words translate to “Schleife” in German. Interestingly, both students who said they had no prior programming experience could give a basic definition of loops as “something that gets repeated”. In the post-questionnaire, all students were able to give a basic definition of loops and gave answers such as “needs less space” or “one does not have to write everything numerous times” when asked for its advantages. They therefore constructed a first understanding of loops for themselves. Even though they are limited, the definitions and advantages given by the students seem age-appropriate. However, when conducting such a unit, one must be careful not to create misconceptions or one-sighted views of a concept, in our case loops. When Benton et al. (2018) asked students of grade 6, who had only worked with code blocks so far, to describe how a given algorithm could be extended to a more complex problem, some could not answer this question in words as was expected but instead chose to draw code blocks as an answer. After our unit, we made a similar observation as one student focused mainly on the block aspect and wrote that a loop was “a block that repeats things”. It is thus important to at least mention that a loop does not have to be a block. Keeping this aspect in mind, however, the lesson seems appropriate to let the students construct their first concept of loops. We also asked students about outdoor learning. Most indicated that they only rarely, if at all, went outside for a lesson in their school. When asked if they liked having lessons outdoors most agreed at least partially before our lesson. One student who answered “don’t know” and one who partially agreed both completely agreed in the post-questionnaire. Apart from one student, all completely agreed that they had had fun during our lesson. Since all students had produced working code at the end of the lesson and also indicated to have had fun, the motivational aspect seems to be high as well. However, this can probably at least partly be attributed to the general novelty aspect of the lesson.

In accordance with the design, test, adjust cycle as described by Bell et al. (2009), the lesson should be analyzed and, where necessary, adapted. In a study executed by Benton et al. (2018), students of grade 6 where asked to compare algorithms and many struggled to recognize that shorter and more readable algorithms should be preferred over long and complicated ones. Due to the limited space on the task sheet in our unit, most students realized that producing numerous lines of code is not desirable if it can be avoided. Providing the students with a sheet to write their code on can thus create the inherent need for loops and seems favorable. However, we would not give out sheets in foil again, since the writing got smeared after some time and thus almost unreadable. It is also important to consider ways to deal with students who are very fast in the initial design phase. One possibility could be not to assign the groups in advance but to always let the fastest three, the second fastest three and so on form a group and allow them to immediately start their first test run.

The division of responsibilities within each team into different roles (developer, compiler and Turtle) worked mostly well. Some problems arose when students had to read out loop blocks. Despite a demonstration at the beginning of the lesson how they could do this, most of the students had some difficulties during this translation process. To avoid this source of confusion and reduce the probability for errors, it might be helpful to discuss loops further at the beginning of the lesson and give more examples how they can be read out by the student who simulates the compiler. The division of roles also requires a high level of discipline from the student who acts as the developer to not interfere during the execution of the code. A possible adaption could be the addition of a forth student to each group acting as a referee and supervising the whole process. Whenever needed, this student could remind the developer not to interfere. This adaption would also offer some benefits when conducting this lesson in a class with more students as it would either reduce the total number of groups or would provide some flexibility allowing groups of both three and four students.

CONCLUSION AND OUTLOOK

In alignment with the presumption of Benton et al. (2018), embodying the Turtle and executing its actions themselves helped students to improve their understanding of direction and, as a consequence, their algorithms. At first, most made mistakes related to direction, however, one could clearly observe how they made use of their body when correcting their code. This is a clear gain from doing this unit outdoors where there was enough space for students to move and turn and thus experience the movements on a deeper level. The same unit inside using a pencil and piece of squared paper might not have had the same effect. The proposed lesson seems appropriate for young children to make first experiences with CT. An outdoor unplugged lesson like the one proposed can thus serve as a playful and motivating incentive for students to engage with CT already at a younger age without the need for computers or other expensive equipment, opposed to other approaches where students are engaged in coding outside using technology (Hitron et al, 2017; Offer et al., 2018). As we only conducted the lesson with six students, however, it is necessary to do a second test run in the future with a whole class of grade five or six students and to review whether results especially regarding the knowledge gain can be reproduced.

In future work, it could also be interesting to further focus on the outdoor aspect and how it can help foster the core CT skills. In this regard, it might be interesting to compare the effects on students' CT skills of an outdoor lesson like the one we proposed and a similar lesson conducted inside. This would however require a reliable tool to measure students' skills in CT. A possible variation of our proposed lesson to stronger emphasize the aspect of outdoor learning could also be to ask students to find interesting shapes outside such as paving pattern or special wall structures and then to design a program with which a Turtle could reproduce these shapes.

References

Bell, T., Alexander, J., Freeman, I., & Grimley, M. (2009). Computer science unplugged: School students doing real computing without computers. *The New Zealand Journal of Applied Computing and Information Technology*, 13(1), 20-29.

Benton, L., Kalas, I., Saunders, P., Hoyles, C. & Noss, R. (2018). Beyond jam sandwiches and cups of tea: An exploration of primary pupils' algorithm-evaluation strategies. *Wiley Journal of Computer Assisted Learning*, 34, 590-601.

Bocconi, S., Chioccariello, A., Dettori, G., Ferrari, A. & Engelhardt, K. (2016). *Developing Computational Thinking in Compulsory Education*. Seville: JRC Science Hub.

Hitron, T., Apelblat, I., Wald, I., Moriano, E., Grishko, A., David, I., Bar, A. & Zuckerman, O. (2017). Scratch Nodes: Coding Outdoor Play Experiences to enhance Social-Physical Interaction. In *Proceedings of the 2017 Conference on Interaction Design and Children* (pp. 601-607).

Hoppe, H.U. & Werneburg, S. (2019). Computational Thinking—More Than a Variant of Scientific Inquiry! In S.-C. Kong & H. Abelson (Eds.), *Computational Thinking Education* (pp. 13-30). Singapore: Springer.

International Society for Technology in Education [ISTE] (2016). *ISTE Standards for Students*. Retrieved from https://www.iste.org/standards/for-students [02.03.2020].

Maloney, J., Resnick, M., Rusk, N., Silverman, B. & Eastmond, E. (2010). The Scratch Programming Language and Environment. *ACM Transactions on Computing Education*, 10(4), 1-15.

Ofer, N., Erel, H., David, I., Hitron, T., & Zuckerman, O. (2018). A little bit of coding goes a long way: effects of coding on outdoor play. In *Proceedings of the 17th ACM Conference on Interaction Design and Children* (pp. 599-604).

Pappert, S. (1980). *Mindstorms: Children, Computers and Powerful Ideas*. New York: Basic Books Inc.

Tall, D. (2003). Using technology to support an embodied approach to learning concepts in mathematics. *Historia e tecnologia no Ensino da Matemática*, 1, 1-28.

Wing, J. (2006). Computational Thinking. *Communications of the ACM*, 49(3), 33-35.

POSTERS

USING MATH TRAILS AS A TRAVEL GUIDE

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Abstract. *This work proposes a different way of getting to know the cities. It is intended that the involvement with cities is done through a math trail. Using mathematics and technology, through the MCM App, we intend that people visit the emblematic places of the cities. Users get to know the city by solving mathematical tasks. These tasks are only possible to be solved with the data collected in the local. At the same time that people visit the cities, people are running a math trail that guides them along the route and turns the cities' visit into a game. We use, as an example, the city of Porto.*

Key words: MathCityMap, math trails, math tasks, cities, travel guide.

OUTDOOR EDUCATION AND MATH TRAILS

In 1986, Simon Priest defined outdoor education based on six points: is a method for learning; is experiential; takes place primarily in the outdoors; requires use of all senses and domains; is based upon interdisciplinary curriculum matter and is a matter of relationships involving people and natural resources (Priest 1986).

Math trails are an example of outdoor education. Running a math trail is one way of experiencing outdoor mathematics. Shoaf, Pollak & Schneider (2004, pp. 4) states that: "*A mathematics trail is a walk to discover mathematics. A math trail can be almost anywhere - a neighborhood, a business district or shopping mall, a park, a zoo, a library, even a government building. The math trail map or guide points to places where walkers formulate, discuss, and solve interesting mathematical problems. Anyone can walk a math trail alone, with the family, or with another group. Walkers cooperate along the trail as they talk about the problems*.".

Math trails can help to connect math content learned in the classroom with the real world and help to discover that math is everywhere. When we run a math trail, we establish connections between mathematics and the real world, we use tools such as diagrams and proportionality and algebraic thinking to solve problems, we update and revise our data collection methods to improve accuracy, and we collaborate to clearly communicate our thinking. The opportunities offered by math trails can help connect students and teachers, families or tourists to the local environment and, in doing so, support the development of rich and meaningful mathematical modeling skills (Druken & Frazin, 2018).

MATHCITYMAP - A TOOL TO ENGAGE WITH THE CITIES

The MathCityMap (MCM - https://mathcitymap.eu/en/) project was created at Goethe University in 2012 in an educational and technological context and aims to develop a central and global platform for the creation and simplified use of tasks that are part of a math trail. In its technical aspect, two components stand out: a web portal, as a tool to create and view tasks and routes, and an application for smartphones, as a tool to execute routes. MathCityMap App (MCP App) is to teachers, students and anyone interested in trying out mathematical routes in the environment around them. MCM App guides users to tasks via GPS, presents the tasks and gives an immediate feedback and a possible resolution. It also provides suggestions for solving tasks. MathCityMap web portal is to teachers and creative users, who want to create their own tasks and routes (Ludwig & Jablonski, 2019).

Caldeira, A., Viamonte, A. J., Figueiredo, I. & Brás, H. (2020). Using Math Trails as a Travel Guide. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 197-200). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.24

Firstly, the math trail idea was developed as a family vacation activity. Later, some schools took advantage of the trails, integrating them into their math learning programs. The success of this idea allowed this program to be adapted and applied in different locations, and the math trail projects later appeared in several cities (Cahyono & Ludwig, 2019).

In this work, we present an idea that combines tourism with math trails. Our scenario will be the cities and their emblematic places, and the objective is to get to know the cities with a special look - a mathematical look. The idea is to go through the main tourist points of the cities and, in each of these points, there is a mathematical task to solve. The task is only possible to be solved at the tourist point because there are data that can only be obtained at that location. This will make mandatory the trip to that local. A brief historical summary is presented in the App to better understand the place.

To exemplify our idea, we used the city of Porto (Portugal). The *Jardins do Palácio de Cristal* (Gardens of the Crystal Palace) are a pleasant green space located in this city. We present a math trail in order to get to know these gardens more intimately. As a tool, we use the MCM.

A TOURIST MATH TRAIL

To visit the *Jardins do Palácio de Cristal*, we use the public route of the MCM App with the code: 141980. This route has a length of 700 meters and an estimated duration of 2 hours and 10 minutes, and includes 10 tasks. Gamification is used in this route, it is displayed sample solutions and hits, and the user' solution is validated (see Gurjanow et al., 2019).

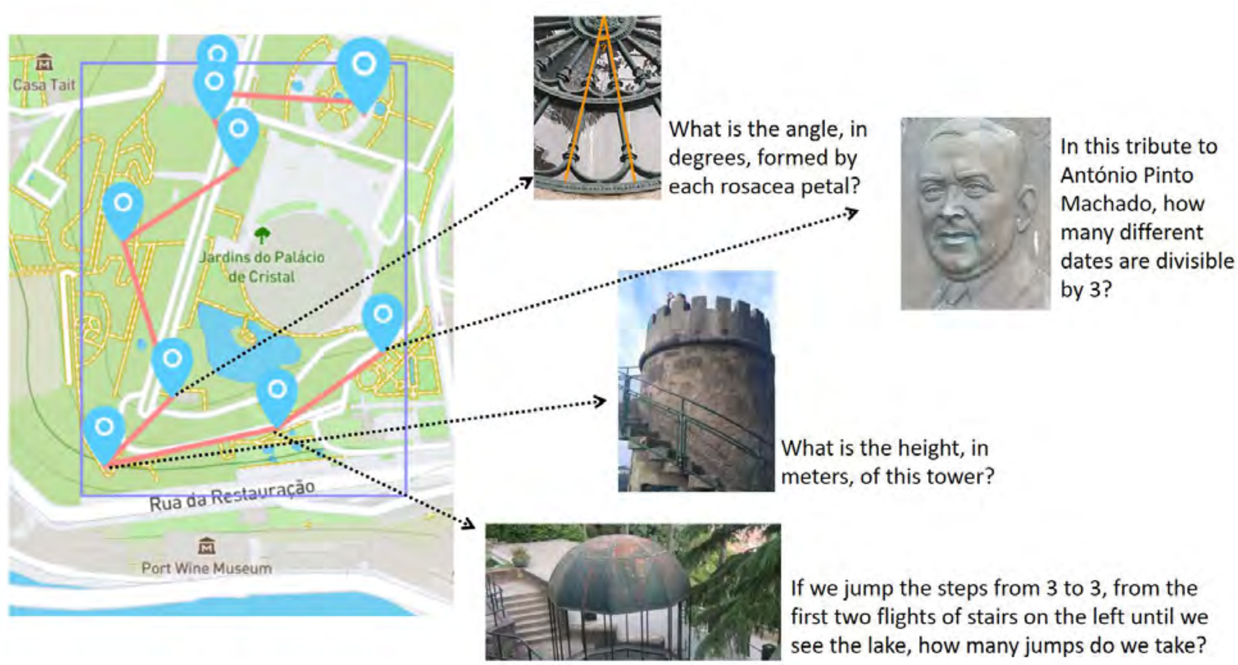

Figure 1: A tourist math trail in *Jardins do Palácio de Cristal*, and example of some tasks.

In Figure 1 it can be seen tourist math trail in *Jardins do Palácio de Cristal* and examples of some tasks.

In the MCM portal, it is possible to put information about the object or the place where the task is and also information about the route. It is possible to take advantage of this field to put historical information, or some funny information, or some feature of the place or object. Thus, it is possible to build a tourist mathematical route. It is also possible to create narratives on the MCM portal. Currently, the only possible narrative is the *Pirate narrative*.

CONCLUSIONS

The concept of math trails has been greatly enriched thanks to the possibilities offered by mobile devices to provide automatic feedback and provide guidance throughout the route.

In this work we intended to present an idea that use math trails and combines tourism and family relationships with mathematics. We proposed that tourists took a trip around the Porto with a mathematical look. Our aim is that people visit the main tourist points of the city and that in each of these points they solve a task. The goal is to get to know the city in a playful way and using their mathematical knowledge. All activities were developed to be carried out only at the tourist spot itself. In addition, in each tourist spot a brief historical summary is presented so that the tourist gets to know that tourist spot better.

As a tool, we use MathCityMap, where users can create tasks and trails and share them amongst themselves or with the public to run through its App. MCM guides users to tasks via GPS, presents tasks and gives immediate feedback and possible resolution. It also provides suggestions for solving tasks.

References

Cahyono, A. N. & Ludwig, M. (2019). Teaching and Learning Mathematics around the City Supported by the Use of Digital Technology. *Eurasia Journal of Mathematics, Science and Technology Education, 15(1)*, em1654. https://doi.org/10.29333/ejmste/99514

Druken, B., Frazin, S. (2018). Modeling with Math Trails. Retrived from

https://www.researchgate.net/publication/334330282_Modeling_with_Math_Trails/citations

Gurjanow, I., Oliveira, M., Zender, J., Santos, P. A., & Ludwig, M. (2019). Shallow and deep gamification in mathematics trails. In M. Gentile, M. Allegra & H. Söbke (Eds.), *Games and learning Alliance - 2019: 7th International conference*, *GALA 2018, Palermo, Italy, December 5-7, 2018, Proceedings* (pp. 364–374). Springer International Publishing.

Ludwig, M. & Jablonski, S. (2019). Haciendo matemáticas al aire libre con MathCityMap. Presented at Jornadas para el Aprendizaje y la Enseñanza de las Matemáticas (19JAEM), 3 a 5 julio 2019, A Coruña, Spain.

Priest, S. (1986) Redefining Outdoor Education: A Matter of Many Relationships, *The Journal of Environmental Education, 17 (3)*, 13-15, DOI: 10.1080/00958964.1986.9941413

Shoaf, M., Pollak, H., & Schneider, J. (2004). *Math Trails*. Lexington: COMAP.

Using math trails as a travel guide

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Abstract. This work proposes a different way of getting to know the cities. It is intended that the involvement with cities is done through a math trail. Using mathematics and technology, through the MCM App, we intend that people visit the emblematic places of the cities. Users get to know the city by solving mathematical tasks. These tasks are only possible to be solved with the data collected in the local. At the same time that people visit the cities, people are running a math trail that guides them along the route and turns the cities' visit into a game. We used, as an example, the city of Porto.
Keywords: *MathCityMap | math trails | math tasks | cities | travel guide*

OUTDOOR EDUCATION AND MATH TRAILS

- Outdoor education is a method for learning; is experiential; takes place primarily in the outdoors; requires use of all senses and domains; is based upon interdisciplinary curriculum matter and is a matter of relationships involving people and natural resources (Priest 1986)
- Math trails are an example of outdoor education, and they can help to connect math content learned in the classroom with the real world and help you to discover that math can be found everywhere (Shoaf, Pollak, & Schneider, 2004).
- The opportunities offered by math trails can help connect students and teachers, families, or tourists to the local environment and, in doing so, support the development of rich and meaningful mathematical modeling skills (Druken & Frazin, 2018).

A TOURIST MATH TRAIL

- **Local**: *Jardins do Palácio de Cristal*. Porto, Portugal
- **App**: MathCityMap
- Public route of the MCM App with the code: 141980
- 10 tasks in a route of 700 meters
- Duration of 2 hours and 10 minutes
- With gamification

MATH CITY MAP A TOOL TO ENGAGE WITH THE CITIES

- The goal of MathCityMap (MCM - https://mathcitymap.eu/en/) project, was to develop a web portal (a trail management system), to create mathematical tasks and routes and an application for smartphones, to execute the tasks of the routes, related to mathematics on real world objects, and places.
- The App guides users to tasks via GPS, presents the tasks, and gives an immediate feedback, a possible resolution and suggestions for solving tasks.
- We intend to present a strategy that combines tourism with mathematics. Our scenario are the cities and their emblematic places, and the objective is to get to know the cities with a special look - a mathematical look. The idea is to go through the main tourist points of the cities where is a mathematical task to solve. The task is only possible to be solved at the tourist point and a brief historical summary is presented to better understand the place.

CONCLUSIONS

In this work we present an initiative that use math trails (outdoor education) and combines tourism and family relationships with learning mathematics. It is intended that they visit the main tourist points of the city and that in each of these points they solve a task! The goal is to get to know the city in a playful way and using their mathematical knowledge.

References

- Cahyono, A. N., Ludwig, M. (2019). Teaching and Learning Mathematics around the City Supported by the Use of Digital Technology. Eurasia Journal of Mathematics, Science and Technology Education, 15(1), em1654. https://doi.org/10.29333/ejmste/99514
- Druken, B., Frazin, S. (2018). Modeling with Math Trails. Retrieved from https://www.researchgate.net/publication/334330282_Modeling_with_Math_Trails/citations
- Gurjanow, I., Oliveira, M., Zender, J., Santos, P. A., & Ludwig, M. (2019). Shallow and deep gamification in mathematics trails. In M. Gentile, M. Allegra & H. Söbke (Eds.), Games and learning Alliance - 2018: 7th international conference, GALA 2018, Palermo, Italy, December 5-7, 2018, Proceedings (pp. 364–374). Springer International Publishing.
- Ludwig, M. & Jablonski, S. (2019). Haciendo matemáticas al aire libre con MathCityMap. Presented at Jornadas para el Aprendizaje y la Enseñanza de las Matemáticas (JAEM), 3 a 5 julio 2019, A Coruña, Spain.
- MathCityMap https://mathcitymap.eu/
- Priest, S. (1986) Redefining Outdoor Education: A Matter of Many Relationships, The Journal of Environmental Education, 17:3, 13-15, DOI: 10.1080/00958964.1986.9941413
- Shoaf, M., Pollak, H. & Schneider, J. (2004). Math Trails. Lexington: COMAP.

MATHCITYMAP GENERIC TASKS OBJECTS IN PORTUGAL AND IN SLOVAKIA

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[2] Constantine the Philosopher University in Nitra, Slovakia

Abstract. *Portal MathCityMap is the technical support and database of tasks and trails within the Europe and the whole world. Every composed task is based on exploitation of some real object and is identified with several attributes: real object characteristics, key words of mathematics topic which the solution of the tasks can be placed in, school grade which mathematical competences should allow to solve the task. Generic tasks represent grouping the single tasks about almost the same objects into one category. Five categories of MathCityMap generic tasks are introduced and illustrated by real objects localized in tasks in Portugal and in Slovakia.*

Key words: MathCityMap portal, Mobile Math Trials in Europe project, generic task.

MATHCITYMAP GENERIC TASKS

MathCityMap portal (MCM - https://mathcitymap.eu/en/) is a math trail management system, where users can create tasks and trails and share them amongst themselves or with the public to run through its App. It is a very important tool to be used by mathematic teachers, as it brings together an important recipe for the success of their students: outdoor education and technology.

The concept of math trails lately has been greatly enriched thanks to the possibilities offered by mobile devices to provide automatic feedback and provide guidance throughout the route. However, this requires theoretical and empirical considerations, provided by **Mobile Math Trails in Europe** (MoMaTrE, www.momatre.eu) (see for example: Ludwig & Jablonski, 2019; Cahyono & Ludwig, 2019; Gurjanow et al., 2019 and the references in it). The MoMaTrE project is an Erasmus + project started in 2017, lasting three years. Five countries are partners in this project: Slovakia, France, Spain, Portugal and Germany.

One of the project MoMaTrE outputs is categorizing tasks into **generic tasks** categories. A mathematical topic indicator characterizes each category of generic tasks. A generic tasks is a mathematical tasks which is very typical but also very general, for example, to calculate the slope of a ramp or a handrail of a stairway you can find them everywhere in Europe in different styles. Generic tasks are the most frequent tasks in MCM trails. In different locations on planet Earth, we can find the same objects or have the same scenario. Five generic tasks categories are introduced and illustrated with the most interesting MCM tasks objects localized in Portugal and Slovakia.

Table 1 shows some examples of generic tasks in Slovakia and Portugal. These tasks can be found everywhere: the ramp slope, the building height or statue height, a bicycle stand, etc.

Generic tasks categories

- Counting/Combinatory;
- Quantities;
- Height;

Caldeira, A. & Ceretkova, S. (2020). MathCityMap Generic Tasks Objects in Portugal and in Slovakia. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 201-204). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.25

- Slope;
- Volume.

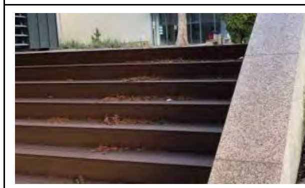
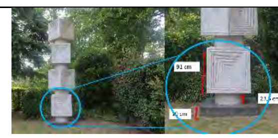
	Slovakia	Portugal
Counting/Combinatory	 **Stairs** How many possibilities exist to go upstairs if one can take one or two steps within each move? The step sequences can be combined?	 **Parking a bicycle** Three bicycles will park at this location. Each rim can only have one bicycle (it does not matter whether it is parked on the left or right of the rim, facing the street or not). How many possibilities are there to park three bikes?
Quantities	 **Information board** How many posters in A5 format (15 cm x 21 cm) can be placed on the information board? All the posters must be visible and they cannot overlap each other.	 **Footbridge** It is intended to pave the passage. All slabs (with dimensions of 42x60 cm) must be placed in the same orientation and, in the case of being cut, the leftovers are wasted. What is the minimum number of slabs needed to pave the passage?
Height	 **Staircase height** How deep (high) is the staircase on the left side of the music pavilion? Give the result in centimetres.	 **Statue height** How tall is the statue? (Give the result in meters)
Slope	 **Railing** Determine the slope of the railing. Give the result in percentage!	 **Ramp** In order to know if the ramp can be comfortably used by a wheelchair person, determine an approximate value for the ramp slope (in percentage).
Volume	 **Flowers container** How many cubic meters of soil are there in a big concrete area/ field for planting flowers?	 **Mass of a tree trunk** Determine the volume/mass of the tree trunk.

Table 1: Examples of generic tasks in Slovakia and in Portugal.

In the counting /combinatory category, we can use, for example, stairs or bicycle stand, and use counting techniques - come up with an enumeration strategy and compute. Here we can use any collection not too large of well identified objects (steps in a staircase) or calculate how many possible combinations exist for different situations, or ... . In the quantities category we can use an information board or a footbridge and we can apply our knowledge about polygons areas - decomposition of a polygon into several polygons and calculation of areas. In the height category, we can use stairs, or a statue or even a building to calculate their height. We can calculate a distance through scaling, through counting number of some patterns or we can measure two non-corresponding distances in order to establish a scale. In the slope category, we can use a railing or a ramp to calculate slopes – a constant slope with easy horizontal and vertical segments. In the volume category, we can use a flowers container or a tree trunk. It is necessary to identify a cuboid, measure its sides or identify a cylinder, its base and height, and choose the correct units for the volume or mass of the object calculation

CONCLUSIONS

The aim of this work was to show that, with the support of generic tasks, available in MCM, is much easier for a math teacher to quickly create tasks for their students because there is a range of ideas and best-practice examples. The generic tasks can inspire teachers to create local tasks or even textbook tasks for regular mathematics lessons. Generic task description and examples, as the project MoMaTrE output, is a very useful tool, available on the project web page to download.

References

Cahyono, A. N., Ludwig, M. (2019). Teaching and Learning Mathematics around the City Supported by the Use of Digital Technology. *Eurasia Journal of Mathematics, Science and Technology Education*, 15(1), em1654. https://doi.org/10.29333/ejmste/99514

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Ludwig, M. & Jablonski, S. (2019). Haciendo matemáticas al aire libre con MathCityMap. Presented at Jornadas para el Aprendizaje y la Enseñanza de las Matemáticas (19JAEM), 3 a 5 julio 2019, A Coruña, Spain.

MCM portal. (2020, March 20). *Tutorials*. Retrieved from https://mathcitymap.eu/en/portal-en/

Mobile Math Trials in Europe. (2020, March 20). *Downloads.* Retrived from http://momatre.eu/downloads/

MathCityMap Generic Tasks Objects in Portugal and in Slovakia

ROSETA
RESEARCH ON OUTDOOR STEM EDUCATION IN THE DIGITAL AGE

Amélia Caldeira, Instituto Superior de Engenharia do Politécnico do Porto, Portugal
Sona CeretKova, Constantine the Philosopher University in Nitra, Slovakia

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Keywords: *MathCityMap Portal | math trails | math tasks | cities | travel guide*

MathCityMap GENERIC TASKS

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One of the project MoMaTrE outputs is categorizing tasks into **generic tasks** categories. A mathematical topic indicator characterizes each category of generic tasks. A generic tasks is a mathematical tasks which is very typical but also very general, for example, to calculate the slope of a ramp or a handrail of a stairway you can find them everywhere in Europe in different styles. Generic tasks are the most frequent tasks in MCM trails. In different locations on planet Earth, we can find the same objects or have the same scenario. Five generic tasks categories are introduced and illustrated with the most interesting MCM tasks objects localized in Portugal and Slovakia.

Generic tasks categories

- Counting/Combinatory;
- Quantities;
- Height;
- Slope;
- Volume.

MoMaTrE MathCityMap isep

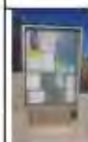
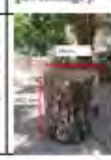
	Slovakia	Portugal
Counting/Combinatory	 **Stairs** How many possibilities exist to go upstairs if one can take one or two steps within each move? The step sequences can be combined?	 **Parking a bicycle** Three bicycles will park at this location. Each rim can only have one bicycle (it does not matter whether it is parked on the left or right of the rim, facing the street or not). How many possibilities are there to park three bikes?
Quantities	 **Information board** How many posters in A5 format (15 cm x 21 cm) can be placed on the information board? All the posters must be visible and they cannot overlap each other.	 **Footbridge** It is intended to pave the passage. All slabs (with dimensions of 42x60 cm) must be placed in the same orientation and, in the case of being cut, the leftovers are wasted. What is the minimum number of slabs needed to pave the passage?
Height	 **Staircase height** How deep (high) is the staircase on the left side of the music pavilion? Give the result in centimetres.	 **Statue height** How tall is the statue? (Give the result in meters)
Slope	 **Railing** Determine the slope of the railing. Give the result in percentage!	 **Ramp** In order to know if the ramp can be comfortably used by a wheelchair person, determine an approximate value for the ramp slope (in percentage).
Volume	 **Flowers container** How many cubic meters of soil are there in a big concrete area/ field for planting flowers?	 **Mass of a tree trunk** Determine the volume/mass of the tree trunk.

Table 1: Examples of generic tasks in Slovakia and in Portugal

CONCLUSIONS

The aim of this work was to show that, with the support of generic tasks, available in MCM, is much easier for a math teacher to quickly create tasks for their students because there is a range of ideas and best-practice examples. The generic tasks can inspire teachers to create local tasks or even textbook tasks for regular mathematics lessons. It is a very useful tool.

References

Cahyono, A. N., Ludwig, M. (2019). Teaching and Learning Mathematics around the City Supported by the Use of Digital Technology. Eurasia Journal of Mathematics, Science and Technology Education, 15(1), em1654. https://doi.org/10.29333/ejmste/99514

Gurjanow, I., Oliveira, M., Zender, J., Santos, P. A., & Ludwig, M. (2019). Shallow and deep gamification in mathematics trails. In M. Gentile, M. Allegra & H. Söbke (Eds.), *Games and learning Alliance - 2019: 7th International conference, GALA 2018, Palermo, Italy, December 5-7, 2018, Proceedings* (pp. 364–374). Springer International Publishing.

Ludwig, M. & Jablonski, S. (2019). Haciendo matemáticas al aire libre con MathCityMap. Presented at Jornadas para el Aprendizaje y la Enseñanza de las Matemáticas (19JAEM), 3 a 5 julio 2019, A Coruña, Spain.

MCM portal. (2020, March 20). *Tutorials*. Retrieved from https://mathcitymap.eu/en/portal-en/

Mobile Math Trials in Europe. (2020, March 20). *Downloads*. Retrived from http://momatre.eu/downloads/

"CSI VIENNA" – DESIGN AND EVALUATION OF THE OUT-OF-SCHOOL LAB *ELKE*

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Abstract. *In the course of PISA and TIMSS a lot of out-of-school lab days have been established; as non-formal extracurricular learning environments they shall provide an efficient addition to formal learning at school. The ELKE project is a non-formal extracurricular learning lab that aims at linking curricular content and competence-centred activities. This makes it an effective addition to chemistry education at school. Additionally, ELKE is a teaching/learning-environment for university teacher training. The poster illustrates the general concept as well as the contextual design of "CSI Vienna", the Viennese experimentation day. Furthermore, it shows first results concerning the effectiveness of the ELKE student lab on learning. These results will also be related to the outcomes of studies on ELKE as a teaching/learning-environment for future chemistry teachers.*

Key words: Out-of-School Lab Days, Competence Orientation, Teaching and Learning Chemistry

OUT-OF-SCHOOL LAB DAYS – THE CONCEPT OF *ELKE*

Out-of-school labs are unquestionably an enrichment to regular formal learning environments (e.g. Itzek–Greulich et al., 2017; Bell et al., 2003). Particularly the activity-oriented approach of many out-of-school labs allows students to acquire knowledge and competences in various contexts and consequently develop scientific literacy. In order to achieve optimal learning gains from these non-formal learning environments, however, it is necessary to connect them with the formal in-school learning environment. Itzek-Greulich et al. (2017) showed that linking out-of-school lab days and school had a positive effect on the development of students' content knowledge. With particular regard to scientific literacy and acquisition of scientific competences Rehm & Parchmann (2015) claim that a reasonable and systematic interconnectedness of these types of learning environments is crucial to ensure optimal learning outcomes. Therefore, it has to be examined how non-formal learning environments like out-of-school lab days can contribute to establish a link to the formal learning environment of school.

The extracurricular learning lab *ELKE* systematically connects formal and non-formal learning environments, providing benefits for students as well as pre-service chemistry teachers by combining a classical student learning lab and a teaching/learning lab (Groß & Schumacher, 2018). The German acronym *ELKE* stands for experimenting (German: 'Experimentieren'), learning (German: 'Lernen') and acquiring competences (German: 'Kompetenzen Erwerben'). Integrating these principles into the out-of-school lab experience and dealing with content that is aligned to the respective governmental curriculum, which can be prepared and revisited by teachers in school lessons, *ELKE* successfully establishes the connection between a non-formal learning environment and regular chemistry teaching at school. In order to be able to do so, materials that teachers can use for preparation and follow-up activities during their lessons are provided.

As a classical student learning lab *ELKE* offers a student-centered and competence-oriented design. Aiming at increasing the scientific interest and curiosity of the students participating in the program, each *ELKE* day is embedded in a context that school students might find interesting; by taking new roles (like being a scientist or criminal investigator)

Gross, K. & Harmer, S. P. (2020). "CSI VIENNA" – Design and Evaluation of the Out-Of-School Lab ELKE. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 205-208). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.26

and having to solve more or less realistic scientific problems, students can improve their scientific literacy in a playful way. Moreover, the use of various digital media and forms of experimentation allows it to consider students' diversity adequately and, subsequently, to deepen their learning (e.g. Seibert et al., 2019). The *ELKE* lab day "A day full of chemistry – Student reporter in lab", for example, makes use of digital forms of documenting experiments. After running through an experimental circuit, the students use digital tools to document the results of their experiments. Using video documentation, the experiments are not reduced to their rational logical findings, but allow a process-oriented deeper reflection of the conducted experiments, as the recording of a video documentation requires more detailed consideration of the subject-specific content. Additionally, using their own smartphones for this type of documentation allows students to return to prior experiments and findings any time and increases motivation by adding a fun aspect. At the same time *ELKE* as a teaching/learning lab offers pre-service (chemistry) teachers the chance to acquire scientific and didactic expertise and reflect upon their professional development (Gross & Pawlak 2020). The topics dealt with in the out-of-school lab *ELKE* cover all types of schools and age groups. So far two experimentation days for primary schools, six experimental days for lower and three for upper secondary have been designed (Groß & Schumacher, 2018).

"CSI VIENNA" – AN EXPERIMENTAL MURDER MYSTERY

The new experimentation day "CSI Vienna" is set within the context of a murder mystery. It gives the students the chance to study and conduct experiments autonomously within the field of nutrition and analyses. During their time at the lab the students take up the roles of investigators who help a police detective to solve the fictional murder of Mr. Argon who has obviously been poisoned. In the course of their investigations the students learn about various components of their daily diet: they learn about the chemical structures of various nutrients as well as their functions for nutrition. Furthermore, the students also get to know how to detect these substances. Conducting their own experiments, the students are able to reduce the number of suspects step by step and to solve the murder case in the end.

In order to gain insights into the effectiveness of this experimentation day for learning, the students and their chemistry teachers are asked to fill in a semi-structured questionnaire assessing the leaning outcomes and the way of learning. First results show that the students could build up chemical content knowledge within the field of nutrition (e.g. "that you can analyse and reconstruct what a person has eaten from the gastric content"). Regarding the practical work in the student lab the participating students claimed that they liked conducting experiments autonomously (e.g. "that we could do experiments ourselves") and that they received support from the supervising pre-service chemistry teachers in case they had comprehension problems (e.g. "that the tutors helped us, whenever we had questions"). Furthermore, the contextualisation of the experimentation day within a murder mystery increased the students' motivation.

The chemistry teachers stated that the participation in the experimentation day lead to the increase of competences in the field of chemistry as well as linguistic competences (e.g. "talking about findings and results") and social competences (e.g. "teamwork, strategic planning").

FURTHER ADAPTIONS OF "CSI VIENNA" – EXPERIENTAL EDUCATION ASPECTS

Due to its overall conception the whole *ELKE* out-of-school lab experience covers a lot of experiential education aspects. The embedding of the lab day into a murder mystery case attributes the whole day features of a modern live action roleplay. *ELKE* focuses on activity-oriented learning and creative problem-solving. In order to emphasise the experiential education experience, it is possible to add another step to the procedure and transfer parts of the murder mystery in an outdoor learning environment; after a brief introduction to the murder case the students are equipped with an investigation box, containing e.g. rubber gloves, plastic bags, tweezers, a fingerprint set and a tablet computer, and are guided towards the crime scene where the corpse of Mr. Argon had been found earlier. The students find the leftovers of an obviously romantic picnic in the park and are asked to collect potential evidence like crumbs, a napkin with a suspicious red stain on it (Is it blood? Is it ketchup? Or is it just lipstick?) or fingerprints. The students are encouraged to use the tablet computer to take pictures of the crime scene and document their proceedings. This step shall help the students to get "into" the murder case, on the one hand, but also to foster their strategic planning competence. At the same time the experience becomes more holistic, as the students not only think about what they possibly have to consider (brains), they also have to collect potential samples (hands) and might even have to overcome disgust when collecting samples of half-eaten food (heart). In the course of the lab day the students will analyse their samples, again, using the tablet computer to document their findings and results. Future *ELKE* days may focus even stronger on doing chemistry outdoors like taking water samples from the Danube and analysing them in a mobile lab, we never know which villain will try to harm the Viennese water system….

References

Bell, R., Blair, M., Crawford, B., & Lederman, N. (2003). Just do it? Impact of a science apprenticeship program on high school students' understandings of the nature of science and scientific inquiry. *Journal of Research in Science Teaching*, 40, 487-509.

Groß, K., & Schumacher, A. (2018). ELKE - Eine Möglichkeit der systematischen Vernetzung eines außerschulischen Lernortes mit dem Chemieunterricht. *MNU-Journal*, 71, 414-420.

Gross, K., & Pawlak, F. (2020). Using Video Documentation in Out-Of-School Lab Days as an ICT Learning and Diagnostic Tool. *World Journal of Chemical Education*, 8(1), 52-60. DOI: 10.12691/wjce-8-1-7

Itzek–Greulich, H., Flunger, B., Vollmer, C., Nagengast, B., Rehm, M., & Trautwein, U. (2017). Effectiveness of lab–work learning environments in and out of school: A cluster randomized study. *Contemporary Educational Psychology*, 48, 98-115.

Rehm, M. & Parchmann, I. (2015). Lernvielfalt Naturwissenschaften. Ziele und Ansätze einer systematischen Vernetzung schulischen und außerschulischen Lernens. *NiU-Chemie*, 26(147), 2−7.

Seibert, J., Kay, C. W. M., & Huwer, J. (2019). EXPlainistry: Creating Documentation, Explanations, and Animated Visualizations of Chemistry Experiments Supported by Information and Communication Technology To Help School Students Understand Molecular-Level Interactions. *Journal of Chemical Education*, 96, 2503-2509.

"CSI VIENNA" – DESIGN AND EVALUATION OF THE OUT-OF-SCHOOL LAB *ELKE*

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Theoretical Background

The potential of out-of-school labs is out of question. Visiting such an out-of-school lab is an additional opportunity to arouse students' interest in chemistry and encounter typical scientific working methods. Numerous didactic studies have shown that visits in such out-of-school labs foster students' interests in natural sciences [1,2]. It is of major relevance to allow students to experiment autonomously, dealing with relevant, up-to-date topics in authentic scientific learning environments [a.o. 3]. Nonetheless there are hardly any research results that allow generalizable conclusions on the effectiveness of such non-formal learning environments, due to the large variety of different scientific out-of-school learning environments. In a quasi-experimental intervention study Itzek-Greulich et al. (2014) were able to show that linking out-of-school labs and school itself had a positive effect on the development of students' content knowledge [4]. Therefore it can be expected that relating these different learning environments is crucial for optimal learning gains [a.o. 5].

The Concept of *ELKE*

The *ELKE* concept is a new interpretation of an out-of-school lab day for school students at the University of Vienna. The content fits the Austrian curriculum and the materials provided for teachers allow preparation and follow-up activities during the school lessons. Additionally, the competence-oriented design aims at combining the out-of-school learning experience with the traditional chemistry school lessons systematically [6].

The German acronym *ELKE* stands for experimenting (German: 'Experimentieren'), learning (German: 'Lernen') and acquiring competences (German: 'Kompetenzen Erwerben'). In the sense of the Bundesverband der Schülerlabore e.V. "LernortLabor", it can be seen as a classical student lab for school students as well as a *teaching/learning Lab* for pre-service chemistry teachers [7]. Each *ELKE* day aims at integrating the underlying principles of experimenting, learning and acquiring competences into the out-of-school lab experience (see figure 1).

The *ELKE* content design

The content range of the classical student lab and teaching/learning lab *ELKE* comprises all types of schools and age groups. So far there are two experimental days for primary level (*ELKE*[Junior]), six experimental days for lower secondary (*ELKE*) and three experimental days for upper secondary (*ELKE*[Oberstufe]) (see table 1).

Table 1: Examples of various *ELKE* experimental days (overview: [8])

ELKE	Content
A day full of chemistry – Student reporter in lab [9]	Planning and realizing separation processes based on different properties of substances
Running, sweating ... drinking?! [6]	Detecting ions and assessing so-called sports drinks
Chemistry escape – Find your way (out)! [10]	Telling properties of substances on the basis of structure and verifying these hypotheses; planning and realizing esterification

***ELKE* is adaptable**

Focusing on specific scientific and chemical aspects of didactics, the non-formal learning environments *ELKE* allows to connect relevant topics of chemistry didactics systematically. So each *ELKE* content can get a particular didactic focus: *ELKE S* for example concentrates on language sensitive science and chemistry teaching. *ELKE I* on the other hand, emphasises the development, design and implementation of inclusive learning environments.

Figure 1: The Concept of *ELKE*. Experimenting (WHAT?) – Learning (HOW?) – Acquiring Competences (WHY?).

"CSI Vienna – Identifying Evidence Chemically"

Student Lab *ELKE* at the University of Vienna

Aims and objectives of the lab day:

For one day the students take the roles of investigators who have to help a police inspector solving the fictional murder mystery of the poisoned Mr. Argon. The students learn about the nutrients of their daily diet (e.g. carbohydrates, proteins, lipids, antioxidants, alcohol or ascorbic acid), as well as the structure and importance of these nutrients. Furthermore, they get familiar with various chemical methods of detecting these nutrients. Additional experiments like the visualisation of fingerprints allow the students to identify themselves with their roles as investigators in the fictional murder mystery [11].

Procedure:

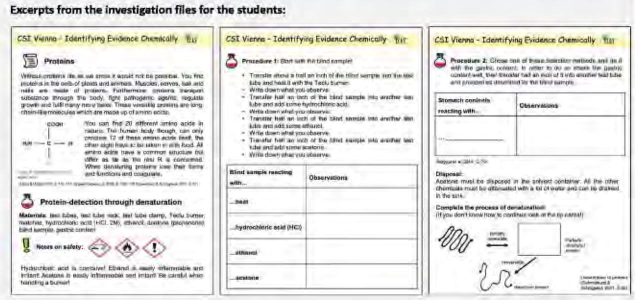
- **Start** – Introduction to the criminal case (podcast)
- **Lab phase 1** – Analysis of carbohydrates, starch, proteins and lipids
- **Emergency meeting I** – Restriction of the suspects
- **Lab phase 2** – Analysis of further ingredients (i.a. calcium, sulphate, alcohol, antioxidants)
- **Emergency meeting II** – Identification of the culprit

Introduction of the criminal case:

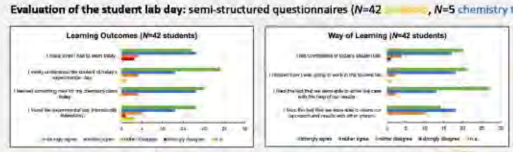
Probably Mr. Argon was poisoned by one of his female neighbours during a romantic picnic for two. After questioning the three main suspects, we know what the three ladies had for dinner on the evening of the murder.
This information is our starting point, my dear investigators! The coroner gave us a sample of Mr. Argon's gastric content. Now we want to analyse the substances from Mr. Argon's stomach and draw conclusions what he might have eaten, because if we know that, we will know who spent the last evening with Mr. Argon and therefore is the culprit!

The Suspects:

Excerpts from the investigation files for the students:



Evaluation of the student lab day: semi-structured questionnaires (*N*=42 students, *N*=5 chemistry teachers)



"[...] you compare blind samples with real things to find out."

"I have understood how to detect different nutrients."

„[...] that you can analyse and reconstruct what a person has eaten from the gastric content."

"[Students could increase their competences concerning] usage of chemicals and laboratory equipment, analysing and evaluating results."

"It was really nice, since we did not only conduct experiments, but could also solve a 'case'."

"[Students could increase their social competences like] teamwork, strategic planning."

LITERATURE:

[1] C. Pawek (2009). Schülerlabore als interessefördernde außerschulische Lernumgebungen für Schülerinnen und Schüler aus der Mittel- und Oberstufe. www.dlr.de/schoollab/en/Portaldata/24/Resources/dokumente/Diss_Pawek.pdf (22.03.2017). [2] P. Guderian (2007). Wirksamkeitsanalyse außerschulischer Lernorte – Der Einfluss mehrmaliger Schülerbesuche eines Schülerlabors auf die Entwicklung des Interesses an Physik. http://edoc.hu-berlin.de/dissertationen /guderianpascal-2007-02-12/PDF/guderian.pdf (22.03.2017). [3] I. Schmidt, D.S. Di Fuccia & B. Ralle (2011). Außerschulische Lernstandorte. MNU, 64(6), 362–369. [4] H. Itzek-Greulich, B. Flunger, C. Vollmer, B. Nagengast, M. Rehm & U. Trautwein (2014). The impact of a science center outreach lab workshop on german 9th graders' achievement in science. In ESERA (Hrsg.), 10th Conference of the European Science Education Research Association Proceedings, 97–106. [5] M. Rehm & I. Parchmann (2015). Lernvielfalt Naturwissenschaften. Ziele und Ansätze einer systematischen Vernetzung schulischen und außerschulischen Lernens. NiU-Chemie, 26(147), 2–7. [6] K. Groß & A. Schumacher (2018). ELKE - Eine Möglichkeit der systematischen Vernetzung eines außerschulischen Lernortes mit dem Chemieunterricht. MNU 71(6), 414-420. [7] O. Haupt, J. Domjahn, U. Martin, P. Skiebe-Corrette, S. Vorst, W. Zehren & R. Hempelmann (2013). Schülerlabor – Begriffsschärfung und Kategorisierung. MNU 66(6), 324-330 . [8] Institut für Didaktik der Chemie, Universität Wien (2020). ELKE Austria. https://chemiedidaktik.univie.ac.at/forschung/elke-austria/ (17.01.2020). [9] K. Groß & F. Pawlak (2020). Using video documentation in out-of-school lab days as an ICT learning and diagnostic tool. World Journal of Chemical Education, 8(1), 52-60. DOI: 10.12691/wjce-8-1-7 [10] K. Groß & A. Schumacher (2020). Chemie einmal anders erleben im Lehr-/Lernlabor ELKE. Chemistry escape – Finde den Weg!. CiuZ 54/2, 126-130. [11] L.S. Winkler & K. Groß (2020). CSI – Chemisch Spuren identifizieren! – Ein Experimentiertag im Schülerlabor ELKE AUSTRIA, Chemie & Schule, 35(1), 14-17.

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POPULARISATION OF STEM SUBJECTS BY THE MEANS OF STUDY PROGRAMMES FOR UPPER-SECONDARY STUDENTS

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Abstract. *Interdisciplinary teaching is considered as an effective teaching approach. When designing the problems for interdisciplinary investigations the personal relevance for students should be considered. In the poster we present an example of the project presented by two students taking part in the program 'Discover the world of science' for upper-secondary by the university.*

Key words: Mathematics education. Physics education. Interdisciplinary teaching. Mathematical modelling.

INTERDISCIPLINARY TEACHING

It is almost impossible to understand the world around us by the means of only one scientific discipline (Maass et al., 2011). According to (St. Clair and Hough, 1992), interdisciplinary teaching is in line with current research findings on learning styles and students' needs in secondary education and promotes a holistic problem-solving approach and gives students a more comprehensive view of the world. It develops the students' ability to solve the problem by pointing it out from multiple points of view. Nikitina (2006) divided strategies of interdisciplinary teaching into three groups: (1) Contextualization means setting the content of the discipline in the broader context. Main advantage of contextualizing strategy is offering the student to gain theoretical, methodological, epistemological and historical connections among disciplines, to make mathematics and science more accessible. But, we have to be careful while implementing, because it is not aimed to turn the mathematics classroom into philosophical debate. (2) Conceptualization, i.e. working the core concepts which are central for two or more disciplines (e.g. linearity, exponential growth). This strategy aims to understand essential natural laws which are valid without human intervention. It proceeds from the empirical data to more general knowledge. Instead of philosophical issues characteristic for contextualizing, the conceptualizing connections need strong standards of verification, replication and mathematical expression. These links in practice usually need particular effort, they are not intuitive, and students usually do not see the connections. The role of the teacher in this kind of approach is really crucial. (3) Problem-centering is pragmatic, real-life oriented pedagogy. In order to solve (usually) ill-structured problems, the concepts, processes and ideas from different disciplines have to be used. In contrast with the previous two strategies, its aim is not to build coherence between different ideas, but to create tangible outcomes or products. The epistemological goal of this strategy is not so much to advance the knowledge, but to use tools of different disciplines to "fight" with the difficult problem. Disciplines here are used precisely, but only particular parts necessary for attaching the problem. Students in problem-centering classes may acquire specific disciplinary knowledge, but classes like this should be supplemented by broader context and content to obtain the consistent and personally meaningful knowledge of each discipline.

One possible strategy for interdisciplinary teaching is the creation of pupils' projects, i.e. project-based learning (Novotna et al., 2016; Pavlovičová et al., 2011). According to Suryana et al. (2018) STEM/STEAM is learning approach that builds the students not only

Medová, J. & Valovičová, L. (2020). Popularisation of STEM Subjects by the Means of Study Programmes for Upper Secondary Students. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 209-212). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.27

reliable in theory but how to apply the theory to solve the problem. There is some evidence that project-based learning in STEM subjects increases the academic performance of students (Han et al., 2016).

The aim of this paper is to describe the activity focused on popularization of STEAM subject and to provide the narrative about two students taking part in the activity.

THE PROGRAMME FOR STEM POPULARISATION

The program Discover the world of science (DiSci) was aimed at popularization of STEM subjects through out-of-school programmes for students. The summer camp was organized for lower-secondary students. During the week in which they were held, the students were divided into groups and worked on an assigned project. The second activity, an unusual study program Discover the World of Natural Sciences, was designed for upper-secondary school pupils of the Nitra region. There are several similar projects in Slovakia and word-wide. However, curriculum in DiSci programme is specific in several aspects. The curriculum consists of five subjects: mathematics, chemistry, physics, geography and biology. Students of this program attend the university during the lecture period of two semesters. At the beginning of the study the pair of students chose the topic of the final project. During the first semester they were purposefully educated, lectures were more or less related to the topic of their project. The content of the second semester was focused on the development of competencies necessary for the writing and presentation of the final thesis. Students attended a short seminar on statistical methods and mathematical modelling. The real-life context can enhance students' modelling abilities (Plothová, 2017). Students presented their final work at students' conferences held at the end of their two-semester study. After the successful defense of the final thesis, a graduation ceremony took place and they were awarded the title Young Scientist.

EXAMPLE OF THE STUDENTS' PROJECT

One of the final projects connected together mathematics, physics, biology and engineering, the project about ergonomic artificial lighting. Both students participating in this project were students from grade 10, year one of vocational schools. One of the students attended a hotel academy (five-year programme with final high-stake exam) and stayed in the students' dormitory, the second one was a future confectioner (three-year programme without matura).

According to their interest, students chose the two rooms to consider the ergonomics of the artificial lighting, the dormitory room of one of the students and the freshly-renovated kitchen of the house of the second student. In the project they summarized the influence of artificial lightning for human organisms, the fatigue and influence for the human eyes. They also described the basic characteristics of different sources of light, e.g. color rendering index or color temperature. The intensity of light was calculated in the chosen rooms using the formula $E_n = \dfrac{100.P}{A.P^*} \cdot k$ where P is the light input of sources, A is area of the room and k is the correction factor considering efficiency of particular light source.

They found that the artificial lighting in the kitchen is not satisfactory and the lighting in the dormitory room fulfils the criteria given in the norm. The number of light bulbs needed for the kitchen was estimated by the means of GeoGebra and plotting the intensity of light as the function of the number of the bulbs. The parents of the student bought more lamps according to the calculation of their son.

Even after the presentation of the project the students persisted in verifying whether the different places have an appropriate level of light sources, including kitchens in different restaurants and patisseries where they attended their practical part of the study.

CONCLUSIONS

The project DiSci aimed at popularization of mathematics and science among the students in Nitra region. The presented example illustrates that even students whose carriers are not focused in STEM subjects can be involved in investigations and studying subjects usually out of their interest when they problems are set to the context suitable for students' interest and connected to their everyday life. Even though they lacked deeper knowledge and understanding of mathematics, they were able to solve the mathematical problem using the technology, particularly GeoGebra.

References

Han, S., Capraro, R. M., & Capraro, M. M. (2016). How science, technology, engineering, and mathematics project based learning affects high-need students in the US. *Learning and Individual Differences*, 51, 157-166.

Maass, K. et al. (2011). *Enhancing mathematics and science learning through interdisciplinary enquiry*. Nitra: UKF, 2011. ISBN 978-80-8094-986-0

Nikitina, S. 2006. Three strategies for interdisciplinary teaching: contextualizing, conceptualizing, and problem-centering. *Journal of Curriculum Studies*, 38(3), 251-271

Novotna, J., Jančařik, A., & Rusek, M. (2016). STEM and project-based education. In D. Rychtarikova, D. Szarkova and L. Balko (Eds.), *APLIMAT 2016 - 15th Conference on Applied Mathematics 2016*, Bratislava, SK: STU, pp. 876.

Pavlovičová, G., Švecová, V., Rumanová, L. (2011). Podpora medzipredmetových vzťahov prostredníctvom projektového vyučovania [Supporting interdisciplinarity through project-based learning]. *Cieľom vyučovania matematiky je šťastný človek*. Žilina: EDIS, p. 193-200.

Plothová, L. et al. (2017) An Analysis of Students' Use of Mathematical Models in Solving Tasks with Real-life Context. APLIMAT 2017 : Proceedings from 16th Conference on Applied Mathematics, Bratislava, SK: STU, 2017, p. 1207-1223.

St. Clair, B. & Hough, D.L. (1992). *Interdisciplinary teaching: A review of literature. Information analyses*. ERIC digest. (ED373056)

Suryana, A., Sinaga, P., & Suwarma, I. R. (2018). Analysis of the ability of junior high school students' performance in science in STEM project-based learning. *In Journal of Physics: Conference Series* (Vol. 1013, No. 1, p. 012059). IOP Publishing.

Popularisation of STEM subjects by the means of study programmes for upper-secondary students

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INTERDISCIPLINARY TEACHING

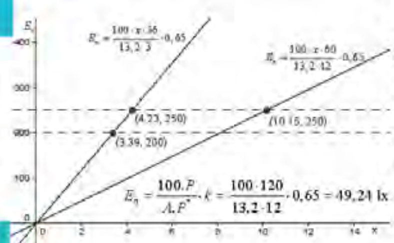
It is almost impossible to understand the world around us by the means of only one scientific discipline (Maass et al., 2011). According to (St. Clair and Hough, 1992) interdisciplinary teaching is in line with current research findings on students' needs in secondary education. It promotes a holistic problem-solving approach and gives students a more comprehensive view. One possible strategy for interdisciplinary teaching is the creation of pupil projects (Pavlovičová et al., 2011).
Nikitina (2006) divided strategies of interdisciplinary teaching into three groups: (1) setting the content of the discipline in the broader context, (2) working the core concepts which are central for two or more disciplines and
(3) problem-centering (in order to solve open, divergent problems, the concepts, processes and ideas from different disciplines have to be used). The reasonable combination of the strategies should be implemented.
According to Suryana et al. (2018) STEM/STEAM is learning approach that builds the students not only reliable in theory but how to apply the theory to solve the problem. There is some evidence that project-based learning in STEM subjects increases the academic performance of students (Han et al., 2016).

THE PROGRAMME FOR STEM POPULARISATION

The program Discover the world of science (DiSci) was aimed at popularization of STEM subjects through out-of-school programmes for students. Study program Discover the World of Natural Sciences, was designed for upper-secondary school pupils of the Nitra region.
The curriculum consists of five subjects: mathematics, chemistry, physics, geography and biology. Students of the program attended the university during the lecture period of two semesters. At the beginning of the study the pair of students chose the topic of the final project. During the first semester they were purposefully educated, lectures were related to the topic of their project. The content of the second semester was focused on the development of competencies necessary for the writing and presentation of the final thesis. Students attended a short seminar on statistical methods and mathematical modelling. Students presented their final work at students' conferences held at the end of their two-semester study. After the successful defense of the final thesis, a graduation ceremony took place and they were awarded the title Young Scientist.

STUDENTS' PROJECT

One of the final projects connected together mathematics, physics, biology and engineering, the project about ergonomic artificial lighting. Both students participating in this project were students from grade 10, year one of vocational schools.
One of the students attended a hotel academy (five-year programme with final high-stake exam) and stayed in the students' dormitory, the second one was a future confectioner (three-year programme without matura).



According to their interest, students chose the two rooms to consider the ergonomics of the artificial lighting, the dormitory room of one of the students and the freshly-renovated kitchen of the house of the second student. In the project they summarized the influence of artificial lightning for human organisms, the fatigue and influence for the human eyes. They also described the basic characteristics of different sources of light, e.g. color rendering index or color temperature. The intensity of light was calculated in the chosen rooms using the formula where P is the light input of sources, A is area of the room and k is the correction factor considering efficiency of particular light source.
They found that the artificial lighting in the kitchen is not satisfactory and the lighting in the dormitory room fulfils the criteria given in the norm. The number of light bulbs needed for the kitchen was estimated by the means of GeoGebra and plotting the intensity of light as the function of the number of the bulbs. The parents of the student bought more lamps according to the calculation of their son. Even after the presentation of the project the students persisted in verifying whether the different places have an appropriate level of light sources, including kitchens in different restaurants and patisseries where they attended their practical part of the study

CONCLUSIONS

The project DiSci aimed at popularization of mathematics and science among the students in Nitra region. The presented example illustrates that even students whose carriers are not focused in STEM subjects can be involved in investigations and studying subjects usually out of their interest when they problems are set to the context suitable for students' interest and connected to their everyday life.

REFERENCES

Han, S., Capraro, R. M., & Capraro, M. M. (2016). How science, technology, engineering, and mathematics project based learning affects high-need students in the US. *Learning and Individual Differences*, 51, 157-166.
Maass, K. et al. (2011). *Enhancing mathematics and science learning through interdisciplinary enquiry*. Nitra: FPV UKF v Nitre, 2011.
Nikitina, S. 2006. Three strategies for interdisciplinary teaching: contextualizing, conceptualizing, and problem-centering. *Journal of Curriculum Studies*, 38(3), 251-271
Novotna, J., Jančařík, A., & Rusek, M. (2016). STEM and project-based education. In D. Rychtarikova, D. Szarkova and L. Balko (Eds.), *APLIMAT 2016 - 15th Conference on Applied Mathematics 2016*, Bratislava, SK: STU, pp. 876.
Pavlovičová, G., Švecová, V., Rumanová, L. (2011). Podpora medzipredmetových vzťahov prostredníctvom projektového vyučovania [Supporting interdisciplinarity through project-based learning]. *Cieľom vyučovania matematiky je šťastný človek*. Žilina: EDIS, p. 193-200.
St. Clair, B. & Hough, D.L. 1992. *Interdisciplinary teaching: A review of literature. Information analyses.* ERIC digest. (ED373056)
Suryana, A., Sinaga, P., & Suwarma, I. R. (2018). Analysis of the ability of junior high school students' performance in science in STEM project-based learning. *In Journal of Physics: Conference Series* (Vol. 1013, No. 1, p. 012059). IOP Publishing.

DEVELOPING AND ASSESSING E-LEARNING SETTINGS BY DIGITAL TECHNOLOGIES

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Abstract. *The project is part of the Master program for chemistry teacher education at the university. The German acronym "E-lement" stands for "developing e-learning (German:* ***e-l****earning* ***e****ntwickeln) including assessing (German:* ***m****itsamt* ***E****valuation) by digital technologies (German: durch* ***n****eue* ***T****echniken). It consists of three elements: a theoretical course, an exercise and a practical course. The aim of the module is to deepen the content knowledge of selected school relevant topics, to reflect them didactically, to transform them into e-learning settings by using new technologies, to implement them in out-of-school lab days and to assess the outcomes of the pupils by using digital tools. The poster presents an overview of the elements of the module, the methodology and first results of the project.*

Key words: content knowledge, pedagogical knowledge, digital technologies, out-of-school labs, chemistry teacher education.

THE PROJECT E-LEMENT

Education in the digital age is a new challenge not only in the classroom but also in teacher education. It requires to widen the framework for teaching which has been described by Lee Shulman (1986). Pedagogical content knowledge (PCK) has to be extended to TPCK (TPACK) to include technological knowledge (Koehler & Mishra, 2009). Apart from the difficulty to define TPACK, it is important that social and contextual factors can make the relationship between teaching and technology more difficult. Teachers are often left alone with the new technologies and do not know how and where to integrate them (Koehler & Mishra, 2009). In order to overcome these obstacles we designed a project which is based on the general idea that learners (here teacher students) are much more motivated to acquire new knowledge when they are aware of the future relevance of their knowledge. And furthermore the combination of knowing and applying which is crucial for the project is in accordance with the notion of competence, one of the central challenges of education since PISA (Reiners, 2017).

These ideas are fundamental for a module in a Master program for chemistry teachers which consists of three elements.

In a theoretical course selected topics which have already been taught in subject-matter courses during their studies are taken up again and reflected as specific content for teaching at school. The reflection implies the analysis of the content, its curricular relevance, discussions of preconceptions, the design of possible learning contexts and the development of appropriate experiments.

Based on this course the teacher students are invited to use the result of the reflection and to develop adequate learning environments with digital technologies (Clark & Mayer, 2016.), like PowerPoint (Banerji, 2017). In order to support them they are introduced to e-learning units for classroom like blended learning (Keengwe, 2019), flipped classroom (Bergman & Sams, 2014), and game-based learning (Van Eck, 2006). Furthermore they should construct digital learning environments by themselves, like videos, audio recordings

Reiners, C. S., Schmitz, L. & Mueller, S. (2020). Developing and Assessing E-Learning Settings by Digital Technologies. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), *Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020* (pp. 213-216). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.28

and animations and they also reflect on advantages and disadvantages of e-learning scenarios.

In the final practical course they are offered the opportunity to test the designed learning settings authentically, i.e. with real pupils in out-of-school lab days. In order to assess the learning outcomes the teacher students develop research questions by themselves, collect data by using digital tools like tablets and videos and analyze them qualitatively. Thus out-of-school lab days serve several functions: for the pupils they are innovative learning environments, for the teacher students they are authentic teaching environments and a platform to apply their knowledge. Furthermore the competence-oriented lab days offer the opportunity to collect first experiences in doing research and support the future teachers in making up their mind on the use of new technologies.

The main aim of the practical units, i.e. exercise and practical course is to support them on the way from mere recipients of new technologies to active constructors and thus make learning more meaningful to them. In order to find out whether this transformation was successful the module was evaluated. Data was collected by questionnaires which the teacher students had to fill out after the module. The questionnaires contain open-ended questions (Denzin & Lincoln, 2011) addressing their overall experiences (for example: "What are you taking out of the module? and "What else would you have wished for?") as well as structured questions to figure out their assessment of selected digital tools and scenarios.

FIRST RESULTS

Answers to the open-ended questions from a total of 21 participants were analysed using the qualitative content analysis according to Mayring (2015). The inductively formed categories indicate that after the module many teacher students now consider PowerPoint as an effective tool to develop e-learning arrangements (12 entries). For example, when asked what he learned from the module, one participant responds:

"Integration of digital learning environments; learning how to program with an ordinary program." [All German quotations in this article were translated by the authors.]

In addition, some participants (7 entries) think that using the program is an opportunity to design chemistry lessons. This is underlined by the following statement of a teacher student who developed an digital teaching unit after participating in "E-lement" and afterwards used it in her own chemistry class:

"I learned how to use the animation function of Powerpoint for school and even created a small unit on my own on induced dipoles and used it in class."

However, the teacher students are also concerned that the development of such a digital learning unit is very time-consuming (11 entries):

"You can also use PowerPoint in a different way, but it is very time consuming. Whether and when it can be used in school is questionable."

Regarding the structured questions, 11 out of 19 students stated that they would use the learning environment developed within the "E-lement" module in their own chemistry lessons, while six are undecided. In addition, 14 students see themselves in a position to create digital learning environments in the future (four undecided) and seven plan to do so in their future teaching (six undecided).

OUTLOOK

First results of the evaluation indicate that after participating in the module "E-lement" teacher students are prepared to develop and assess e-learning environments for digital chemistry lessons. Especially PowerPoint seems to be a useful tool for them. This can be regarded as a first step to support future teachers on their way from mere recipients of new technologies to active constructors. Though it turned out that many participants consider the development of digital learning units to be time-consuming, the fundamental potential of the module to become active constructors of new technologies is an important advantage in recent times of "home schooling".

References

Banerji, A. (2017). Teaching Chemistry 2.0 – Creating Digital Learning Environments with Powerpoint and Prezi. In O. E. Finlayson, E. McLoughlin, S. Erduran & P. Childs (Eds.), *Electronic Proceedings of the ESERA 2017 Conference. Research, Practice and Collaboration in Science Education, Part 4/4* (co-ed. K. Juuti & E. A. Kyza), (pp. 630–636). Dublin, Ireland: Dublin City University.

Bergmann, J. & Sams, A. (2014). Flipping for mastery. *Educational Leadership*, *71*(4), 24–29.

Clark, R. C., & Mayer, R. E. (2016). *E-learning and the science of instruction: Proven guidelines for consumers and designers of multimedia learning* (4th ed.). Hoboken, NJ: Wiley.

Denzin, N. K., & Lincoln, Y. S. (Hrsg.). (2011). *The Sage handbook of qualitative research* (4th ed). Thousand Oaks: Sage.

Keengwe, J. (Ed.). (2019). *Handbook of research on blended learning pedagogies and professional development in higher education*. Hershey, PA: IGI Global.

Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research*, *32*(2), 131–152.

Mayring, P. (2015). Qualitative Content Analysis. Theoretical background and procedures. In A. Bikner-Ahsbahs, C. Knipping & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education. Examples of methodology and methods* (pp. 365-380). New York: Springer.

Reiners, Ch. S (2017). *Chemie vermitteln: Fachdidaktische Grundlagen und Implikationen*. Berlin: Springer Spektrum.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher, 15(2)*, 4-14.

Van Eck, R. (2006). Digital game-based learning: It's not just the digital natives who are restless. *EDUCAUSE review*, *41*(2), 16.

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Figure 1: Screenshots and video from the students developed environments

Theoretical Background: Education in the digital age is a new challenge not only in the classroom but also in teacher education. It requires to widen the framework for teaching which has been described by Lee Shulman. Pedagogical content knowledge (PCK) has to be extended to TPCK (TPACK) to include technological knowledge (Koehler & Mishra, 2009). Apart from the difficulty to define TPACK, it is important that social and contextual factors can make the relationship between teaching and technology more difficult. Teachers are often left alone with the new technologies and do not know how and where to integrate them (Koehler & Mishra, 2009). In order to overcome these obstacles we designed a project which is based on the general idea that learners (here teacher students) are much more motivated to acquire new knowledge when they are aware of the future relevance of their knowledge. And furthermore the combination of knowing and applying which is crucial for the project is in accordance with the notion of competence, one of the central challenges of education since PISA (Reiners, 2017).

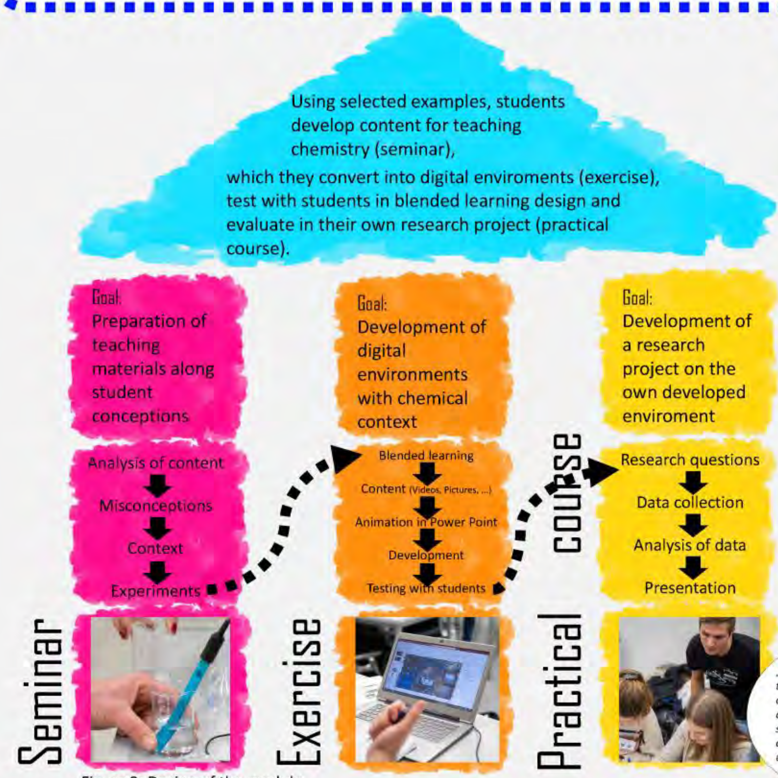

Figure 2: Design of the module

What do students think after the module about PowerPoint for e-learning?

Methodology: Data was collected by questionnaires which the teacher students had to fill out after the module. The questionnaires contain open-ended questions (Denzin & Lincoln, 2011) addressing their overall experiences as well as structured questions to figure out their assessment of selected digital tools and scenarios. A total of 21 participants have taken part in the evaluation of the module so far. The data were analysed using inductive category formation according to Mayring (2015).

Results: Twelve students consider Power-point a useful program for the development of e-learning environments. In addition, students see e-learning based on PowerPoint as a possibility for planning chemistry lessons. However, many consider the development of digital learning environments to be very time-consuming.

„I learned how to use the animation function of Powerpoint for school and even created a small unit on my own on induced dipoles and used it in class." Student

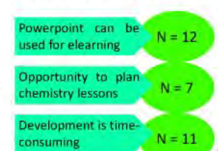

Figure 3: Category system

Outlook: First results of the evaluation indicate that after participating in the module "E-lement" teacher students are prepared to develop and assess e-learning environments for digital chemistry lessons. Especially PowerPoint seems to be a useful tool for them. This can be regarded as a first step to support future teachers on their way from mere recipients of new technologies to active constructors. Though it turned out that many participants consider the development of digital learning units to be time-consuming, the fundamental potential of the module to become active constructors of new technologies is an important advantage in recent times of "home schooling".

References:
Denzin, N. K., & Lincoln, Y. S. (Hrsg.). (2011). *The Sage handbook of qualitative research* (4th ed). Thousand Oaks: Sage.
Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research*, 32(2), 131–152.
Mayring, Ph. (2015). Qualitative Content Analysis. Theoretical background and procedures. In A. Bikner-Ahsbahs, C. Knipping & N. Presmeg (Eds.), Approaches to qualitative research in mathematics education. Examples of methodology and methods (pp. 365-380). New York: Springer.
Reiners, Ch. S (2017). *Chemie vermitteln: Fachdidaktische Grundlagen und Implikationen*. Berlin: Springer Spektrum.
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, 15(2), 4-14.

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Education outside the classroom becomes more relevant and connects school contents with real world problems. Especially the combination of outdoor education with digital tools allows new dimensions of mobility which outdoor education asks for and seems contemporary.

This volume contains the papers presented at the "Research on Outdoor STEM Education in the digital Age" (ROSETA) Online Conference, held in June 2020. The proceedings summarize and connect theoretical considerations, practical experiences and ideas, as well as empirical research results for outdoor education with digital tools. The papers' focus is on the STEM subjects Science, Technology, Engineering and Mathematics.

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